CHAMBERS'S EDUCATIONAL COURSE—EDITED BY W. AND R. CHAMBERS.

NATURAL PHILOSOPHY.

IN TWO VOLUMES.

VOLUME I.

MATTER AND MOTION; MECHANICS AND MACHINERY; HYDROSTATICS, HYDRAULICS, AND PNEUMATICS; ACOUSTICS.

WILLIAM AND ROBERT CHAMBERS,
LONDON AND EDINBURGH.

Edinburgh :
Printed by W. and R. Chambers.

NATURAL PHILOSOPHY.

FIRST TREATISE.

LAWS OF MATTER AND MOTION.

NOTICE.

THE following Treatises, taken in order, constitute the Course of Natural Philosophy: —

I. LAWS OF MATTER AND MOTION.
II. MECHANICS—MACHINERY.
III. HYDROSTATICS—HYDRAULICS—PNEUMATICS.
IV. ACOUSTICS.
V. OPTICS.
VI. ASTRONOMY.
VII. ELECTRICITY.
VIII. METEOROLOGY.

After having been issued separately, the whole are now embraced in two volumes.

The first Treatise, as will be observed, is devoted to the LAWS OF MATTER AND MOTION, a subject usually treated under the title of *Statics*, *Pyronomics* (or Heat), and *Dynamics*, and forming not only the proper introduction to Natural Science, but that particular department of it with which it is of the most importance that all should be made familiar.

In exercising a class in this and the succeeding branches, it will be found to be of considerable importance to cause each paragraph to be mastered, or thoroughly understood, before proceeding to what follows; for the whole constitutes a structure in which each part rests on what has gone before it. The pupil should also not only *read*, but be induced to *think* on the nature of the principles which are unfolded, and led to find examples of their action in the everyday concerns of life, and the common phenomena of the universe.

CONTENTS.

MATTER AND MOTION.

INTRODUCTION.

1. As man contemplates the bodies that make up the universe, and the endless movements and changes they undergo, he becomes impressed with the conviction that these *phenomena* (appearances), as they are called, are not simply a collection of individual things ruled by chance, but that there are fixed connections—in other words, order and uniformity among them; and he feels irresistibly impelled to trace out these connections wherever they can be discovered. In this pursuit we have Natural Philosophy in its widest sense.

2. When we have found out an unchangeable link of connection between two or more phenomena, we are said to have discovered or established a *law* of nature. It is observed, for instance, that whenever matter is heated, it becomes enlarged in bulk; it is therefore recorded as a law of nature, that 'heat expands bodies.'

3. When, again, we can shew that some other phenomenon, seemingly widely different, is really, though indirectly, caused by the operation of the same law, we are said to *explain* that phenomenon. Thus we explain the fact that a clock is apt to go slower in summer than in winter, by first establishing that a clock goes slower the longer the pendulum is, and then inferring from the law of expansion by heat, that the pendulum must be longer in summer than in winter.

4. Some phenomena depend upon the peculiar kind of substance of which the body manifesting them is composed, and consist in changes of its constitution; as when sulphur, at a certain temperature, takes fire—that is, unites with the oxygen of the atmosphere, and forms a suffocating gas, changing permanently its constitution and properties. The facts of this class form the separate science of *Chemistry*.

5. Organised bodies—that is, plants and animals—also manifest a peculiar set of appearances which are summed up in the word *life*. The consideration of *vital* phenomena belongs to the department of science called *Physiology*, sometimes *Biology*.

6. But there is a large and important class of phenomena of a much less special kind, and which belong to matter in general, and to all bodies composed of it, whatever be their peculiar constitution, and whether organic or inorganic. Thus, a stone, a piece of sulphur, a plant, an animal, all fall to the earth if unsupported, are all capable of being divided into small parts, all reflect more or less light, &c. It is the investigation of universal laws of this kind, where no change of constitution is concerned, that constitutes Natural Philosophy, in its narrower sense; for which the term *Physics* is now more generally used, as being more precise.

7. Of those physical phenomena, again, some have a higher generality than others, and it is these most general laws of the material world that naturally fall to be discussed in this introductory treatise. They may be arranged under the heads of *General Properties of Matter*, *Motion and Forces*, and *Heat*.

GENERAL PROPERTIES OF MATTER.

8. Matter, or that which composes all bodies, has certain *properties;* by which is meant, that it has the power of making certain impressions upon our senses, or of exciting in us *sensations*. Through these sensations we are said to have a *perception* of matter and bodies; but as to what matter is in itself, beyond its power of affecting our senses, we know nothing. The something, whatever it is, in which this power is conceived to reside, is called *substance*. Some philosophers deny the existence of anything beyond the properties; but though we have no direct evidence of anything else, it is difficult, if not impossible, to get rid of the notion, that there is a substance in which the properties inhere. So far as natural science is affected, the question is of no moment; what really concerns us is, how matter appears and acts, and not what it is.

9. The more important of the properties of matter are—Impenetrability, Extension, Divisibility, Inertia, Porosity, Compressibility, Elasticity, Attraction, States of Aggregation, Malleability, &c. We shall describe and illustrate them in succession, classing such qualities together as seem to be

naturally connected. Extension and Impenetrability claim precedence as being essential to our very notion of a body. We cannot conceive a body that does not extend over or occupy a portion of space, however small, and that does not exclude all other bodies from occupying the same space while it is there. This quality of obstinately shutting out other portions of matter from its own room, seems really what we chiefly mean by substance.

10. *Impenetrability* is that quality of bodies by virtue of which each occupies a certain portion of space, to the exclusion of all other bodies; it expresses the fact that two bodies cannot be in the same place at the same time. The term impenetrability is not a happy one, though it is difficult to find a better. In the popular sense of the word, matter is anything but impenetrable. The hand can be thrust into water, a nail can be driven into wood, and even the hardest substances are pierced by others that are more or equally hard. But all these are instances merely of displacement, or of removing part of one body to make room for another. There is no wood where the nail is, nor are the particles of the removed wood driven into one another, so to speak; they are merely forced closer together, as those of a sponge are when squeezed. In some cases it might seem at first sight that something like interpenetration of substances actually takes place. Water will rise in vapour, and yet the portion of air in which it disappears may not occupy more room than it did before. A measure of water and one of sulphuric acid mixed together, occupy less space than the two did separately. But in such cases we must conceive the particles of the one substance as finding room in the intervals between the particles of the other, as will be more fully spoken of under Divisibility and Porosity.

11. That the most movable and unsubstantial substances, when displacement is prevented, occupy space as effectually as the most solid, is seen in a blown bladder, or in an air-cushion. This property of air is taken advantage of in the diving-bell. An easy illustration is got by pressing a common glass tumbler, mouth downwards, into a vessel of water. Though the water ascends more or less according to the depth, the air makes good its claim at ordinary depths to the greater part of the space, and even though sunk to the bottom of the sea, the water would never get quite to the top. If a small lighted taper, floating on a bit of cork, be carried down with the tumbler, the singular appearance may be beheld of a light burning under water.

12. *Extension*, or *Magnitude*, and *Form*.—Magnitude or size is one of those simple ideas that do not require or admit

of explanation, because there is nothing simpler to explain them by. It is chiefly by their extension that bodies make themselves known to our senses; and when we try to think of those minute particles of matter that elude the senses, we can conceive them only as extended or having a certain magnitude.

13. Bodies are extended in three directions, or have three *dimensions*—namely, length, breadth, and depth. Width is the same dimension as breadth; and for depth we often use height, and sometimes thickness. The way in which these dimensions are bounded gives each body its peculiar form or shape. This is equally true of a block of stone, a sheet of paper, a hair, a particle of dust.

14. In a line we consider only length, or linear magnitude; and the quantity of it is expressed in numbers of some convenient unit, as an inch, a foot, a mile. In a surface we consider both length and breadth, or superficial magnitude. This, which is sometimes called *area*, is expressed in *square* inches, *square* feet, &c. Solid magnitude has length, breadth, and depth. The quantity of solid magnitude in any body makes its size, bulk, or *volume;* and is expressed in *cubic* inches, *cubic* feet, &c.

15. *Surfaces* are the boundaries of solid bodies, and *lines* are the boundaries of surfaces. Thus, a box is bounded by six plane surfaces, and these surfaces are bounded or separated from one another by lines or edges. A sphere is bounded by one *curved* surface.—There is no magnitude so great that we cannot conceive a greater; and none so small that we cannot conceive a smaller.

16. *Divisibility.*—There is no known limit to the divisibility of matter. A chip of marble may be broken from a block, and that chip may be crushed to powder. The smallest particle of this powder discernible by the naked eye, when examined by the microscope, is seen to be a block having all the qualities of the original marble, and capable, by finer instruments, of being divided into still smaller blocks, which may be again divided; and so on, with no other limit than the fineness of our senses and instruments.

17. The unlimited degree to which matter may be comminuted is yet more strikingly seen in other ways. The thinnest part of a soap-bubble, just before it bursts, was shewn by Newton not to exceed the 2,500,000th part of an inch in thickness. And yet it is evident that the ultimate particles of the water, if there are such, must be of much less diameter than this.

18. The arts present numerous instances of this quality of

matter. The goldbeater produces leaves of which there are 282,000 in a pile of an inch deep. In making the gilt silver-wire used in embroidery, a rod of silver is covered with a small proportion of gold, and then drawn out to a fine wire, in which the gold retains the same proportion to the silver as at first. A portion of this wire, on which the gold is only the 432,000,000,000th part of an ounce, may be seen by a microscope to be covered with a continuous coating of the metal, having all the appearance of solid gold. The thinness here far exceeds that of goldleaf.

19. Still more minute must be the division when a substance is dissolved in a liquid, or water rises in vapour, since the particles in this case become so minute as to be invisible with the most powerful magnifiers.

20. The microscope has revealed the existence of animals, a million of which would not occupy more space than a grain of sand. Yet these animalcules, as they are called, have limbs and organs, and display all the appearances of vitality. How shall we conceive the smallness of the tubes or vessels in which their fluids circulate, and the minuteness of the particles of matter composing these tubes and fluids!

21. Divisibility thus extends far beyond the limits perceptible to the senses. Are we, therefore, to assume that it is without limits—that matter is infinitely divisible? This would be a rash assumption. On the contrary, there are many reasons for believing that there is a limit somewhere, and that there are ultimate particles, of a determinate size and shape, incapable of further subdivision. These assumed ultimate particles are called *atoms*, from a Greek word signifying indivisible. Their existence is inferred from a number of facts connected chiefly with crystallisation and chemical combination, which cannot be otherwise explained.

22. But whether the ultimate component particles of bodies have a fixed size and shape or not, we know that they are *indestructible*. This is not, indeed, what a first impression suggests, for nothing is more common than for bodies to decay, dissolve, evaporate, and disappear. But it can be proved that in no case is anything lost. The structure or form is destroyed, the materials remain. Water, mercury, and many other substances disappear in invisible vapour when heated; but if the vapour is carefully collected and cooled, the water or mercury reappears without loss of weight.

23. When a piece of wood is heated in a close vessel, such as a retort, we obtain water, an acid, several kinds of gas, and there remains a black, porous substance, called charcoal. The wood is thus decomposed or destroyed, and its particles take

a new arrangement and assume new forms; but that nothing is lost is proved by the fact, that if the water, acid, gases, and charcoal be collected and weighed, they will be found exactly as heavy as the wood was before distillation. In the same manner, the substance of the coal burnt in our fires is not annihilated; it is only dispersed in the form of smoke or particles of soot, gas, and ashes or dust. Bones, flesh, and other animal substances, may in the same manner be made to assume new forms, without losing a particle of the matter which they originally contained. The decay of animal or vegetable bodies in the open air, or in the ground, is only a process by which the molecules of which they were composed change their places, and assume new forms.

24. The decay and decomposition of animal and vegetable substances beneath the surface of the earth fertilise the soil, which nourishes the growth of living plants; and these, in their turn, form the nutriment of animals. Thus is there a perpetual change from death to life, and from life to death, and as constant a succession in the forms and places which the particles of matter assume. Nothing is lost; and not a particle of matter is struck out of existence. The same matter of which every living animal and every vegetable was formed in the earliest ages, is still in existence. As nothing is lost or annihilated, so it is probable that nothing has been added, and that we ourselves are composed of particles of matter as old as the creation. In time, we must in our turn suffer decomposition, as all forms have done before us, and thus resign the matter of which we are composed to form new existences.

25. *Inertia*, or *Inactivity*.—This is one of the most important qualities of matter, and deserves careful consideration from the number of phenomena it enables us to explain. It is at the foundation of nearly all that concerns motion and forces. The term inertia or inactivity is meant to express the fact, that an inorganic body has no power to change its state. If it is at rest, it cannot put itself in motion; if it is in motion, it cannot bring itself to rest; any change must come from some external cause. It hardly gives a correct notion to say that bodies are quite passive to a change of state; for they *resist* the change, with a force depending upon the mass of matter they contain and the amount of motion sought to be given to them or taken away. The expression *vis inertiæ*, force of inactivity, though involving a sort of contradiction, is sometimes used to indicate this quality of resistance. No one term conveys all that is meant; *persistence* has been suggested as less objectionable than inertia, inactivity, or passiveness. Bodies may be

said to have a tendency to persist in their actual state, whether of motion or rest.

26. The following instances illustrate the action of this property of matter:—When the sails of a ship are loosened to the breeze, slowly and heavily at first the vessel gets into motion, but gradually its speed increases as the force by which it is impelled overcomes the inertia of its mass. A great force is necessary at first to set a carriage in motion; but when once this is effected, it goes onward with comparative ease; so that, in fact, a strong effort is necessary before it can be stopped. If a person be standing in it when it is suddenly set agoing, his feet are pulled forward, whilst his body, obeying the law of inertia, remains where it was, and he accordingly falls backwards. On the other hand, if the vehicle be suddenly stopped, and the individual be standing in the same position as formerly, the tendency which his body has to move forward—for it acquired the same motion as the carriage by which it was borne along—will cause him to fall in the opposite direction. Casualties of this description frequently occur to persons on horseback, who are thrown over the necks of their steeds, or fall behind them, according as the animal stands still suddenly, or starts off unexpectedly. A man jumping from a coach at full speed will certainly fall prostrate on the ground, if he leaps down as if he were descending from a body at rest, to one which is in the same state; for when he makes the attempt, his body has the same motion as the coach; and when the feet arrive at the ground, the motion in the lower part is arrested, while it continues in the upper part; and thus he finds himself thrown down.

27. Another familiar example of the inertia of matter is this:—Upon the tip of the finger let a card be balanced, and a piece of money—say a shilling—laid upon it. Let the card then be smartly struck, and it will fly from beneath the coin, leaving it supported upon the finger. This arises from the inertia of the metal being greater than the friction of the card which passes from beneath it.

28. The process of beating a carpet, or dusting a book, rests on the same principle as the experiment with the shilling and the card. The carpet being struck, is suddenly put in motion, while the particles of dust remain where they were, their inertia being sufficient to overcome the slight force with which they adhere to the surface of the carpet. When a dusty book is struck against a table, the book and the dust are first brought into rapid motion together, and the book being then arrested by the table, the dust continues in motion by its inertia, and is thus detached.

29. Coursing, or hare-hunting, affords a striking illustration of inertia. In that field-sport, the hare seems to possess an instinctive consciousness of the existence of this law of matter. When pursued by the greyhound, it does not run in a straight line to the cover, but in a zigzag one. It *doubles;* that is, suddenly changes the direction of its course, and turns back at an acute angle with the direction in which it had been running. The greyhound being unprepared to make the turn, and therefore unable to resist the tendency to persevere in the rapid motion which it has acquired, is impelled a considerable distance forward before it can check its speed and return to the pursuit. But, in the meantime, the hare has been enabled to shoot far ahead in the other direction; and although a hare is much less fleet than a greyhound, by this scientific manœuvring it often escapes its pursuer. Those who have witnessed horseracing, may have observed that the horses shoot far past the winning-post before their speed can be arrested. This is also owing to the inertia of their bodies.

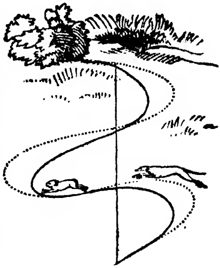


Fig. 1.

30. It is a common impression with those who have never reflected upon the subject, that bodies are more inclined to rest than to motion. This arises from the fact, that while no body begins to move or increase its speed, without some cause for the change being apparent, all the motions that come within our observation on the surface of the earth do actually come to an end, most of them gradually and without any very apparent cause. Thus the notion is begotten that rest is the natural state to which all matter, when left to itself, seeks, as it were, to return.

31. But a little consideration shews us that the retardation and stoppage of motion are as dependent on causes as its beginning is. A ball rolled on the rough earth soon stops; on a wooden floor, it continues longer; and on smooth ice, longer still. This shews that one cause of the arrest of terrestrial motions is friction. Another constant impediment is the resistance of the air. A common top continues to spin a greater length of time in a space from which the air has been extracted by an air-pump, than it usually does. A pendulum set in motion in an exhausted receiver, will continue to swing, without the help of clock-work, for a whole day, having nothing to

resist its motion but the small amount of friction at its point of suspension. Finding thus that motion is prolonged in proportion as we diminish obstructions to it, though we can never completely remove them, we conclude, that if they were removed, motion once begun would go on for ever.

32. It is in the heavenly bodies, however, that we find complete proof of this truth. They move without friction, and unresisted by any fluid like air, and no observation can detect any slackening of their speed; they retain the amount of motion they had from the beginning.

33. *Compressibility, Contractibility; Expansibility, Dilatability.*—When a body is forced by mechanical pressure into less space than it previously occupied, it is said to be *compressed;* when any cause not mechanical, such as loss of heat, causes its volume to diminish, it is said to be *contracted. Expansion* and *dilatation* are used to express enlargement of volume or bulk. Now, all bodies whatever are liable to these two opposite kinds of change—to have their volume, from a variety of causes, at one time enlarged, at another time diminished, and that without any addition to, or deduction from, the matter composing them. To say nothing of sponge, cork, wood, &c., the compressibility of which we experience every day; even those substances which we consider as types of solidity, are compressible. A piece of iron, when squeezed in a vice or hammered, loses in bulk, and becomes more compact. The compression of the foundation-stones of buildings can be detected; and a coin or medal, after suffering the pressure of the die, is sensibly less than the piece of blank metal was before.

34. The most compressible substances are air and other gases. A moderate pressure will force a quantity of air confined in a vessel into half its volume; and as the pressure is increased, the volume goes on diminishing almost without limit. By pressure and cold combined, several gases have been reduced to the liquid form.

35. Compared with gases, or even solids, liquids have little compressibility. For all practical purposes, they are assumed to be completely incompressible, and water is so considered in hydrostatics and hydraulics. But experiment has proved this not to be strictly true; for, when submitted to a pressure of 15,000 pounds on the square inch, water is found to lose 1-20th of its volume.

36. Expansion or dilatation is chiefly seen in the case of heat being applied to bodies. When a bar of iron is heated, it becomes sensibly longer and thicker. Liquids expand still more by being heated than solids; and, though they resist compression by mechanical force, they readily contract when

their temperature is lowered. The expansions and contractions depending on heat will be noticed more fully afterwards. Gases are unlimitedly dilatable. If a room were completely exhausted of air, and a cubic inch of any gas introduced, it would instantly diffuse itself through the whole room, so that no space, even the smallest appreciable, would be completely void. The only reason why the air of our atmosphere does not diffuse itself throughout space, is that it has weight, which sets a limit to its dilatation.

37. *Elasticity.*—Some bodies, when compressed, recover their former size when the pressure is removed. Such bodies are called *elastic;* and those which remain as the compressing force put them, are *non-elastic.* Air and other gases afford the best examples of elasticity. Caoutchouc, ivory, and steel, are among the most elastic of solid substances. But no solid body is perfectly elastic, nor are any completely non-elastic; so that elasticity may be considered as general a property of matter as compressibility. Liquids, in so far as they are compressible, are also elastic, for they recover their former volume when relieved from pressure. When a steel spring is bent, the concave side is compressed, and the convex side is dilated; and the elastic quality of the steel makes both sides strive to regain their former state, and thus restore the shape of the body. The cause of elasticity is not well understood; it is supposed to be some peculiar relation between the forces of attraction and repulsion among the atoms.

38. *Porosity* and *Density.*—In common language, a pore is a small hollow space or interstice between the particles of a body, large enough to be seen, or to admit the passage of liquids or gases. In this sense, some substances, such as sponge, wood, sugar, &c., are called *porous,* and others are contrasted with them as *solid.* But experiment and reflection lead us to the conclusion that all bodies are porous. We have seen that bodies are made up of indefinitely small atoms; and the fact that all bodies admit of compression and expansion, makes us believe, that in no case do these atoms fill the whole space occupied by the body, but have interstices of greater or less size between them; so that when a body is compressed, its atoms are only more closely packed. There is nothing, then, that is not porous, in this sense; and one body is more dense or solid than another, only because it is less porous. *Density* thus means the comparative closeness of the atoms of a body; and a dense body contains, bulk for bulk, more atoms, that is, more matter, than one that is less dense, or, in other words, more porous. As weight depends upon the quantity of matter, density and weight thus go together.

39. In comparing the densities of different substances, the density of water is taken as a standard, and called 1. If a cubic inch, then, of any substance weigh twice as much as a cubic inch of water, its comparative density is expressed by 2; and this is generally called its *specific gravity*. The following table exhibits the specific weights of a few of the more familiar substances:—

Platinum, coined,	22·100	Porcelain, china,	2·384
" wire,	19·267	Sulphur, natural,	2·033
Gold, coined,	19·325	Ivory,	1·917
Mercury,	13·598	Boxwood,	1·330
Lead,	11·352	Oak, old,	1·170
Silver,	10·474	Amber,	1·078
Copper, hammered,	8·878	Mahogany,	1·060
" fused,	7·788	Milk,	1·030
Steel,	7·816	Sea-water,	1·026
Iron, wrought,	7·788	Water, distilled,	1·
" cast,	7·207	Claret,	·994
Tin,	7·291	Alcohol, absolute,	·793
Antimony,	6·712	Linseed Oil,	·953
Diamond,	3·520	Ash-wood, dry,	·644
Flint Glass,	3·375	Beech, dry,	·590
Marble,	2·873	Cork,	·240

40. Porosity, even in the sense of admitting the passage of liquids and gases, exists to a greater extent than is generally supposed. If a wooden cask full of spirits is sunk in water for a time, the cask will be found filled with water, and the spirits gone. The spirits escape, and the water enters through the pores of the wood. A hollow globe of gold filled with water, and closed with a screw, was once submitted to great pressure; when the surface of the gold became covered with dew, the water being forced through its pores. The important process of filtration depends on porosity.

41. *Attraction* and *Repulsion.*—The term attraction is applied to a great many phenomena, which we must regard as of different kinds, or produced by different causes. The force, whatever it is, that makes a stone fall to the earth, is called the *attraction of gravitation,* because it is the cause of *gravity* or weight. Sir Isaac Newton demonstrated that the same force acts on the moon, drawing it towards the earth; and on the earth, drawing it towards the sun: or rather, that the attraction between any two heavenly bodies is mutual, making them approach each other. It is now established as a fundamental law, not only of our globe, but of the universe, that every atom of matter is attracted towards every other atom. The effects of this law, in causing

the fall of bodies and weight, will be considered under Terrestrial Gravity. See also ASTRONOMY.

42. *Magnetic Attraction.*—There is a certain ore of iron, a piece of which, being suspended by a thread, will always turn the same side to the north; this is called the *loadstone*, or *natural magnet*. When it is brought near a piece of iron or steel, a mutual attraction takes place, and under certain circumstances the two bodies will come together, and adhere to each other; this is known as *magnetic attraction*. When a piece of steel or iron is rubbed with a magnet, the same virtue is communicated to the steel, and it will attract other pieces of steel; and if suspended by a string, one of its ends will constantly point towards the north, while the other of course points towards the south. This is called an *artificial* magnet. The *magnetic needle* is a piece of steel first touched with the loadstone, and then suspended so as to turn easily on a point. By means of this northward-pointing instrument, which is called the Mariner's Compass, the sailor is enabled to guide his ship through the pathless ocean.

43. *Electrical Attraction.*—All nature appears to be pervaded by a mysterious affection which bears the name of *Electricity*, in consequence of its having been supposed by the ancients to reside exclusively in *electron*, or amber.

44. In its ordinary state, electricity is invisible; but when excited, it assumes the appearance of a bright and subtile fluid. It is sometimes excited in very tremendous forms in the atmosphere; but it can be produced in less extent by mechanical means, particularly by the rubbing of amber, glass, silk, and many other bodies.

45. When a piece of glass or sealing-wax is rubbed with the dry hand or a piece of cloth, and then held towards any light substance, such as hair or thread, the light body will be attracted by it, and will adhere for a moment to the glass or wax. The influence which thus moves the light body is called *electrical attraction*. When the light body has adhered to the surface of the glass for a moment, it is again thrown off or repelled, and this is called *electrical repulsion*. (See ELECTRICITY —MAGNETISM).

46. These forms of attraction act between bodies at a distance as well as near. We know, for instance, that gravitation retains Neptune in his orbit at the distance of 2854 millions of miles from the sun. But there are forces at work in matter which act only at insensible distances, and between the adjacent molecules; they are hence called *molecular forces*. These play an exceedingly important part in nature, being the causes of a number of the most interesting phenomena. They are spoken

of under the names of Cohesion, Adhesion, Repulsion, and Chemical Attraction.

47. *Cohesion* is that force that binds together particles of the same kind of matter, so as to form masses or bodies. Without some force to hold the atoms or molecules together, we could not have bodies, but mere heaps, as of sand. Some of the ancient philosophers imagined that atoms were provided with hooks that made them stick to one another. But as we now know that distant bodies act upon one another with nothing between—draw one another, as it were, without ropes—we can conceive atoms holding one another, when near, without hooks.

48. Cohesion acts only when the atoms are at distances so minute as to be insensible to us: when removed beyond that distance, it has no influence whatever; and when the atoms of a solid body are once separated, it is in most cases impossible to bring them near enough again to make them cohere. Two fresh cut surfaces of lead may be made to cohere with some force; but a slight film of rust or of grease will completely prevent the necessary nearness of the metallic molecules. Interrupted cohesion is easily restored when the body is in a fluid or half-fluid state, owing to the mobility of the molecules; as when a broken stick of sealing-wax is mended by melting the two ends and pressing them together, or two pieces of iron are joined by welding.

49. The three *states of aggregation*, as they are called—that is, the *solid*, *liquid*, and *aeriform*—are owing to differences in the strength and manner of acting of cohesion. It is commonly said that its force is greatest in solids, less in liquids, and altogether wanting in gases. But this account does not explain all the differences of these states. If the smallest quantity of air, or any other gas, is admitted into the exhausted receiver of an air-pump, it does not remain at the bottom, but spreads itself instantly and uniformly through the whole space, as if its particles wished to remove from one another as far as possible. There seems no limit to the space over which the smallest portion of gas will thus spread itself, so that it shall be found in every part of it. We cannot help inferring from this, that the atoms of a gas, instead of attracting, actually *repel* one another with a force sufficient to overcome their own weight —for gases have weight as well as solids and liquids, as will be afterwards shewn. The dilatation of solid bodies by heat, and the recoil of elastic bodies after compression, would also seem to imply some repulsive force at work.

50. Again, some gases, when compressed with great force, have their atoms forced so near that they become liquid; and in this condition they are seen to cohere and form drops, thus

shewing that they are not destitute of attractive force. The natural conclusion from these and other observed facts is, that among the atoms of all bodies there are two opposing forces at work—an attractive force, and a repulsive force—that when attraction considerably predominates over repulsion, a solid body is the result; when there is almost a balance of the two forces, we have a liquid; and when the repulsive force has the upper hand, we have a gas.

51. Many substances are seen to assume all three forms in turn. Liquid water turns at one time into solid ice, at another into vapour or steam. Greater extremes of cold and heat have the same effects on mercury. Several gases, by applying great pressure and cold, have been rendered liquid, and one at least even solid; and thus it becomes probable that all substances are capable of existing in any one of the three states under certain conditions.

52. From the fact that the increase of heat regularly increases the energy of repulsion, heat and the repulsive force may be considered one and the same thing. The subject of HEAT will be more fully considered under a separate head.

53. In solids, the cohesion is exerted in such a way as not only to keep the atoms from separating, but also to retain them in the same relative position; in liquids, on the contrary, the atoms, while still kept from separating, are allowed to move or slide freely upon one another in all directions. This free motion of the molecules upon one another, rather than want of cohesion, is the grand characteristic of liquids as well as gases; it is one of the causes of drops being spherical. Differences of cohesion give rise to such distinctive qualities of bodies as the following:—

54. *Hardness.*—This quality depends not so much upon the force with which the particles resist separation, as upon their resisting displacement or alteration of relative position. Softness implies the opposite. Hardness is tested by one body scratching another. It does not correspond with density. Thus, glass scratches gold, and even platinum. Steel has its hardness modified by tempering.

55. *Malleability* is a distinguishing quality of some metals, and means the capability of being extended into thin plates or leaves by hammering. It depends upon the union of softness and tenacity; the particles shift their position without separating. The chief malleable metals are gold, silver, copper, iron, zinc at the temperature of boiling water, and lead. Gold may be reduced to leaves so thin as to be translucent.

56. *Ductility* is the property by which a metal admits of

being drawn into wire. The most malleable metals are not the most ductile. Iron is much more ductile than tin or lead, though not so malleable. The most ductile metal is platinum.

57. *Tenacity* expresses the quality by which a body resists being torn asunder, and depends upon the intensity of the cohesive force. It is not the opposite of *brittleness*. Brittleness is associated with hardness and unyieldingness, except within narrow limits. Glass is brittle—that is, is easily broken by bending or crushing—but a glass-rod will sustain a great weight without being *torn* asunder. Thus it is both brittle and tenacious. Fibrous substances, as silk and flax, possess great tenacity. The most tenacious of all substances is steel.

58. *Adhesion*, or, as it is sometimes called, heterogeneous cohesion, is the term applied to the attraction which makes two different substances stick to one another by their surfaces. Cohesion acts between the particles of the same kind of substance; adhesion, between dissimilar kinds of matter. Though the same force probably causes both, yet the effects are so different that it is convenient to consider them separately, and give them different names.

59. *Adhesion between solids.*—Particles of dust on an upright pane of glass, chalk-marks on a wall, sealing-wax on paper, cement, are all instances of adhesion between substances of different kinds. Two polished plates of brass laid on one another, require considerable force to separate them. (Strictly speaking, this is a case of cohesion; but where the attraction is between the surfaces of two distinct bodies, it may be considered as adhesion, notwithstanding that the matter is of the same kind in both.) If one of the plates be of steel, the adhesion will be less; and generally similar surfaces adhere more than dissimilar.

60. Adhesion between surfaces is the chief cause of friction, and unctuous substances are interposed to prevent it. It is the adhesion between the ring of the wheel and the rail that enables the wheel of a locomotive to *bite*, as it is termed—that is, turn round without slipping. When the rails are wet, still more when they are covered with hoarfrost or greasy, the wheels often turn round without advancing.

61. *Adhesion of liquids to solids* takes place much more readily than that of solids to solids, because in the case of a liquid and a solid the surfaces come into more complete contact. When the hand or a rod of metal is dipped into water, a film of the water adheres to the surface, and is borne up against its own weight; nor can any force shake it all off. Plunge a bit of gold, or silver, or lead, into mercury, and a portion of the mercury will in like manner adhere. Wherever

we have *wetting*, we have a case of adhesion of a liquid to a solid. It is the cause that in pouring water over the edge of a vessel, the water is apt to run down the side of the vessel rather than fall perpendicularly.

62. But liquids do not always wet solids, or adhere to them. A rod coated with grease, or the wing of a water-fowl, remains dry when plunged in water. Mercury does not adhere to a porcelain cup, or to a rod of iron or platinum. The explanation is simple. There is in every case an attraction between the solid surface and the liquid, but it is opposed by the attraction of the particles of the liquid for one another, and there can be actual adhesion only when the first is stronger than the other. When the adhesive force is able to overcome the attraction of the liquid for its own particles, a part of it is separated and carried off on the surface of the solid; if the cohesion of the liquid is the stronger of the two, there is no wetting of the surface.

63. *Capillary attraction* is only a particular effect of adhesion. A tube with a small bore, like a hair, is called a capillary tube, from *capilla*, the Latin word for a hair. If the end of such a glass-tube is dipped in water, the water is seen to rise in the tube above the level of the rest of the surface. In a series of tubes of different diameters, the liquid ascends highest in the smallest; or the heights are inversely as the diameters. Water will be seen to rise in a similar way between two glass-plates placed as in the figure, with two of the upright edges touching, and the other two slightly apart. The sustained film rises higher as the plates approach, assuming the form of a particular curve. The fluid rises also slightly on the outside of the tubes and plates, and the surface of the sustained column within the tube is seen to be hollow like a cup.

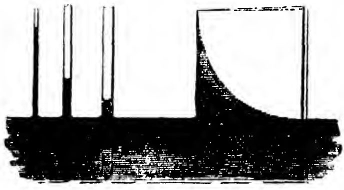


Fig. 2.

64. But liquids do not always ascend in narrow tubes or spaces; it is only when they wet the solid substance that they do so. If a greasy glass-tube is dipped in water; or, still better, if a clean glass-tube is dipped in mercury, the liquid inside, instead of rising, sinks below the general level; the surface of the column, too, becomes convex instead of concave.

65. The rise or the depression depends upon the adjustment between the forces of adhesion and cohesion, as in the case of

wetting. When the liquid wets the tube, the particles next its surface have part of their weight taken away or supported by adhesion, and thus a longer column is required to balance the pressure of the rest of the fluid. In cases where the cohesive attraction of the liquid particles within the tube for one another is too strong to permit them to adhere to its surface, that cohesion tends to draw them away from it, and downwards; while the tube prevents them from receiving the support they would have from the liquid particles around them, if it were not there. 'Mathematicians have shewn, that if the adhesion between the solid and the liquid be equal to half the cohesion of the particles of the fluid, the surface at the point of contact will be neither elevated nor depressed; if the adhesion between the two be more than half the cohesion, elevation will occur; and if it be less than half, the surface will be depressed and convex.'

66. Capillary attraction is exemplified in many familiar appearances, and plays an important part in nature. If a piece of sponge or a lump of sugar be placed so that its lower corner touches the water, the fluid will rise up and wet the whole mass. In the same manner, the wick of a lamp will carry up the oil to supply the flame, though the flame is several inches above the level of the oil. If one end of a towel happens to be left in a basin of water, while the other hangs over below the level of the water, the basin will be emptied of its contents; and, on the same principle, when a dry wedge of wood is driven into the crevice of a rock, and afterwards moistened with water, it will absorb the water, swell, and sometimes split the rock.

67. A striking illustration of this subject is given thus:—Place a wine or drinking glass on a book on the table, and set another close by it, so as to be on a lower level. Pour some water and some oil into the higher glass; then moisten a piece of cotton-wick in water, and drop an end of it into each glass, so as to reach near the bottom and form a bridge between them. The water which was below the oil in the one glass will in an hour or two be found transferred to the other, leaving the oil behind. If the wick be moistened with oil, the oil will be transferred, leaving the water.

68. When two light bodies, such as two bits of cork, are left to float on water, near each other, they soon come together, moving at last with a rush. This is sometimes given as an example of the gravitation that draws the planets to the sun; but it is really owing to this attraction of adhesion that we are considering. When the liquid wets the floating bodies, it rises slightly all round them, and this sustained liquid hangs as a

weight on them on all sides. So long as it rises equally, there is no motion; but when the bodies come near each other, the space between them becomes like part of the inside of a capillary tube, the water rises higher than on the opposite sides, and the bodies move towards the sides that are most strongly pulled. When the floating bodies are not wetted by the liquid, there is a semblance of repulsion between them and it; the surface between the two bodies is depressed, and they are pushed together by the greater repulsion on the outside. If one of two bodies floating on water is smeared with oil, so as to prevent the water from adhering, instead of coming together, the two will recede from each other, for reasons analogous to the above.

69. *Endosmose or Osmose.*—Connected with capillary attraction is *endosmose.* If two liquids be separated by a piece of ox-bladder, the one below the membrane being pure water, the other above being a solution, say of carbonate of soda, the water will pass through the membrane against gravity, and raise the solution above its former level. A smaller portion of the solution finds its way into the water. This remarkable phenomenon is known as *endosmose* and *exosmose*, or simply as *osmose.* It is not yet fully understood, but is believed to play a part in the passage of the fluids through the membranes of living animals and plants.

70. Adhesion depends upon surface alone. Now, it is calculated that a cubic inch of charcoal when reduced to powder exposes a surface of about forty-two square feet; and as this substance has an energetic attraction for colouring-matters and animal and vegetable impurities, it is found that coloured and impure liquids filtered through powdered charcoal become limpid and pure.

71. *Adhesion between two liquids.*—The lower half of a tube may be filled with water, and the top with alcohol or spirits, with little disturbance at first; but after a time the two liquids will be found equally *diffused*—part of the heavier liquid rising up, and of the lighter being drawn down, by the force of adhesion between the two. Where there is no adhesion between two liquids, as in the case of water and oil, they do not diffuse or mix.

72. *Adhesion between solids and gases.*—Dry iron filings, and even small needles, when gently laid on the surface of water, will float, though eight times heavier than the water, because each has a film of air adhering to it so strongly that even when it does sink it carries a portion of the air along with it. In making barometers, it is found that air adheres so firmly to the surface of the glass, that the mercury must be boiled in the

tube before it can be expelled. Some porous solids, such as charcoal, absorb air and other gases to an amount many times their own bulk, the force of adhesion condensing the gases on the surface of their molecules. When a lump of sugar is dropped into a cup of tea, the atmosphere of air which surrounds the particles does not quit them till they are dissolved; bubbles are seen rising till all the sugar has disappeared.

73. *Chemical attraction* is a molecular force, whose effects are of a different kind from those of cohesion. If we divide a piece of marble by breaking it into parts however small, each part is still marble. But the chemist takes it to pieces in a different way. Out of a piece of marble he will produce three distinct substances, altogether unlike the original body—a metal not unlike silver, a black body called carbon, and a gas resembling air. Most of the substances of which our earth is made up are thus composed of two or more different substances. When the chemist tries still further to analyse or take to pieces any of the three substances he found in the marble, he finds he cannot; out of the carbon he can make nothing but carbon, and so with the others. When a substance thus resists being divided into other substances, it is said to be a simple substance or *element*. Of these elements there are in all about sixty known; and these combine in unions of two, three, or more, to form most of the bodies that we see around us. Thus, water is composed of two gases like air in appearance; vermilion, of sulphur and quicksilver. Such unions are chemical unions, and the attractive force that produces them is *chemical attraction*. Cohesion and adhesion produce unions, but they leave the united substances with their qualities unchanged; chemical attraction, in uniting two substances, changes their properties, and produces a new substance, with new properties. The investigation of changes that thus alter the constitution of bodies belongs to the science of CHEMISTRY.

74. The properties of matter above described have been divided into *essential* and *accidental*. A better distinction is that of *general* properties and *specific* properties. The first would include those properties that are common to all bodies, such as Magnitude, Impenetrability, Divisibility, Inertia, Porosity, Compressibility, Elasticity, Gravity, Cohesion, &c.; the second would include such as are peculiar to, or distinguish certain species or kinds of matter, as Density, Hardness, Malleability, &c. But it is often difficult to draw the line of distinction. Thus, elasticity is perhaps as much a specific property as malleability; and density, more or less, is possessed by all matter whatever; rare is only less dense. Such distinctions are therefore in a great degree arbitrary.

MOTION AND FORCES.

75. Motion is change of place, and its opposite is rest. Motion in any one body has always reference to the place of other bodies, and various distinctive terms are used indicative of this reference. A man sitting on the deck of a ship has a *common* motion with it; if walking on the deck, he has *relative* motion to the vessel. If a boat sail against a stream exactly as fast as the stream flows, it is at rest relatively to the bottom and banks, but in motion as respects the water. *Absolute* motion means change of place with respect to space itself. But we have no means of marking a fixed point in space, and therefore can never observe such a motion; we know only relative motions. As little do we know of absolute rest. The earth is in constant rotation and also revolution round the sun; and the sun himself is in motion, we know not whither. Motion, and not rest, is the great law of the universe.

76. *Velocity*, or speed of motion, is measured by the space passed over in a given unit of time; as when we say that a man walks three miles an hour; or that sound travels 1120 feet in a second. The velocity is *uniform*, when equal spaces are always passed over in equal times; it is *accelerated*, when gradually increased, and *retarded*, when gradually diminished. If the increase or diminution is equal in equal times, the motion is said to be *uniformly accelerated* or *uniformly retarded*.

77. *Force* is any agency that produces motion in a body; when the body is not free to move, the force exerts a *pressure*. Force and pressure are often used indifferently; and weight being the kind of force with which we are most familiar, the amount of a force or pressure is usually expressed in so much weight; as ounces, pounds.

78. A body when put in motion acquires the power of setting other bodies in motion: in other words, it acquires force; and this force of a moving body is called its *momentum*, or *quantity of motion*. Momentum does not depend upon velocity alone. Of two balls moving at the same rate, but the one having twice the mass of the other, the larger will have just twice the force or momentum. Momentum, then, is made up of velocity and quantity of matter taken together. To get the comparative momenta of two bodies in numbers, we multiply the weight of each by its velocity. If two balls of 6 and 8 pounds move, the first 100, the other 50 yards in a second, the momentum of the first will be to that of the second as 6×100 to 8×50, or as 600 to 400, or as 3 to 2.

LAWS OF MOTION.

79. The leading truths respecting the movements of bodies have been summed up in the shape of a few axioms, or propositions, which were first put into shape by Newton under the name of *Laws of Motion.* Various modifications of these laws have been proposed, with a view of rendering them more explicit. We shall give them as laid down by Newton, and then follow them up with observations on each.

1st, Every body must persevere in its state of rest, or of uniform motion in a straight line, unless it be compelled to change that state by forces impressed upon it.

2d, Every change of motion must be proportional to the impressed force, and must be in the direction of that straight line in which the force is impressed.

3d, Action must always be equal and contrary to reaction; or the actions of two bodies upon each other must be equal, and their directions must be opposite.

FIRST LAW.

80. This law is little else than a definition of the property of Inertia or Inactivity. In fact, all these laws are but expansions of the idea of inertia; the facts they state go to make up that idea, or flow from it as consequences. There are three things to be attended to in this law—namely, the *persistency* of matter, both in rest and motion, when not acted upon by external agency; the *uniform* velocity; and the *straight* direction of motion when once begun and left to itself.

81. That a body at rest will continue at rest unless acted upon, is so evident, that no one perhaps ever thought of uttering the contrary. Why do we not as readily assent to the statement, that a body, once in motion, will never stop unless something stop it? The reason is, that all the motions immediately around us *are* interfered with and stopped. Thus ordinary experience leaves the impression, which was shared in till not long ago even by philosophers, that rest is the natural condition of matter, and that motion requires to be sustained by something external. Reflection on the obvious causes that retard all terrestrial motions, and more extended observation, especially of the heavenly bodies, enable us, as we have seen when speaking of Inertia, to correct this erroneous first impression; so that to the natural philosopher motion is as natural a state as rest, and matter is quite indifferent to either state. Motion, unobstructed, no more requires to be sustained than rest.

82. When we say that unobstructed motion is naturally

uniform, we are only repeating in a different form the truth last illustrated. If we cannot conceive a body beginning to move without a cause, neither can we conceive a moving body beginning of itself to move faster; and if we believe that when a body in motion is stopped, something external stops it, we must believe that when its motion becomes slower, something is retarding it. The uniformity of unobstructed motion is beautifully illustrated in the case of the motions of the heavens. On it depends the regularity with which day and night, summer and winter, and all other vicissitudes, return. The uniform motions produced by man are attained by counteracting the retarding tendencies.

83. Motion is naturally *straight*. If a ball projected forward is seen to bend to the right or the left, we infer at once that something interfered with it. That it bends downwards, we know to be owing to the attraction of the earth. Motion therefore requires force to bend it. It might seem at first sight that a body once set amoving in a circle ought, by the law of persistence, to go on moving in the same path when left to itself. But the *same path* is, in this case, not the *same direction*. A body moving in a circle, or any other curve, is continually changing its direction; it is always turning a corner, as it were.

SECOND LAW.

84. That every change of motion is proportional to the force that produces it, is involved in the way of measuring force, which is by the quantity of motion it gives or takes away. As to the *direction* of the resulting motion, the law requires careful explanation. It has been proposed to express it in the following terms, as less liable to be misunderstood:—'All force, or cause of motion, in any direction, produces its effect in that direction, and in no other.' When the body is previously at rest, it is self-evident that, if unobstructed, it will follow the direction of the force that begins to act upon it; but if it is already in motion in one direction, and a force then comes to act upon it in another direction, it is not so clear what course it will take. To determine the path or line in which a body will move when thus acted upon by more forces than one, is a problem which lies at the foundation of the whole science of mechanics. It is usually treated under the title of the

Composition and Resolution of Motion.

85. To avoid confusion on this subject, it is necessary to attend to the difference between moving in the same straight

line, and moving in the same direction. The ball B in the figure is moving in the same direction, whether it move in the line AC or in any line parallel to AC, as *gf* or ED; in any of these cases it is equally approaching the line CD. In the same way a motion from B towards E, or from *h* towards *i*, or from C towards D, is still in the same direction, because these lines are parallel.

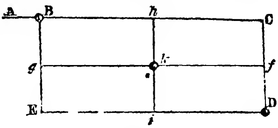


Fig. 3.

86. Now, let the ball be moving along the line AC with a velocity that would carry it from B to C in two seconds, and when at B let it receive a blow that would carry it from B to E in the same time; the question is, How will the ball now move? This is best understood by supposing it placed not on a plane surface, but in a groove in the upper side of a movable bar lying on a table. The ball being now set arolling at the same rate as before along a groove in the bar AC, let the bar be made at the same time to slide across the table, keeping parallel to itself, and carrying the ball along with it, so as to arrive at the position ED in two seconds. The common motion of the bar and the ball will not in any way interfere with the motion of the ball in the groove, any more than the common motion of a ship and a man on board of it interferes with the man in walking across the deck. The ball will be at the end of the groove at the end of the two seconds, just as if the bar had been at rest; it will therefore, as a result of the two movements, be found at the point D.

87. If the position of the ball on the table is observed at the intermediate points, it will be found to describe a straight line from B to D; for since we have supposed both motions uniform, the bar will, at the end of the first second, be in the position *gf*, midway between BC and ED, and the ball will at the same instant be half-way from *g* to *f*, at *k*; and it can be proved that *k* is in a straight line between B and D. The same could be shewn as to any intermediate stage.—When both motions are not uniform, the body moves in a curve, as will be seen in speaking of projectiles.

88. The movable groove is introduced to make the effect of two movements conjoined more readily conceived; to shew palpably, as it were, that a body may be moving in two directions at one and the same time. But if it receive the second impulse by a blow while rolling freely on the table, it will still arrive at D by the same path.

89. In any case, then, when two forces act upon a body, if

we draw two straight lines, AB and AC, in the directions of the two forces, and make the lengths AD and AE in proportion to the velocities that the forces would give to the body if acting separately; then if we draw EF and DF parallel to AD and AE, and join AF, this line, which is called the *diagonal*, gives the direction which the body will move in, *and also its velocity; that is, the body acted on by the two forces moves from A to F in the same time in which it would have moved from A to D by the one force alone, or from A to E by the other force alone.* A figure thus formed represents both the motions and the forces that produce them, and is called the *parallelogram of forces*. AD and AE are called the *components*, and AF the *resultant*.

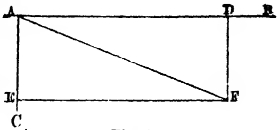


Fig. 4.

90. In figures 3 and 4, the forces are represented as acting

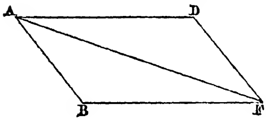


Fig. 5.

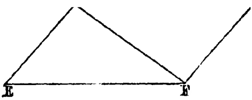


Fig. 6.

at right angles to one another; but the angle may vary, as in figures 5 and 6, and the learner should observe the effect on the resultants as he draws parallelograms with angles still narrower or still wider.

91. We arrive at a similar result if, instead of motions, we consider a set of forces acting against one another so as to prevent motion. When forces thus balance one another, they are said to be in equilibrium; and the investigation of such cases forms the part of Mechanics called *Statics* (from a Greek word signifying 'to stand'), while the consideration of force producing motion belongs to *Dynamics* (from the Greek word for 'power').

92. Two strings fastened to the ring P, and drawn in opposite directions by equal forces, will keep the ring at rest, and the forces are then in equilibrium. The forces A and B, in fig. 7, are represented by two weights, say of six pounds each, suspended over pulleys. Now, it is evident that for one of the two forces we could substitute

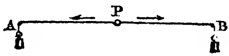


Fig. 7.

two other forces, as in fig. 8, pulling in the directions PC, PD, which, if properly adjusted, would still keep the ring at rest. The effect of the two new forces, C and D, is thus equal to the effect of the one force, B, and the force B is said to be *resolved* into the two, C and D.

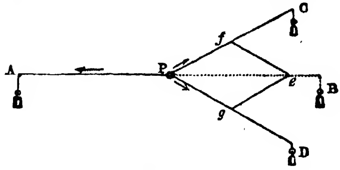


Fig. 8.

93. If the two new forces are equal to one another, the system, when at rest, will be so disposed that the line of direction of the force for which they are substituted will divide the angle equally. If one of them, as D, be greater than the other, the angle will be divided unequally, and the less of the two parts will be next the greater weight, as in fig. 9. The exact proportion of the two to each other and to the one original force, may be determined by setting off a length of six parts along P*e*, to represent the six pounds of the original force, and then drawing *ef* and *eg* parallel to PD and PC. We have then a parallelogram of forces. P*f* represents the magnitude of the force C, P*g* that of the force D, and P*e* is the resultant. It is easy to see that the sum of the two forces at D and C must be together greater than the one force for which they stand, just as the two lines, P*f* and P*g*, are together greater than P*e*. As the two forces pull at an angle to the force they have to counteract, part of their effect is lost.

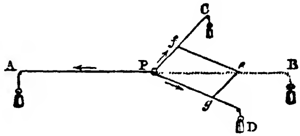


Fig. 9.

94. As *ge* in the last figure is equal to P*f*, and in the same direction, the two component forces and their resultant are represented by the three sides of the triangle P*ge*; and the resolution of forces is thus often represented by the half of the parallelogram of forces, to save drawing the whole.

The following are examples of the resolution of forces :—

95. Let HM be a canal-boat, MP the rope by which it is drawn by a horse attached to it at P. The force of the draught being denoted by MP, it may be resolved into MA and MB, of which only MA is effective in drawing the boat forward; the other force, MB, tends to turn the head of the

boat in the direction MB. This last force must therefore be counteracted, which is effected by means of the helm, HE, turned to an oblique position. When the boat is in motion, the water, being at rest, produces a resistance or pressure against the helm. If CD

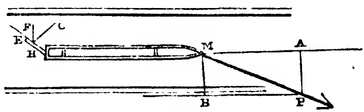


Fig. 10.

denote the resistance, it may be resolved into HD and HC, of which HD produces no effect on the helm; therefore CH is the only effective pressure. Again, CH may be resolved into CF and FH, the latter of which tends to turn the stern of the boat in the direction FH, and thus counteracts the force MB, by tending to turn the boat round in an opposite direction; and the part CF tends to move the boat backwards, and thus, counteracting a part of the force MA, it retards the progress of the vessel. The two forces, FH, MB, would move the boat sideways, or laterally, to the side of the canal; but this can be prevented by giving the helm a little more obliquity, for, from the length and shape of the vessel, it is much more easily moved in the direction of its length than of its breadth.

96. Let TP be a ship, SL its sail, WA the direction of the wind and its pressure on the sail. WA can be resolved into AB perpendicular to the sail, and BW parallel to it, the latter of which has no effect in pressing on the sail; therefore AB is the effective pressure on the sail. Were the vessel round, it would move in the direction BA. Let BA be resolved into CA and BC, the former, CA, acting in the direction of the keel or length of the vessel, or in the direction CA,

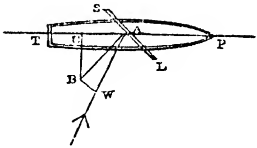


Fig. 11.

and the latter perpendicular to it, or in the direction of the breadth. The former pressure, CA, is the only pressure that moves the vessel forward, the other, BC, makes it move sideways. From the form of the vessel, however, this latter force, BC, produces comparatively little lateral motion; any that it does occasion is called *leeway*.

97. By a similar double resolution, we may represent how much of the actual force of the wind is effective in turning round the arms of a windmill.

THIRD LAW.

98. When one stone is dashed against another stone at rest, the moving stone is hit as hard, and is as likely to break, as the one at rest; and when one person knocks his head against his neighbour's, it is difficult to say which is most hurt. The hand pressed against a fixed body is equally pressed in its turn. If a man standing in a boat attempt to push off another boat of the same size that is alongside, both boats will recede equally from each other; if he pull the other boat towards him, his own boat advances half-way to meet it. A magnet draws a piece of iron towards it; but the magnet is also drawn towards the iron, as is seen when they are both suspended so as to move freely. In all these cases, we see that the body that we consider as acting upon the other, is itself acted upon in turn, and in the opposite direction: this is what is meant by *reaction*. But to determine more exactly the equality of the action and reaction in all cases, it is necessary to advert to the way in which action is measured.

99. We have seen that the effect of a force in producing motion is not estimated by the velocity merely which it produces, but by the velocity combined with the mass of the body moved. Mass multiplied by velocity, which is called the *momentum* or *moving force*, or quantity of motion, is the measure of the force which has been exerted to set a body in motion. Now, such a body becomes in turn capable of setting other bodies in motion, and its power of doing so also depends upon its momentum; it can give only what it received. Motion received thus, implies motion either taken away or given in an opposite direction.

100. To recur to the examples of reaction formerly cited. If the magnet and the piece of iron are of the same weight, they move to meet each other with equal velocities, for thus only can the momentum be the same in both cases. If the magnet is three times the weight of the piece of iron, the iron must move with three times the velocity of the magnet to make the momentum the same; and so it is found to do. In the case of the boats, suppose the one in which the man is seated to be ten times the weight of the other, then for every ten feet that the light one moves off, the heavy one will recede one foot. The two will thus have the same momentum; and if each were to strike an object, the same shock would be given in both cases.

101. In the last case, both motions would still be visible. But let a boat of a ton weight be pushed away from the side of a ship of a thousand tons' weight, and then only one seems

to move; for while the boat moves off a yard, the ship recedes only the thousandth part of a yard, which it would require minute observation and measurement to render apparent. From this we can pass to the extreme case of a boat pushed off from shore. Where is the evidence of reaction here? We see none, it is true; still, the consideration of the cases already adduced, and of a thousand similar, lead us irresistibly to believe that the shore, if it is free to move, must recede from the boat. But the shore can move only by carrying the earth with it; and considering the vast mass of the earth compared with that of the boat, the space moved over would defy measurement, even if we had any fixed mark to count from. We cannot help believing, then, that when a stone falls—in other words, when the earth draws a stone towards it—the earth is itself drawn, or falls, towards the stone.

102. Other exemplifications of the law of action and reaction are the following:—The force of the gunpowder that propels a ball, reacts upon the gun and makes it *recoil* with a momentum equal to that of the ball. The mass of the gun, however, being many times that of the ball, the velocity of the recoil is so many times less. Hence the use of the great weight given to guns and mortars, and their carriages.

103. Similar to the recoil of a gun is that produced by water flowing in a jet from a hole in the side of a full vessel; if such a vessel be set on a floating plank, the plank will move in the opposite direction to the jet. On the same principle, a man floating in a small boat may propel himself in one direction by blowing with a bellows, or even with his mouth, in the opposite direction. It is in the same way that a sky-rocket ascends; a constant stream of gases is rushing with force from the burning end of the rocket, and the rocket recoils with equal force.

104. In case of a carriage being run away with, persons have been known to lay hold of the sides to hold it back; they forget that while pulling back with their hands they are pushing forwards with their feet, and that the action and reaction, being equal and contrary, destroy each other.

Communication of Momentum.

105. When two bodies come in collision, there is always communication of momentum to one and a destruction of it in the other; and the laws are different according as the bodies are elastic or inelastic.

106. *Collision of inelastic bodies.*—Let a pound, say of clay, moving 10 feet in a second, encounter another mass of clay of a pound weight, at rest. The two pounds will now move on

as one mass, but evidently with slackened speed. The collision can *create* no motion; the momentum remains the same; and as the moving mass is now double, the velocity will be only 5 feet. The first pound has therefore lost half its momentum, and has given it to the second.

107. If the mass in motion weighs 12 pounds, and that at rest 4 pounds, let us consider what the velocity will be after striking. The original momentum is expressed by the weight of the moving body multiplied by its velocity, or $12 \times 10 = 120$. But the same is the momentum of the two bodies moving in one mass; and as that mass is 16 pounds, the velocity must now be less; it must be 120 divided by 16, or $7\frac{1}{2}$ feet a second, since 16 multiplied by $7\frac{1}{2}$ gives 120, the same momentum as before. If the smaller of the two masses is made the moving body at first, the velocity after impact will be 40 divided by 16, or $2\frac{1}{2}$ feet a second.

108. Next suppose both bodies in motion in the same direction; let, for instance, a body of 4 pounds, moving 15 feet, overtake one of 12 pounds moving 10 feet. Here the momentum of the one body is 60, and that of the other is 120; and none of this is lost when the bodies unite, because it is all in one direction. The momentum of the united mass is therefore 180, which, divided by 16, gives $11\frac{1}{4}$ feet as the velocity.

109. If both bodies are moving with equal momenta in opposite directions, they instantly destroy one another's motions, and are brought to rest; if the momenta are different, the motion of the two bodies is continued in the direction of the greater. Let the two motions last supposed be contrary; then the momentum of 60 in one direction destroys half the momentum of 120 in the other, leaving a momentum of 60 to carry on both bodies, which, dividing by the mass, gives $1\frac{3}{4}$ feet for the velocity.

110. To judge of the *shock* occasioned by collision, we are to look, not to the velocities of the separate bodies, but to the *relative* velocity of the two—that is, the rate at which they are approaching to, or receding from, one another. Thus, in the case of the two motions above considered, when they were in the same direction, the bodies were approaching each other at the rate of 5 feet a second; when in opposite directions, the rate was 25 feet. When two opposite trains, the one moving 30 miles, and the other 20 miles, come in collision, it gives the same shock to both as if one of them had been at rest, and the other had run into it at the rate of 50 miles.

111. *Collision of elastic bodies.*—When one billiard-ball strikes another of equal size directly, the first remains at

rest, and the one struck moves on with the same velocity as the other had. This is different from the action of two inelastic bodies, as two masses of clay, and the difference has been thus explained: 'The approaching ball, at a certain point of time, has just given half of its motion to the other equal ball, and if both were of soft clay, they would then proceed together with half the original velocity; but as they are elastic, the touching parts at the moment supposed are compressed like a spring between the balls, and by then expanding and exerting force equally both ways, they double the velocity of the foremost ball, and destroy altogether the motion of that behind.'

112. This peculiar action of elastic bodies appears when a number of ivory balls are placed close in a row, and the outermost at one end is smartly struck against the next; none of them moves sensibly from its place, except the outermost at the other end of the row. Each ball in turn receives the whole motion from the one that precedes it, and gives it away entire to the next. The last becomes thus the vehicle of the whole motion. Instead of placing the balls on a table, they may be suspended as in the figure.

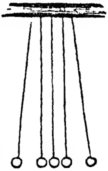


Fig. 12.

113. Action and reaction, in the case of elastic bodies, is thus double of what it is in inelastic; and all the results of collision will be modified accordingly. Thus, two elastic bodies moving with equal momenta in opposite directions, will not merely destroy one another's motions, but will recoil, each in the contrary direction. If the elasticity were perfect, the velocities in recoiling would be exactly equal to what they were in advancing. No bodies, however, are perfectly elastic, nor are any altogether without elasticity; so that the statements above made are in practice true only to a certain extent.

REFLECTED MOTION.

114. When a body strikes a fixed plane, if it is completely inelastic, its motion is destroyed, and it remains on the plane. But this is true only of soft masses, all hard solids have more or less elasticity, and rebound or are *reflected* from the surface, and this reflection follows a regular law of direction. If an ivory ball, for instance, be dropped, as from L, on a level marble slab at K, it will rebound in the same perpendicular line, and, being almost perfectly elastic, will rise again nearly to L. But if the ball is thrown

obliquely from H to K, it will rebound or be reflected in the direction KI, so as to make the angle HKL, called the *angle of incidence*, equal to the *angle of reflection* LKI. This is a result of the law of the resolution and composition of motion. Of the motion of the ball from H to K, a part is directed perpendicularly towards the plane, and a part parallel to it. When the ball reaches the plane, the downward perpendicular part of the motion is converted into an upward perpendicular motion, while the horizontal part remains unaffected; the resultant of these two tendencies is the direction KI. The motions of billiard-balls are mostly guided by this law of reflection. When the reflection takes place from the surface of a ball, a tangent to the ball, at the point of impact, enables us to judge of the angles.

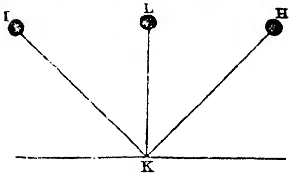


Fig. 13.

COMMON MOTION.

115. Bodies receive a common motion in various ways. The same force may act on each separately, as when a number of bodies are dropped at the same instant, and fall towards the earth by the force of gravity. In this case, the lines of motion are all parallel; and though the velocities are accelerated, they are all accelerated alike; the bodies therefore keep the same position relative to one another. The same would be true of a number of billiard-balls, if they could be all struck with exactly the same force and in parallel lines.

116. But the motion which the objects on board a ship, or in a carriage, or on a tea-tray, have in common, is communicated in a different way. The force, in these cases, does not act on all the objects of the system, but only on parts or points of one or more of them. Now, when a pressure acts against a part of a body, that part has a tendency, as we may say, to move on and leave the rest of the body behind. This brings the force of cohesion among the particles and parts of the body into play, and thus the original pressure is *propagated* to every particle, and the whole coherent mass moves on together. This accounts for the body of the ship moving along with the mast; but why should a loose object on deck move? what force acts on it? The force of *friction*, in this case, supplies the place of cohesion. A block of wood lying on deck while the ship is at rest, requires a

certain force to make it slide along. Now, when the ship begins to move, the log cannot remain behind without sliding along the deck, so that it is actually pulled along with a force equal to that which is necessary to make it slide on the deck. Place one book above another on a table, and draw the lower one along pretty smartly—the upper one will accompany it. Put four small wheels or two rollers between the books, and when the lower is pulled along, the upper one falls behind—a proof that it is the force of friction that transmits the motion of a body to those resting on it.

117. When a system of bodies has thus, from any cause, received a common motion, their positions, with regard to one another, remain unaffected by it, and in considering their relative motions, we may leave their common motion out of account; everything is as if the system were at rest. This arises from the property of inertia, as already expounded. When a ship is going steadily, all things go on in the cabin as in a room. On deck, a ball thrown upwards comes down in the same spot; the deck has not slipped away from under it. The *motal inertia* of the ball carried it on while in the air with the motion it had acquired from the ship.

118. The feats of equestrians depend upon this law of common motion. Riding at full speed, they throw up balls, which return into the hand. The rider springs right up from the saddle, and alights on it again without falling behind the horse. When a bar is held in his way above the horse, he does not require to leap forward in surmounting it; he springs directly upwards, and this upward motion, combined with the forward motion he has in common with the horse, results in carrying him in a curve over the obstacle, and planting him on the very spot of the horse's back he sprung from. The action of *centrifugal force* in such cases will be considered afterwards.

119. If a ship, sailing westward at the rate of 10 feet a second, fire a ball northward at a fixed object, at such a distance, we shall suppose, that the ball would reach it in two seconds; then, if the gun was pointed directly to the mark, the ball will strike 20 feet to the west of it. For, while flying northward, it retains the motion of the ship, which carries it west at the rate of 10 feet a second. If the mark were a ship sailing westward at the same rate, the ball would hit as if both ships were at rest.

120. All bodies, at any part of the earth's surface, have a common motion eastward along with the earth in its rotation. The velocity of that motion is greatest at the equator, and grows less towards the poles, as the circles described grow less and less. When a current of air, then, or of water in the

ocean, is moving from any place in the northern hemisphere towards the equator, it carries the eastward velocity of the place it starts from to places where that velocity is greater; it thus has a tendency to fall behind the surface of the solid earth, which gives it the semblance of being deflected from its direct south course towards the west.

TERRESTRIAL GRAVITY.

121. It is a law, as we have seen, wide as the universe, that every particle of matter acts upon every other particle by a force known as *gravitation*, which draws them the one towards the other. We are now to consider the laws of this force, and explain the more important phenomena it gives rise to.

122. The attraction of gravitation is always in proportion to the mass of the attracting body. This will be readily admitted. A force is exerted by each atom, and the more atoms the more force. It is not the volume or bulk of the body, but its quantity of matter that decides its effect.

123. Gravitation acts at all distances, but with diminished effect as the distance increases. The decrease, however, does not go on in a simple proportion; twice the distance does not give simply half the attraction. If, at the distance of a foot, the force of attraction were a pound, at the distance of 2 feet, it would be $\frac{1}{4}$ of a pound; at 3 feet, $\frac{1}{9}$ of a pound; at 4 feet, $\frac{1}{16}$ of a pound. This is expressed by saying that attraction varies inversely as the squares of the distances between the bodies.

124. This law holds with many other influences besides gravitation; with light, for instance, and whatever is diffused from a centre in all directions. By considering the annexed figure, we see that the same quantity of light which, at a yard from a candle, is diffused over a board of one foot square, is, at the distance of two yards, diffused over a board having a side of two feet, but containing four squares of one foot to the side.

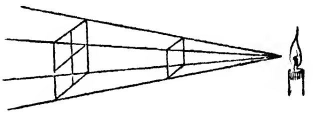


Fig. 14.

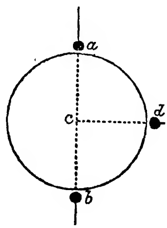
125. The distance between two mutually attracting globes is measured from centre to centre. Each particle attracts in a line pointing to itself; but the result can be shewn by composition of forces to be the same as if the whole force were concentrated in a sort of mean point, which in regularly shaped bodies is the same as the centre of magnitude. The attraction

of the earth, then, on a body at its surface is exerted at the distance of about 4000 miles, which is the length of its *semi-diameter;* at the height of 4000 miles above the surface, or at the distance of two semi-diameters from the centre, the attraction is reduced to one-fourth. The moon being at the distance of sixty semi-diameters, the earth draws it with a force which is only a 3600th part of what it would be, were the moon at the earth's surface. It belongs to astronomy to treat of gravitation as ruling the planetary motions; we have here to speak of the phenomena it produces on bodies at or near the earth's surface.

126. The attraction of the earth for bodies on its surface is so strong, owing to its overwhelming mass, that it overpowers their attraction for one another, and renders it in most cases inappreciable. But a plumb-line suspended near a large mountain is sensibly drawn from the perpendicular; and by a delicate instrument, called the torsion-balance, the action of a large ball upon a small one has been measured.

127. The *weight* or gravity of a body is another name for the force with which it is drawn towards the earth. As each atom is drawn, the more atoms there are in a body, the greater the force or weight must be; and thus weight and quantity of matter come to be looked upon as the same. All matter has weight; even the lightest gases, as can be shewn by experiment. (See PNEUMATICS.) Weight, or the force of terrestrial gravity, being the force with which we are best acquainted, is used as the measure or standard for all other kinds of forces.

128. The force of gravity is directed towards the earth's centre. A plumb-line indicates the direction, and we usually call it *downward;* but *down* and *up* are no fixed directions; they vary for every part of the earth's surface, and what is *down* to us, is *up* to our antipodes. This is shewn in the figure, which represents balls falling towards the earth at three different places on the surface. At points, however, not far distant from one another, downward or vertical lines do not differ sensibly from parallel lines, owing to the great distance of the centre.



Falling Bodies.

129. Perhaps nothing explains more phenomena than the complete understanding of what takes place when a body falls;

for half the motions in the world are caused, directly or indirectly, by falling. Falling is the best instance of a *uniformly accelerated* motion. Gravity not only puts a body in motion, but *continues* to act on it with equal force after it is in motion, and thus is constantly adding to its speed. It is not with the force of gravity as it is with that of a locomotive. A locomotive, too, continues to exert its force on the train; but after reaching a certain speed, the friction becomes so great as to consume the whole force in overcoming it, and the motion is no longer accelerated. But, except the trifling resistance of the air, a falling body has no friction to overcome; whatever motion it has once received, it retains, and would retain, if gravity were to cease. As the force of gravity continues unabated, it must give an addition to the speed during every second equal to what it gave during the preceding second, and thus the motion is *uniformly accelerated.*

130. This acceleration is seen in every body dropped from a height through the air; and no less in a rock rolling down a declivity, which continually gathers force as it descends, till its momentum is sufficient to shatter itself and everything it encounters to pieces. In a water-fall, the increasing velocity breaks off the lower part of the descending column in drops, which separate more and more as they descend. When a viscid liquid, like molasses, is poured out from a height, the bulky sluggish stream becomes gradually rapid and smaller, and is at last reduced to a thread; but wherever a vessel is held into the stream, it fills equally fast.

131. All bodies, heavy and light, would fall equally fast, if the resistance of the air were removed. A ball of lead two pounds' weight does not fall faster than a ball of one pound. A gold leaf, however, will fall considerably slower than the same gold in a solid state, because it exposes much more surface to the air. That this is the cause of the difference, is shewn by an experiment with the air-pump, in which a guinea and a feather are dropped in a vacuum, and fall to the bottom together.

132. It has been well ascertained by observation, that when a body begins to fall from a state of rest, it descends 16 feet 1 inch in the first second of time. Suppose that no fall had ever been observed beyond that point, and that we had to find by reasoning what the body would do during the next second. The first consideration is, How far would the body fall, if gravity were to cease?—in other words, What velocity has it acquired? It would not be unnatural to say 16 feet (we omit the 1 inch), since that is the space it fell in the second that is ended; but reflection shews that this cannot be the case.

During the first half of the second, it made very little way, and the greater part of the distance was passed over in the last half; 16 feet expresses the mean velocity, or the velocity it had at the middle of the second, and at the end it must have been moving with a velocity just double the mean, or 32 feet.

133. The velocity acquired, then, at the end of the first second is 32 feet; and if gravity were to cease, it would, by the law of inertia, move over 32 feet in the next second. But gravity still acting, will make it fall another 16 feet in addition to the 32, or 48 feet in all; and will create as much velocity in addition as it created in the first second, so that if it were to cease at the end of the second second, the body would move 64 feet in the third second. Thus, during the second second, the body will fall through three spaces of 16 feet, and at the end of it will have its velocity double of what it was at the end of the first second. The whole space fallen during the two seconds will thus be four spaces of 16 feet.

134. We arrive at this result by reasoning, and observation proves it correct. Without giving the steps of the investigation for the succeeding spaces of time, the results may be exhibited in the following table:—

Number of Seconds.	1	2	3	4	5	&c.
Velocities at the end of number of seconds, in spaces of 16 feet, .	2	4	6	8	10	&c.
Spaces fallen through during each successive second, ...	1	3	5	7	9	&c.
Spaces counted from beginning of fall, ...	1	4	9	16	25	&c.

135. For example: The third column, headed 3 (seconds), informs us that at the end of 3 seconds a falling body is moving (for the instant) at the rate of 6 spaces of 16 feet—that is, of 96 feet—a second; that during the third second it falls 5 spaces, or 80 feet; and that the whole distance fallen during all the three seconds is 9 spaces, or 144 feet.

136. The distances that bodies fall, then, do not increase simply as the times, but as the *squares* of the times. To find, therefore, how far a body will fall in any number of seconds, multiply the number of seconds by itself, and that product by 16. Thus, in 7 seconds, a body will fall $7 \times 7 \times 16 = 784$ feet. The height of a precipice might be roughly measured in this way, by observing how many seconds a stone takes to reach the bottom.

137. As a body in descending to the earth receives increasing accessions to its velocity during every successive second, so

when a body is projected upwards from the surface of the earth, its velocity decreases in the same proportion till it comes to a state of momentary rest, when it instantly begins to descend with a gradually increasing velocity, which at any point in the descent is equal to its velocity at the same point when ascending. In this calculation, however, we omit the influence of the atmosphere, which would cause the final velocity in the descent to be less than the original velocity with which the body was projected upwards.

Projectiles.

138. Any heavy body launched or *projected* into the air with an impulse, is a projectile; and to investigate the motions of such bodies forms a distinct branch of the science of motion. We have seen in the last paragraph the law that a body projected perpendicularly upwards observes. When it is thrown downwards, gravity does not interfere with its direction, any more than in the other case, but immediately begins to add to its motion at the same rate that it would have done had the body begun to move from a state of rest. A stone dropped from the top of a tower will, in 3 seconds, descend 9×16, or 144 feet; if thrown straight down with a velocity of 20 feet a second, it will descend 3×20, or 60 feet in addition —in all, 204 feet.

139. But projectiles are almost always thrown, not vertically, but horizontally, or rather obliquely; and every one is familiar with the kind of curve that they then describe. It is seen in a stone thrown from the hand or in water spouting from a hole in the side of a vessel. Its nature will be understood from the annexed figure. Suppose a cannon-ball fired from the top of a tower, whose height is represented by the line A4′, in the direction of the horizontal line AB. The force of the gunpowder ceases to act as the ball leaves the cannon, and the ball retains a uniform velocity which would carry it over

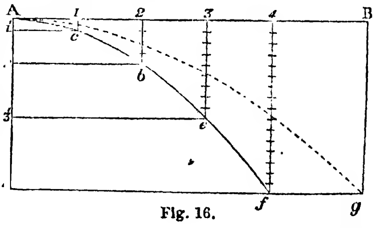


Fig. 16.

equal spaces in equal times. Let A1 be the space the ball would describe in a second, if acted on only by the projectile force, and set off a number of equal spaces along AB. Now from the moment the ball leaves the mouth of the cannon, it is

acted on by gravity; and while moving in the direction of AB, also falls; so that at the end of a second it must be 16 feet lower than it would have been if gravity had not acted—that is, if we take the perpendicular line 1*c* to represent 16 feet, the ball will at the end of the first second be at *c*. The horizontal motion being uniform, the ball at the end of the second second would be at 2, or twice as far towards B; but its downward accelerated motion has in the meantime dragged it down through three spaces more—in all four spaces—of 16 feet below the horizontal line. Making 2*b*, therefore, equal to four times 1*c*, gives *b* as the point where the ball will be at the end of the second second. By similar reasoning, if 3*e* is made equal to nine times 1*c*, 4*f* equal to 16 times 1*c*, &c., according to the law of accelerated motion; *e*, *f*, &c., will be the points at which the ball will be found at the end of the third, fourth, &c., seconds. The resultant of these two dissimilar motions, the one uniform, and the other uniformly accelerated, is a curve of the kind called a parabola, as represented in the figure.

140. From the above investigation, this remarkable consequence follows: that a body projected horizontally from a height above a level plane, comes to the ground as soon as if it were let fall perpendicularly. The ball reaches *f* in the same time that it would fall from A to 4′. A greater velocity of projection would make it take a wider flight; but its horizontal motion does not interfere with its downward motion, and at the end of four seconds it must be at some point in the same horizontal line—at *g*, for example. Two balls, then, projected horizontally from the same height above a level plane, though the one may range only a mile, and the other two miles, will reach the ground at the same time.

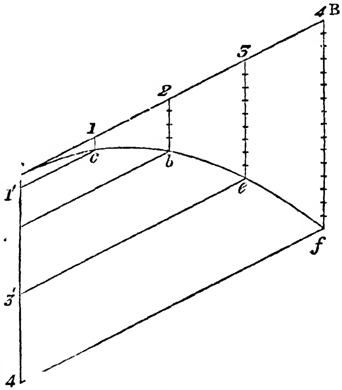


Fig. 17.

141. Projectiles are mostly thrown not horizontally, but in an oblique direction. We arrive, however, at the path described in the same way as in the last case. AB, in fig. 17, is the line of projection as before, and 1, 2, 3, 4, are the points to which the velocity of the ball would carry it in 1, 2, 3, 4

seconds; while 1*c*, 2*b*, 3*e*, 4*f*, represent the distances which gravity will drag it downward from its original direction in the same times respectively. When the elevation of the direction and the velocity are given, Conic Sections teaches us how to find the distance and time of flight of a projectile, the direction of the ground being also supposed known.

142. The laws above arrived at respecting projectiles are strictly true, only on the supposition that the movements are made in empty space. But every projectile has to encounter the resistance of the air; and that resistance becomes so great when the velocity is very high, that in the practice of gunnery, the theory is of little value. With a small velocity, however, a body thrown through the air describes a path not differing much from a parabola.

143. Since the distance of the flight, and the width of the curve described by a projectile, increase with its initial velocity, we can conceive the velocity increased until the curve became as large as that of the earth itself. If this were the case, the projectile, instead of falling, would, if the resistance of the air were removed, continue to go round and round the earth for ever. We thus arrive at the idea of planetary motion. The moon, for instance, is constantly falling towards the earth, like a cannon-ball shot horizontally; but the projectile velocity which it had from the beginning, and which, as it moves in empty space, it retains undiminished, is sufficient to carry it clear round in the curve called its orbit.

Centre of Gravity.

144. Every body is composed of a number of atoms or material points, and the force of gravity acts upon each separate point. If they were like mere grains of sand, each would obey the force independently of the others; and if part were obstructed, the others would still move on. But the result of cohesion is, that all the separate forces of the atoms are compounded into a single force, acting on a single point; so that in a falling body it is as if this point moved with a force equal to the sum of the forces of all the individual points of the body, and dragged all the rest along with it. If this resultant, then, is opposed by an equal force, acting on the same point, the whole body will be kept at rest, or in equilibrium. This point is the centre of gravity, and a knowledge of it is of great importance.

145. When a rod of uniform thickness and density is suspended by its middle point, it remains at rest. There is as much matter on the one side as the other; the atoms balance

one another in pairs; and it is as if the whole were collected at the point of suspension. The centre of gravity of such a rod is the central point at its middle part; and if the substance of the rod is pierced, and this central point rested on a fine needle, the rod will remain immovable in whatever position it is put.

146. Whenever bodies are regularly shaped, and of uniform density, their centres of gravity may be found by mathematical measurement, as in the case of the rod. Thus the centre of gravity of a globe is evidently in its middle point, or centre of dimension. In whatever direction a plane is made to pass through the centre, it cuts the sphere into two equal parts, and every particle in the one half has an equal and corresponding particle in the other similarly situated, so as to balance it. Suspend the centre, therefore, and the globe remains at rest, whichever side is uppermost. Similarly, the centre of gravity of a block of wood, whether cubical or oblong, is in its middle point, which is easily found by measurement.

147. The position of the centre of gravity in a triangular-shaped body is not so obvious. It is thus found. If ABC represent the surface of a thin triangular board, we may suppose it composed of a number of small rods laid side by side, the line AB representing the first, and the others gradually diminishing in length to the top. Now, the centre of gravity of the rod AB is in its middle point D; and if we join the points D and C, the line will pass through the centres of gravity of all the other rods; because it is a property of a line drawn from the top of a triangle to the middle of the opposite side, to bisect or pass through the middle of all lines in the triangle parallel to that side. The centre of gravity of the whole, therefore, must lie somewhere in the line CD. Again, by conceiving the triangle made up of rods parallel to the side CB, and drawing a line from A to the middle point E, we see that the centre of gravity must, in like manner, lie somewhere in the line AE. Now, since it lies also in CD, it can lie only where the two cross—namely, at g. We have thus found the centre of gravity of the surface, as it were; and if the board have any sensible thickness, its actual centre of gravity will lie behind g, at the distance of half the thickness. It is one of the properties of the triangle, that the line Dg, determined as above, is always one-third of the whole line DC; thus the centre of gravity of a triangle is readily found.

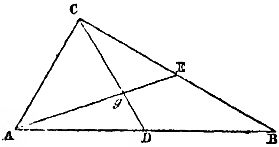


Fig. 18.

148. In a solid cone, the centre of gravity lies in the line

drawn from the top to the centre of the base, at the distance of $\frac{1}{4}$ of the whole line from the bottom (fig. 19.)

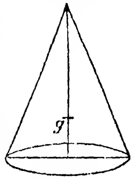


Fig. 19.

149. When two balls are connected by an inflexible rod, they form but one body, and the common centre of gravity is not situated in either, but between them in the rod joining their centres. If the balls are equal in mass, it will evidently be in the middle of the rod; but if they are unequal, the point where they balance each other is found to lie nearer to the larger mass, as will be more fully explained in MECHANICS, when speaking of the lever. If, for example, the balls weigh 3 and 2 pounds, and the line joining their centres be 5 inches long, the centre of gravity will be 3 inches from the smaller ball, and 2 inches from the other; the distances are inversely as the masses. By means of this principle, we may find the common centre of gravity of any number of connected bodies, by first finding it for two, then taking this centre as that of a single body equal to the other two, and so on.

150. When the centre of gravity of a body is supported, it is at rest, or in equilibrium; but according to the way in which the support is given, the equilibrium may be one of three kinds. Let the annexed figure represent a coin or a disk of card-board, of uniform thickness, pierced with three holes *a*, *b*, *c*, one, *a*, being through the centre of gravity. If the disk is suspended on a pin through *a*, it remains at rest in any position given to it; this is *indifferent* equilibrium. If the pin is put through *b*, and the disk moved aside, so as to make *a* describe either half of the dotted curve, as soon as let go, it will swing back, and *a* will finally rest directly below *b*; for the centre of gravity descends, if it can, though other parts should ascend. This is *stable* equilibrium. If the pin passes through the lower hole, the equilibrium is *unstable*; because, unless *a* is directly over *c*, it is unsupported, and descends to the other side.

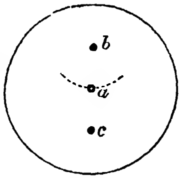


Fig. 20.

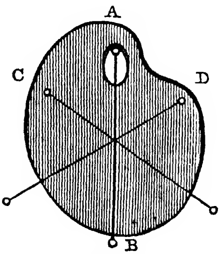


Fig. 21.

151. This tendency of the centre of gravity to seek the lowest point, enables us readily to find the centre of gravity of an irregular body. For example, let a painter's pallet be suspended from the thumb-hole, as in fig. 21;

we know that the centre of gravity must be perpendicularly below the point of suspension, and if we allow a plumb-line or a straight rod to hang down from that point in front of the pallet, and mark the line under it, the centre of gravity must be on that line. Next, suspend the pallet from any other point D, and apply the plummet as before, and another line is found also containing the centre of gravity, which must therefore be where the lines cross. If a third line of direction is tried, it will be found to pass through the same point.

152. The stability of objects resting on the ground depends on the position of the centre of gravity with respect to the base.

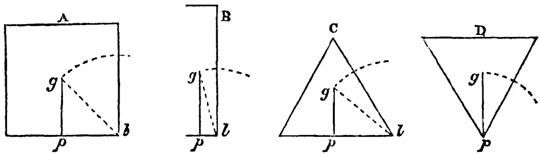


Fig. 22.

In the above figures, representing sections of various solid bodies, *g* marks the centre of gravity, and *gp* the perpendicular, or *line of direction*, in which the centre of gravity presses. It is evident from inspection that so long as the line of direction falls within the base, and that base has any breadth at all, the body will stand, if let alone; for before it can fall over, the centre of gravity must describe a curve, about one corner, as *l*; and as the curve begins by ascending, such a movement is resisted by the weight of the body itself. Bodies so situated require force to overthrow them; and that force is greater the more rapidly the curve ascends at the outset, which depends on the lowness of the centre of gravity and the width of the base. Hence, A is more stable than B. The pyramid C is the most stable of all figures. D is the emblem of instability, and the reasons are obvious.

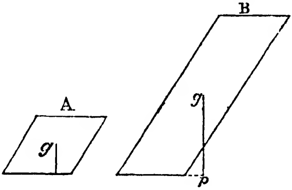


Fig. 23.

153. A structure may overhang its base within certain limits. In A (fig. 23) the centre is still supported, and would have to *rise* before the body could tumble; in B, the centre is unsupported, and moves in a descending curve from the first. The instability

ıere arises from the *height* of the object, for the slope and ɔase are the same in both.

154. This explains why vehicles when loaded high, or made *top-heavy*, are so easily upset. The annexed cut represents a cart crossing an inclined plane; or we may suppose that one wheel is tilted up in passing over a stone. With a load of stones, the centre of gravity would be about C, and the line of direction falling just under the wheel, the cart would escape being upset, though it might be narrowly. But if the load is such as to throw the centre of gravity into the position C′, the vehicle must fall over.

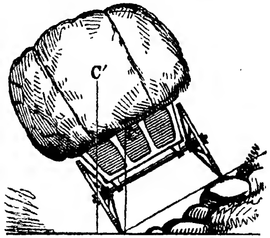


Fig. 24.

155. When a man stands on both legs, the centre of gravity falls between the feet; and the stability is increased by placing the feet apart. In resting on one foot, the body must be thrown over that foot; hence walking requires that the body be swayed slightly from side to side. In persons of a broad build, this oscillating motion is necessarily greater than in others, and hence the gait called a *waddle*. The walk of the duck is an extreme case of the same thing.

156. Animals while standing or walking require a constant succession of efforts and adjustments to keep the centre of gravity above the points of support. We make these efforts ordinarily without attending to them, and by a sort of instinct; but their necessity is seen when a man falls asleep on his legs; he cannot stand without leaning against a support. Drunk persons are incapable of making these adjustments exactly, their perceptions being blunted and deranged, and hence they stagger about.

157. It requires conscious efforts and great art to preserve the equilibrium of the body when the base is greatly narrowed; as in walking on stilts, and still more in rope-dancing. By means of a long pole, loaded at the ends, the rope-dancer can more suddenly restore his balance.

158. Our perceptions and appreciations of the beautiful, both in nature and art, appear very much to depend on the objects we contemplate being constructed in reference to the preservation of their equilibrium or balance. An erect or properly balanced wall, steeple, or pillar, is more grateful to the sense of sight than if it were leaning to a side. We feel as if an object, in leaning, were doing a violence to nature.

The Pendulum.

159. Any heavy body, such as a ball, suspended by a string or rod, so as to swing freely, constitutes a pendulum. Swinging is no less an effect of gravity than falling; and from the importance of the pendulum, as a measurer of time, the phenomenon deserves attentive study.

160. In the accompanying cut, a pendulum of the most common construction is represented. A is the axis or point of suspension; B is the rod; C is the *bob*, consisting of a ball, or a round flattish piece of metal, which is fastened to the rod by a screw behind, and by which screw it can be raised or lowered on the rod; DD′ is the path or arc which the ball traverses in swinging. When the pendulum is at rest, it hangs perpendicularly, as here represented.

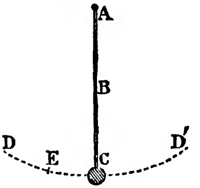


Fig. 25.

161. If the ball is now drawn to one side, as to D, gravity urges it downwards, while the tension of the rod draws it towards A. The composition of the two forces makes it describe a descending curve to C, where gravity is completely counteracted by the opposite pull of the rod. But the ball does not stop here; it goes on, by the law of inertia, in the ascending curve CD′, until gravity destroy its acquired motion. If friction and the resistance of the air did not interfere, it would take as long to destroy the motion as to generate it, and the ball would reach the same elevation as it started from. But these causes gradually render the ascent less and less, and at last bring the pendulum, when it has no maintaining power, to a state of rest.

62. One sweep of a pendulum from D to D′ is called an *oscillation;* and the path it describes, being part of a circle, is called its *arc*. The size or *amplitude* of the oscillation is measured by the number of degrees in the arc, each degree being a 360th part of the whole circle. For reasons which will appear, pendulums are generally made to describe small arcs, not exceeding 5° or 6°.

163. The most remarkable property of the oscillations of the pendulum, and that on which its use as a regulator of movement depends, is that, whether long or short, they are all performed in very nearly the same time. If we observe the vibrations of any body set aswinging, we find that though the arc it describes is continually diminishing, there is no sensible shortening of the time in which the single sweeps are

accomplished. As the journey becomes less, so does the velocity. Galileo is said to have been the first that distinctly noted and investigated this important fact. The cause is easily seen. The wider the sweep of the ball, the steeper is its descent at the beginning, which gives it a greater velocity, and enables it to go over the longer journey in the same time as over a shorter. If we take a moderately short arc, such as CD, the steepness of descent at D is almost exactly double of what it is at E, its middle point; so that a ball beginning its motion at D moves twice as fast as one starting from E, and thus both arrive at C at the same time.

164. But it is only short oscillations that are thus *isochronous*, as it is called; when the arcs are large, the steepness does not increase in exact proportion to the length, and therefore the isochronism is not perfect. Accordingly, pendulums are made to swing in short arcs; and then, though no contrivance could make the extent of the oscillations exactly uniform, the times are virtually equal.

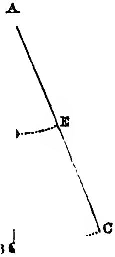


Fig. 26.

165. But though the time of oscillation is not affected by the largeness of the arc, it is by the length of the pendulum itself. Long pendulums vibrate more slowly than short ones. Though the balls B and D (fig. 26) have the same amplitude of vibration, or go over corresponding arcs, the journey of the one is longer than that of the other. But the steepness of descent, or inclination of the path, is the same in both; therefore B must take longer time to perform its journey than D. We must not, however, conclude that when the length of the arc, or, which is the same thing, of the pendulum, is doubled, the time of oscillation is also doubled. The motion of the pendulum is an accelerated motion; and, as in all other uniformly accelerated motions, the spaces described are as the squares of the times. To give double the time of vibration, then, requires the pendulum to be four times as long; treble the time, nine times as long; and so on.

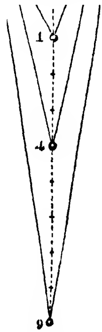


Fig. 27.

166. The truth of this is easily proved by experiment. Suspend three musket-balls on double threads, as in the figure, so that the lengths measured in the dotted line may be as 1, 4, and 9. While the lowest ball makes one oscillation, the highest will be found to make three, and the middle ball two.

167. A pendulum of a little more than 39 inches beats seconds, and one of one-fourth that length beats half-seconds. As a pendulum that beats seconds must always be of the same length, it forms a fixed standard of measure, which can be found again when artificial standards are lost.

168. When we say that the seconds' pendulum is always of the same length, we must be understood to speak of the same place. In different places, its length varies. It will be readily conceived that as the motion of the pendulum is caused by gravity, any alteration in the force of gravity must alter the rate of its movement. Now the force of gravity is different in different parts of the earth. The whole force of the earth's attraction is lodged, as it were, in the centre, and its intensity lessens with distance. But at the equator, owing to the shape of the earth, we are 13 miles further from the centre than at the poles; and therefore the seconds' pendulum must be somewhat shorter at the equator, and grow gradually longer as the latitude increases. Besides the effect of greater distance from the centre, gravity is also lessened towards the equator by the greater centrifugal force which prevails there.

169. The following are the lengths of the seconds' pendulum at a few stations, embracing a considerable range of latitude :—

Spitzbergen, ...	79° 49′ 58″ N. Lat.,	39·2146 inches.
Edinburgh, . . .	55 58 40 "	39·1554 "
London, ...	51 31 08 "	39·1390 "
Jamaica, ...	17 56 07 "	39·0350 "
Sierra Leone, . . .	8 29 28 "	39·0195 "

170. How is the length of a pendulum measured? Is it from the point of suspension to the bottom of the ball, or to what point? This leads us to consider what is called the *centre of oscillation*. We have as yet considered gravity as acting only on the ball, because the greater part of the matter is concentrated there. But the matter composing the rod is concerned in the movement, and were there no ball, the rod would form a pendulum of itself. Now, if each particle of matter in the swinging body were at liberty to swing separately, those particles that are nearer to the axis would move more rapidly than those that are further off. But as the whole cohere in one mass, this tendency is checked; the motion of nearer particles is retarded by the more remote, and the motion of the more remote is accelerated by the nearer. There must, however, be a point dividing the particles that are moving slower than their natural rate from those that are moving faster than their natural rate, a point which is moving exactly as a particle there situated would

move if free to vibrate alone. This point is the *centre of oscillation*.

171. It is evident that the whole of the matter composing a swinging body may be considered as acting at this point; and if it were all concentrated there, the rate of vibration would be the same. The length of a pendulum, then, is measured from the centre of suspension to the centre of oscillation.

172. We may find pretty nearly the centre of oscillation of a common pendulum, or of any pendulous body, by suspending in front of it, and from the same axis, a small ball of lead attached to a fine thread. This ball and thread form what is called a *simple* pendulum, because the weight of the thread may be reckoned as nothing, and the matter is as nearly as possible in one point. Both bodies are now made to swing, and the thread is lengthened or shortened till the two vibrate in exactly the same time; when they come to rest, the centre of the spot on the pendulous body covered by the ball is the centre of oscillation.

173. A bar of wood or metal of uniform thickness, may be suspended as a pendulum, and its centre of oscillation determined in the way described. If its position is now reversed, and the centre of oscillation made the centre of suspension, it is found to vibrate in exactly the same time as before. This is expressed by saying that the 'centres of oscillation and suspension are interchangeable.'

174. By making the centre of oscillation the centre of suspension, we shorten the pendulum, and might therefore expect that the time of vibration would be shortened. But in this arrangement a portion of the matter is thrown above the axis, and this acts as a check, and proportionally retards the movement.

175. A pendulum alone, without wheel-work, would form a time-keeper, if we took the trouble to observe and count its vibrations, and if friction and the resistance of the air did not, after a time, bring its motion to an end. The use of the wheel-work in a clock is to answer these two ends—to count and record the swings of the pendulum, and to act as a *maintaining power*—that is, to supply to the pendulum fresh motion in place of what is constantly being destroyed. It is still the pendulum that measures the time.

176. In ordinary clock-work, the wheels are put in motion by a heavy weight which is attached to a cord wound round a barrel. One of the wheels is so placed that a piece of metal, fixed on the top of the pendulum, and shaped something like

the claw of an anchor, projects its ends, alternately as the pendulum swings, between two teeth of the wheel, first on one side, then on the other. Thus the wheel is held in check, and only one tooth allowed to pass at each swing; and it is evident that an index or seconds' hand fixed to the axis of this wheel will record the vibrations of the pendulum. But while the pendulum thus regulates the rate at which the wheel is allowed to move, the teeth of the wheel in contact with the anchor of the pendulum, give it a push as they are disengaged from its ends, and so communicate just as much moving-power to the swinging part as it is losing by friction. Such a contrivance for bringing the pendulum into connection with the wheel-work, is called an *escapement*, of which there are several varieties.

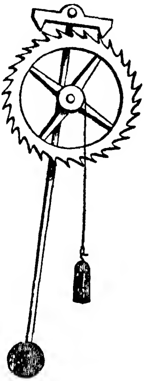


Fig. 28.

177. For adjusting the length of the pendulum, the ball is made to slide on the rod by means of a fine screw. A difference in length of the 1000th part of an inch causes an error of about a second a day. Since all substances are expanded by heat and contracted by cold, changes of temperature must affect the rate of clocks, making them go slower in summer than in winter. *Compensation* pendulums, accordingly, have been contrived, in which expansion in one part is made to counteract expansion in another part.

178. To save space, time-pieces are often regulated by pendulums one-fourth the ordinary length, and therefore beating half-seconds. But a long pendulum, with a heavy bob or ball, is desirable where evenness of rate is the object. A pendulum beating two seconds keeps time much more accurately than one beating single seconds.

CENTRIFUGAL FORCE AND CIRCULAR MOTION.

179. Motion in a circle, or in any other curve, is something constrained. Free motion is naturally *straight*. This, we have seen, is involved in the idea of inertia. To make a body in motion deviate from the straight line, requires a fresh force in a different direction from the straight line in which it is moving; and if we see a body moving in a circle, which is a constant series of deviations from the straight line, we may conclude there is a constant force acting on it to produce the bending, in addition to the impulse or impulses that urge it

onward. Accordingly, when we make a ball whirl round rapidly at the end of a cord, we feel the cord stretched with a sensible force; the ball is pulling outwards, and the hand is pulling inwards. These two opposite pulls must of course be exactly equal, for they are a case of action and reaction. The line of direction of both passes through the centre of the circle, and therefore they are called *central forces*. The outward pull of the ball is called the *centrifugal*, or centre-flying force; the inward pull of the hand, the *centripetal*, or centre-seeking force. The tension of the cord is the measure of both forces.

180. When a body, constrained to move in a circle, is released from the restraint, what is the result? Suppose the ball B retained by the cord AB, and moving in the direction CEG, &c.; if relieved from the restraint of the cord, it does not fly directly away from the centre, in the line AB prolonged. It has still its onward motion in the direction of the curve, and the only effect of the release from the centripetal force is, that that motion ceases to be bent, and goes on straight in the direction it had at the instant. Now, at any point in a circle, the direction of the curve is that of a straight line drawn through the point at right angles to the diameter. Such a straight line is called a *tangent;* and thus motion in a circle, when released from the constraining force, becomes motion in the tangent, or flies off at a tangent. At C, for instance, the direction of the ball, when set free, would be CD; at I it would be IK.

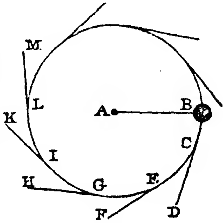


Fig. 29.

181. The tendency to fly off from the centre and at a tangent is exemplified in a multitude of phenomena. A stone let go from a sling is the most familiar instance. In grinding corn, it is the centrifugal tendency that makes the corn find its way from the centre, where it is admitted, gradually towards the circumference, where it escapes as meal. When a vessel with water in it is rapidly turned round like a horizontal wheel, the water recedes from the centre, and rises up round the sides. The potter takes advantage of this in his art: the soft clay is kept rapidly rotating while he is moulding it, and thus it tends of itself to widen out. The same is the case in glass-making, and many other arts.

182. In equestrian performances in a circus, both rider and horse incline their bodies inwards; they are leaning against a force which is impelling them outwards, and which thus

sustains a part of their weight. A skater must do the same in describing a curve. Centrifugal force may be made altogether to counteract gravity: a tumbler of water placed in a sling may, by beginning with short swings, be at last made to go round and round in a vertical circle, where at one part the mouth is downward, without spilling a drop.

183. If a round ball of soft clay be turned rapidly on a spindle, it ceases to be perfectly round in all directions: it bulges out at the middle and shortens in the direction of the spindle. This has happened to our earth; and to a still greater degree to Jupiter and Saturn, which have a more rapid rotation on their axes.

184. The centrifugal force of a body moving in a circle varies in the following way:—1*st*, It increases with the mass of the body; a ball of two pounds' weight resists the bending of its straight course with twice the force that a ball of one pound does. 2*d*, It increases with the circle described. If two balls of equal weight describe their orbits in the same time, but the orbit of the one be twice the diameter of that of the other, it is clear that in any portion of time, as in a second, the first has to be drawn from the straight course twice as far as the second; this requires twice the centripetal force, and hence the centrifugal force is also double. 3*d*, If two equal balls move in equal circles, but the one with double the velocity of the other, or make two revolutions for the other's one, then the centrifugal force is not double, but quadruple. For if we suppose the slower of the two to be deflected from the straight line one inch in a second, the faster suffers two deflections of the same extent in the same time; but to accomplish them in the same time, they must receive each double energy, and thus the two deflections require four times the energy of the one. Thus the centrifugal force increases as the *square* of the number of revolutions.

185. We have seen (par. 139) that the earth's gravitation deflects a projectile from the tangent 16 feet in a second. Now, it can be calculated, by mathematics, how far a body moving in a given circle, at a given rate, is deflected from the tangent in a second, and thus we can compare the weight of the body with its centrifugal force. It can be shewn, for instance, that a body weighing 2 pounds revolving four times in a second in a string 3 feet long, stretches the string with a force of nearly 118 pounds. If any body revolve once in two seconds in a string of that length, its centrifugal force will be about equal to its weight. It is calculated that if the rotation of the earth were seventeen times faster than

it is, centrifugal force at the equator would be equal to gravity; in other words, all bodies would be completely without weight, and a little increase of velocity would throw them off, to circle round like small satellites.

186. If a solid body is pierced by a straight rod, and made to turn on it as on an axis, every particle of matter in the body describes a circle about this axis. All these circles are described in the same time, and therefore the centrifugal force of each particle is in proportion to its distance from the axis. Such a body may be so shaped, that whatever point we take in its axis, the centrifugal forces of all the particles revolving round that point as a centre, balance or neutralise one another. In this case, the axis suffers no strain or pressure from the centrifugal forces, and has no tendency to change its position.

187. Instances of this symmetrical disposition about an axis are furnished by a ring or wheel, a sphere on any diameter, a cone, a cylinder, a cube, a rectangular prism, or pyramid, &c. Without any solid rod or fixed axis, a body of this kind once set agoing continues to rotate about the same imaginary line.

HEAT.

188. To account for the appearances presented by matter in its several forms of solid, liquid, and gas, we have seen it necessary (par. 49) to assume that there is a *repulsive* force at work among its molecules, counteracting and modifying the attractive forces. This repulsive force would seem to be identical with heat. Heat expands bodies; it overcomes the cohesion of their particles, converting solids into liquids, and liquids into gases; without it, there would be only one form of matter, the solid, and all life and motion would cease on the earth.

189. Perhaps no class of natural laws are more interesting in themselves, or more practically important, than the laws of heat. A knowledge of them is of direct application in every art and handicraft, as well as necessary for domestic economy and personal comfort. A full discussion of the subject would require a volume; it is only the most general facts that the limits of this treatise permit to be stated.

EXPANSION.

190. The greater number of bodies, whether solid, liquid, or gaseous, which are not decomposed by heat, are expanded by it: in other words, occupy a greater space when hot than when cold. Solids expand least, gases most, and liquids—speaking generally—are intermediate between them in expansibility.

191. To illustrate the expansion of solids, a rod of iron may be taken, and its length and diameter exactly measured at the temperature of the air. If it be now raised to a red heat, it will be found to have suffered an increase in length, and to be too wide to fit an aperture through which it passed before. When allowed to cool to its original temperature, it will exactly recover its previous dimensions.

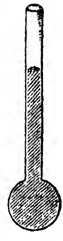


Fig. 30.

192. The expansion of liquids is familiarly illustrated by heating a glass flask filled with any liquid. The liquid rapidly expands, and manifests its expansion by running over. Or a long glass-tube, with a hollow ball at one end of it, may have the ball filled with spirit of wine, or mercury, and then be plunged into hot water. The liquid will indicate its expansion by rapidly rising in the tube or stem of the ball. If it be now removed from the hot water, the enclosed liquid, as it cools, will descend till it reaches the point which it occupied at first.

193. The expansion of gases may be illustrated by inverting a flask with a long neck in water—the water being left standing to some height in the neck of the vessel. If a hot coal or heated plate of metal be now approached to the flask, the liquid will rapidly descend, forced before it by the expanding air. When the heat is withdrawn, the water rises to its previous height.

Fig. 31.

194. Nearly every solid and liquid has an expansibility peculiar to itself. Among solids, the metals are the most expansible bodies. Zinc expands most, platinum probably least, among bodies of the metallic class. Glass, brick, porcelain, marble, and stone, have small expansibilities.

195. If a rod of iron which measures 819 lines in length when as cold as melting ice, is made as hot as boiling water, it is found to measure 820 lines. Between the freezing and boiling points, then, iron increases $\frac{1}{819}$ of its length, and this fraction

is called the *coefficient* of its expansion in length, or of its *linear* expansion. The following are the coefficients of the linear expansion of some of the metals, and of glass:—

Platinum, . . .	about	0·00086	or $\frac{1}{1167}$
Glass, on the average, .	"	0·00087	" $\frac{1}{1147}$
Steel (hard), . . .	"	0·00124	" $\frac{1}{807}$
Iron, ...	"	0·00122	" $\frac{1}{819}$
Copper, ...	"	0·00171	" $\frac{1}{584}$
Tin, ...	"	0·00217	" $\frac{1}{462}$
Lead, ...	"	0·00285	" $\frac{1}{351}$
Zinc, ...	"	0·00294	" $\frac{1}{340}$

196. As the rod of iron expands not only in length, but also in breadth and depth, it is obvious that its bulk or volume must be increased in a greater ratio than its length; an expansion, in fact, of $\frac{1}{819}$ of the length can be shewn to imply an expansion of about $\frac{3}{819}$ of the volume.

197. Among liquids, we find those which are most volatile more expansible than others. Thus, spirit of wine is six times more expansible by heat than mercury. The liquefied gases, the most volatile of known bodies, are more expansible in some cases than even air.

198. Gases, unlike solids and liquids, have not specific expansibilities, but each undergoes almost the same amount of expansion for the addition of the same amount of heat. Thus, if we were to take common air, hydrogen, and carbonic acid, and heat them equally, we should find that they all suffered the same amount of expansion; whereas, if we took any three solids or liquids, and heated them equally, we should find that each expanded to a different extent.

199. When heated from 32° to 212°, water dilates $\frac{1}{23}$ of its volume, mercury $\frac{1}{55}$, and alcohol $\frac{1}{9}$; air or any other gas about $\frac{1}{3}$. An increase of 1° of temperature, therefore, increases a body of air by $\frac{1}{490}$ of its bulk.

200. Water presents a singular irregularity in its expansions and contractions. If boiling water is taken, and allowed gradually to cool, it follows the general law, and goes on contracting until it is within a few degrees of freezing (at 39°); it then begins to dilate, and continues to do so till it come to 32°, the freezing-point. At the moment of becoming solid, it undergoes a sudden enlargement. It is this enlargement of freezing water that causes it to burst pipes and vessels in which it is confined; it is also the reason that ice is lighter than water, and floats on the surface. Ten cubic inches of ice weigh as much as nine cubic inches of water. It thus appears that water at 39° is at its point of greatest density, and will expand whether it be heated or cooled.

201. In laying the rails on a railway, and in all structures where metal is used, allowance must be made for the expansion and contraction of the metal by change of temperature. If iron rails were fastened down with their ends in close contact in a cold day, they would, when it became warm, press against one another, and become bent and twisted. Hence, a small space is left between the ends.—The irregularity in the rate of clocks and watches already mentioned (par. 177), is another instance of inconvenience arising from this law. The expedients by which it is remedied are explained in CHRONOLOGY and HOROLOGY.

202. The practical mechanic avails himself of the law of expansion in putting on the tires or rings of wagon-wheels and iron hoops on vats. Being made a degree too small at first, the tire or hoop is heated to redness, and in this state driven on; it is then cooled with water, when it contracts, and draws the parts of the wheel or vessel together with an irresistible force.

THE THERMOMETER.

203. The thermometer is an instrument in which *temperature*—that is, *the intensity of heat*—is measured by the amount of expansion it produces. The most convenient substances for the construction of thermometers are found to be mercury and alcohol, or spirit of wine. For ordinary temperatures, mercury is preferable; but when too much heat is withdrawn, it freezes or becomes solid, and therefore, for very low temperatures, alcohol is used, which cannot be congealed by any known cold.

204. The mercury or alcohol is enclosed in a glass vessel, consisting of a tube with a hollow bulb at one end, the other being closed; the liquid rises or sinks in the stem according as heat is added or abstracted. But in order that the thermometer may tell us, not only that one body is hotter or colder than another, but *how much* hotter or colder, it must be *graduated*, or have points marked upon it corresponding to certain invariable temperatures. This is done by first plunging it in melting ice, and marking on the glass, or on an ivory scale attached to it, the point at which the mercury comes to stand. This mark is the *freezing-point* of water, for water freezing and ice melting have the same temperature. The thermometer is next placed in boiling water, when the mercury rises to a certain height, and then continues steady. A second mark is here made, which is the *boiling-point*.

205. The space between these two points is then divided into a number of equal parts, called *degrees*, and parts of the same

length are set off above and below the boiling and freezing points, as far as required.

206. In the thermometer used in this country, the space between the freezing and boiling points is divided into 180 equal parts, and we begin counting at 32 degrees below the freezing-point. A cipher is placed there, and it is called the zero or nothing point of the thermometer. The freezing-point of water thus comes to be marked by the number 32°, and the boiling-point, which is 180° higher, by 212°. Thus, we say that a mixture of salt and snow reduces the thermometer to 0°, that the freezing-point of water is 32°, and that its boiling-point is 212°. In the thermometer chiefly used on the continent, the space between the freezing and boiling points of water is divided into 100 equal parts, and the graduation begins at the freezing-point, which is marked 0°, or zero. According to this thermometer, which is called the centigrade, water freezes at 0°, and boils at 100°.

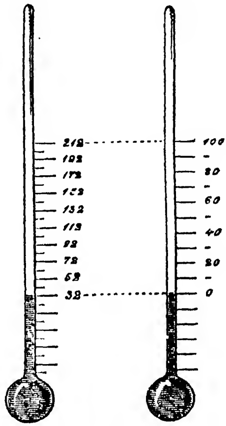


Fig. 32.

207. The centigrade thermometer is now much employed in scientific researches in this country. To prevent any confusion arising from its being mistaken for the thermometer first described, which is called, from its original maker—who was a Dutchman—Fahrenheit's, or the Fahrenheit thermometer, the letter F is placed after temperatures indicated by his thermometer, and the letter C after those denoted by the centigrade. Thus, water freezes at 32° F., or 0° C.; water boils at 212° F., or 100° C.

208. From its zero-point each thermometer counts downwards as well as upwards; and to distinguish the degrees below zero from those above it, the former are distinguished by prefixing to them the minus sign —. Thus, mercury is said to freeze at — 40° F.; that is, at 40° below Fahrenheit's zero.

209. Another scale much used in Germany is that called Reaumur's, in which the freezing-point is marked 0°, and the boiling-point 80°. The degrees on this scale are thus larger than those of Fahrenheit, or even of the centigrade: 9° F. = 4° R.,

or 5° C.; and by means of these proportions, a temperature stated in one scale may be reduced to either of the others, care being taken to allow for Fahrenheit's scale commencing, not at the freezing-point, as the others do, but 32° below it.

210. Examples of converting degrees of one scale into those of another:—

$$20^\circ \text{ C.} = \frac{20 \times 9}{5} + 32 = 68^\circ \text{ F.}$$

$$20^\circ \text{ R.} = \frac{20 \times 9}{4} + 32 = 77^\circ \text{ F.}$$

$$68^\circ \text{ F.} = \frac{(68 - 32) \times 5}{9} = 20^\circ \text{ C.}$$

$$77^\circ \text{ F.} = \frac{(77 - 32) \times 4}{9} = 20^\circ \text{ R.}$$

211. A thermometer, the zero of whose scale is at the freezing-point, is undoubtedly more rational and convenient than Fahrenheit's. Fahrenheit chose the temperature of 32° below freezing as the zero-point of his scale, because it was the lowest that had then been observed, and was considered to indicate the complete absence of heat; but it is now known that there are natural temperatures at least 90° below this; and by artificial mixtures a cold has been produced of — 146° F. This last temperature is said to reduce alcohol to the consistency of melting wax, indicating that it is near its freezing-point.

SPECIFIC HEAT.

212. Different substances require different quantities of heat to raise them to the same temperature. This is expressed by saying that each possesses a *specific capacity* for heat, or, more shortly, a *specific heat.*

213. The fact can be proved in a variety of ways. Thus, if we cause equal quantities of bodies which have all been raised to the same temperature, to melt ice, we shall find that a much greater weight of it will be melted by one body than by another. Thus, mercury at 212° will melt much less ice than an equal quantity of water at the same temperature will, for the mercury has much less heat to give out, so as to produce liquefaction, than the water has.

214. Specific heats are generally stated with reference to equal weights, rather than to equal measures, of bodies. Thus, a pound of water, in rising to a given temperature, absorbs thirty times more heat than a pound of mercury in rising to the same temperature; so that the capacity of water for heat exceeds that of mercury thirty times; and if we call the

specific heat of mercury 1, that of water will be 30. In tables of specific heat, however, the scale is commenced with water, which is called 1000; and as it excels all other bodies in capacity for heat, their specific heats are invariably denoted by some lower number. We subjoin a few examples:

Substances.	Specific Heat of Equal Weights.
Water,	1000
Ice,	513
Wood-charcoal,	241
Sulphur,	203
Glass,	198
Diamond,	147
Iron,	113·79
Copper,	95·15
Silver,	57·01
Iodine,	54·12
Gold,	32·44

215. The great specific heat of water has a most important relation to the welfare of the living creatures on the globe. The sea, which spreads over so large a portion of the earth, cannot in the hot seasons of the year become rapidly raised in temperature, which would prove injurious alike to its inhabitants and to the dwellers on the dry land. Rapid elevation of temperature cannot occur, in consequence of the great amount of heat which must be absorbed by water before it can manifest even a moderate increase in warmth.

216. In the cold seasons of the year, on the other hand, the sea and other great beds of water cool slowly, and, moreover, in cooling, evolve much heat, which equalises the temperature of the air as well as that of the land. An ocean of mercury, even if otherwise habitable by the creatures with which we are familiar, would suffer so great an alteration in temperature during the twenty-four hours of a tropical day and night, as to prove fatal to multitudes, if not to all of them.

217. The specific heat of bodies is diminished by compressing them or making them more dense. By compressing a portion of air in a syringe, its capacity for heat may be so diminished that the quantity it previously contained, which only raised it to the ordinary temperature, is sufficient, after compression, to make it so hot as to set fire to a piece of amadou, or German tinder. On the other hand, when air is dilated by removing pressure from it, as in the receiver of an air-pump, or in the upper regions of the atmosphere, its capacity for heat is increased, and its temperature falls, the same amount of heat no longer sufficing to warm it to the same degree.

PROPAGATION OF HEAT.

218. Heat is transferred from one portion of matter to another in three different ways, which are termed Conduction, Convection, and Radiation.

219. *Conduction* implies the passage of heat from one particle of matter to another in physical contact with it.

220. *Convection* is the conveying or carrying of heat by particles of matter raised in temperature, and set in motion.

221. *Radiation* is the emission of heat by a body such, for example, as a mass of red-hot iron at rest, and not in physical contact with the substances to which it communicates heat. The name has reference to the supposition that the heat passes in radii, or rays, like those of sunlight, which can find their way even through a vacuum, and do not appear to require the stance of ponderable matter to determine their transference.

Conduction.

222. Conduction is best seen in solids, and particularly metals, which are the best conductors. A rod of iron placed with one extremity in the fire speedily becomes hot at the opposite extremity, owing to the conduction of heat from particle to particle along the rod.

223. Dense bodies are generally the best conductors; light and porous ones the worst. Feathers, down, fur, flannel, and most of the fabrics used for winter dresses, owe their so-called warmth to their low-conducting power for heat. Their action is altogether negative, being limited to the prevention of the rapid escape of heat generated by the living beings whose bodies they cover. Hence the best way to preserve ice, is to wrap it in flannel, or other so-called warm covering; for the same means that retard the escape of heat from the living body, retard the access of heat from the air and surrounding objects to the ice.

224. If the conducting power of gold is expressed by 100, that of copper is 90, of iron 37, of lead 18, of marble 2·3, of porcelain 1·2, of brick-earth 1·1. Vessels of porcelain and glass are liable to crack when hot water is suddenly poured into them, because these substances conduct heat slowly, and thus the part first touched by the water becomes expanded, while the parts adjoining are yet cold and unexpanded.

225. Liquids and gases are very bad conductors of heat, although, from the rapidity with which they rise in temperature, when heat is applied to them, they appear to be among the best conductors.

Convection.

226. Liquids and gases rise in temperature chiefly in consequence of the convection, not the conduction of heat by their particles. If a spirit-lamp is applied to the top of a tube filled with water, as represented in the fig., the upper portion of the liquid is soon heated to boiling, while hours will elapse before even a slight degree of heat will reach any distance down the column. But if the lamp is applied at the bottom of the tube, the heat is soon felt at the top, and the whole liquid is made to boil in a few minutes.

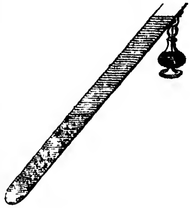


Fig. 33.

227. This remarkable difference is owing, not to any greater readiness that heat itself has to ascend than descend, for a rod of iron heats equally downwards and upwards, but to a motion that takes place among the particles of the liquid. If a pretty large glass flask be taken (fig. 34), and a few fragments of solid blue *litmus** dropped into it, after it has been filled nearly full of water, on applying heat by a small lamp to the bottom of the vessel, a central current of water, rendered distinctly visible by the blue colour it has acquired from the litmus, is seen to ascend till it reaches the surface of the liquid, when it bends over in every direction like the foliage of a palm-tree, and forms a number of descending currents. These keeping pretty near to the walls of the vessel, travel downwards till they reach the heated lower portion of it, when they again ascend as a hot central current. In this way the whole liquid is thrown into circulation,

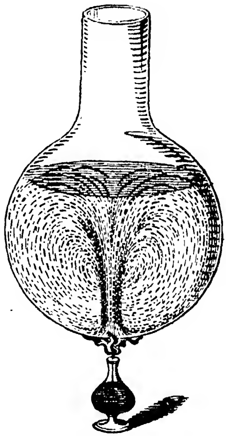


Fig. 34.

for every portion of it in turn becomes heated, ascends, and losing part of its heat in and after its ascent, chiefly by parting with it to the walls of the vessel, the air, and surrounding

* A colouring-matter prepared from certain lichens, and readily procured from any druggist, or dealer in dye-stuffs.

objects, descends as a cold current till it reaches the bottom, when it becomes heated again. The currents which are thus occasioned by heating a liquid are determined by the fact, that when any portion of liquid is warmed, it expands, and has, in consequence, its specific gravity lessened.

228. Air and other gases are raised in temperature in the same way that liquids are. They conduct heat very slowly, as may be proved by applying heat to the top of an air-tight glass vessel with a thermometer suspended a little below the heated portion. But when the heat is applied from below, currents of circulation immediately begin, as in the case of a liquid.

229. It is a popular belief that a large fire in the open air makes the wind rise; and so far it is quite true. The fire can only be maintained by constant lateral and descending streams of cold air to supply the place of the ascending column. When Moscow was burned, the wind rose so high, that men and horses could with difficulty keep their feet whilst passing through the burning streets.

230. Our sensations lead us into many errors regarding the temperature of bodies, which attention to their different conducting and conveying powers enables us to correct. Thus, of the several articles in the same room, the table feels colder than the cloth that covers it, and a marble slab colder than the table. They are, in fact, all of the same temperature –the temperature, namely, of the air in the room; but the marble being the best conductor, robs the hand most quickly of its heat.

231. Take a rod of iron and one of wood, and let both be of the temperature of 50°, as shewn by the thermometer; the iron will feel to the hand much colder than the wood. Let both be next heated, say to 150°; the iron will now feel the warmer of the two. The explanation is this: there is always a tendency to equality or equilibrium of temperature among neighbouring bodies, the colder gaining heat at the expense of the warmer. Now, in the first case, the hand, which, before touching the rods, is at blood-heat, or about 98°, imparts of its heat both to the iron and the wood; but the successive instalments are conducted rapidly from the surface of the iron into the interior substance, while the slower conduction of the wood makes the heat accumulate in its surface, which is thus quickly brought nearly to the temperature of the hand.

232. When the rods, on the other hand, are both warmer than the touching hand, the hand gets the excess of heat from the surface of both; but in the case of the wood, it is only slowly that fresh supplies can come from the interior, and thus

the temperature of its surface is brought to differ but little from that of the hand, while the interior of the iron supplies heat to the surface as fast as it is abstracted.

233. Again, a smooth piece of iron feels colder than a file, because the hand comes into closer contact with the smooth surface than with the file, which it only touches at a point here and there, thus giving but few channels of communication for the heat to pass along. This principle of few points of contact, explains why a fabric of loose soft texture makes a warmer garment than one of the same material that has been made hard and compact by twisting or other means.

234. It also explains partly why water should feel so cold, though we know that it is of the same temperature as the air in which it has stood for any length of time. Water touches the hand at far more points than any solid body can; and this, combined with the fact of its great capacity for heat, enables it instantly to rob the hand of a large amount. The cooling effect is greatly increased if the hand is moved through the water, bringing fresh unwarmed portions in contact with it in rapid succession. It is on this last principle that a degree of cold may be borne with a calm atmosphere, which becomes quite killing with even a moderate wind.

235. When air is confined and prevented from circulating, it forms a non-conductor and a warm protection. Hence the effect of double-windows, and of plaster put on laths, so as to enclose a plate of air between it and the wall, instead of being laid on the wall itself. Woolly coverings and furs imprison the air within their substance, and prevent it from circulating, while they afford but few points of solid contact for the direct conduction of heat. These two circumstances combined give them their remarkable power in arresting the escape of heat; and that power is greater the finer and lighter their texture. Swans' down is said to be the most perfect insulator of heat. From the same causes, snow is an excellent non-conductor, and, like a fleece of wool, protects the earth from any cold much below 32°.

Radiation.

236. Every hot body in the act of cooling, besides losing heat by the conductive and convective action of the solids and fluids in its neighbourhood, parts with much heat by radiation. Rays of heat pass away from the hot body till it has reached the temperature of the air or surrounding medium. In proof of this, it is only necessary to hang a hot body in the vacuum of the air-pump, when it rapidly

cools, although it does not lose heat by either conduction or convection.

237. The rate of cooling of a hot solid body, so far as radiation is concerned, is remarkably influenced by the state of its surface, and in the case of liquids and gases, by the state of the surface of the vessels containing them. Thus, hot water placed in a tin vessel coated externally with lampblack, cools twice as fast as it does in a bright tin vessel. Similarly, if two metallic vessels be taken, the one left bright, and the other covered with linen, hot water will be found to cool much faster in the covered than in the naked vessel.

238. From these observations, it appears that a kettle covered with soot is much less suited for retaining water warm, than if it had a polished metallic surface. So, also, bright metallic covers are the best at table, and metallic tea-pots and coffee-pots are preferable to those of porcelain and stoneware.

239. *Absorption and reflection of rays of heat.*—Rays of heat follow almost the same laws as to reflection, absorption, refraction, &c., as rays of light (see OPTICS). When they fall on the surface of a body, they either enter it or are *reflected.* Those, again, that enter are either *transmitted,* like light through glass, or are retained and *absorbed.* It is only the rays that are absorbed that warm the body; those that are reflected off, as well as those that are transmitted, produce no effect on the temperature.

240. If a red-hot iron ball, B, be placed in the focus of a concave metallic mirror M, its radiant heat will pass from it to the

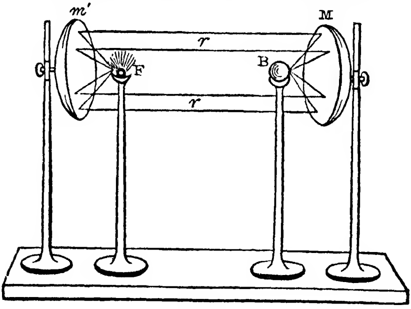


Fig. 35.

mirror, and be reflected in parallel rays r, r. If these rays be intercepted by a second mirror m', placed at the distance of a

few feet, they can be collected into a focus F, and made to act upon a thermometer, ignite tinder, phosphorus, &c. In this manner, it is possible to set fire to substances at a considerable distance from the source of heat. It is not necessary that the radiating body be luminous; when the ball is heated only to 300°, the radiation is still apparent when a thermometer is placed in the focus F. The temperature of the mirrors is scarcely affected by the rays of heat thus playing upon their surfaces; but if M were coated with lampblack, or even gum or paper, the thermometer would instantly fall, shewing that the reflection was arrested, while the mirror would soon grow warm from absorbing the heat.

241. It is found that surfaces that radiate heat best, also imbibe it most readily. If a table, then, is formed of substances according to their power of radiating heat, the same table will serve for their power of absorption. The following is such a table, the radiating and absorbing power of a surface of lampblack being expressed by 100:—

Lampblack,	100	Tin,	14
White lead,	100	Brass, polished,	7
Writing-paper,	98	Copper,	7
Glass,	90	Gold,	3
Polished iron,	23	Silver,	3
Mercury,	23		

Since all the rays not absorbed by a surface must be reflected, this table, read from the bottom upwards, will give the same surfaces in the order of their reflecting powers.

242. The capacity of radiating and absorbing heat depends essentially on the nature of the surface, and its condition as to roughness or smoothness. Heat is radiated and absorbed better by a light porous substance than by a dense—by a surface rough and covered with points, than by one smooth and polished. Colour by itself is not found to have any sensible influence, at least with non-luminous thermal rays.

243. All bodies, even the coldest, are constantly radiating off more or less heat, according to their temperature; all are therefore both giving and receiving rays; but the warmer give more than they receive, the colder receive more than they give. A surface presented towards the open sky, with nothing to radiate or throw back the rays it is emitting, soon becomes cold; but the slightest curtain, such as a net, hung up before it, sensibly arrests the dissipation of heat. It is in this way that clouds act as warm curtains to the earth, and often prevent frosts in spring and autumn nights.

244. The different radiating powers of bodies explains why

dew is sooner deposited on some substances than on others. Those that are good radiators lose their heat most quickly, and thus condense the vapour of the atmosphere.

245. *Transmission of Thermal Rays.*—Some substances, it has been observed, allow heat to pass through them, as light passes through glass; such substances are called *diathermanous.* Bodies are not diathermanous and transparent in the same degree; for black glass transmits heat well, and water, which is highly transparent, is the least diathermanous of liquids. Of solid bodies, rock-salt is the most diathermanous, alum the least so.

246. Air is a highly diathermanous body. The heat of the sun, in radiating towards the earth, passes through the atmosphere without raising it in temperature, except to a very small extent. In consequence of this, the higher regions of the atmosphere, though nearer the sun, are much colder than the lower, which are raised in temperature by the transference to them of heat from the warm earth.

LATENT HEAT.

247. The word fluid includes two species—*liquid* fluids, or liquids, and *elastic* fluids, or gases. Each of these conditions, liquidity and gasëity, is determined by a very remarkable addition of heat to the solid which it converts into a liquid, and to the liquid which it converts into a gas. The subject, therefore, divides itself into two sections—heat of liquidity, and heat of gasëity.

Heat of Liquidity.

248. When a solid body, such as ice, is watched whilst melting, a large quantity of heat is observed to enter it without raising its temperature in the slightest degree. This heat which enters the body serves only to melt or liquefy it, without rendering the liquid the least hotter than the solid was which yielded it. The water which flows from the melting ice is no warmer than the ice. The heat which thus renders a body liquid without warming it, is called *latent,* or *insensible* heat, because it does not affect our sensations, and does not raise the thermometer.

249. The fact of heat becoming latent, is most decisively demonstrated by mixing a certain quantity—say an ounce by weight—of ice, or still better, from its state of division, of snow at 32°, with an ounce of water at 172°. The result will be found to be, that the snow is all melted, and two ounces of water are procured at the temperature of 32°. The hot water,

in cooling from 172° to 32°, has lost 140° of heat, which changes the snow into water, but does not raise its temperature above that originally possessed by the snow.

250. What we have illustrated here with ice, holds good for all solids. Each one of them renders latent a certain quantity of heat in becoming liquid, and retains that heat so long as it remains liquid. The following table shews the amount of heat which disappears during the liquefaction of several solids, and which is distinguished as their latent heat:

	Latent Heat.
Water, ...	140 degrees.
Sulphur, ...	145 "
Lead, ...	162 "
Bees-wax, . . .	175 "
Tin, ...	500 "

251. The heat thus absorbed is not lost; when the liquid solidifies, it is again given up. Thus, when water freezes, the 140 degrees of latent heat all abandon it, and manifest themselves as sensible heat. It is this necessity of absorbing such a quantity of heat, and getting rid of it again, that makes the processes of thawing and freezing go on so slowly. The heat developed in slaking lime, is the latent heat of fluidity becoming sensible. Water poured on burnt limestones does not *wet* them; it combines with them, and becomes a solid, and thus gives out its heat of fluidity.

252. Every solid has a certain temperature at which it fuses or becomes liquid. Thus, platina fuses at 3082° F.; wrought iron, 2912°; steel, 2372°; gold, 2282°; silver, 1832°; zinc, 700°; lead, 590°; tin, 442°; sulphur, 230°; an alloy of 1 part of lead, 1 of tin, and 4 of bismuth, 201°; tallow, 92°; ice, 32°; mercury, —38·2°.

Latent Heat of Gaseïty—Vaporisation.

253. In boiling off a pound of water, or converting it into vapour, it can be shewn by experiment that as much heat is absorbed as would have raised its temperature about 1000°, if it had not gone off in steam. Yet the water rises no higher than 212°, however hot the fire is, and the steam is of the very same temperature as the water it rises from. Thus, 1000° have disappeared or become latent in the steam; and before the steam can be condensed into water again, all this heat must be given out. (See STEAM-ENGINE.)

254. The same is true of the vapour that rises slowly and silently from water at temperatures below boiling (see METEOROLOGY). This absorption of latent heat is the cause of the cold which always accompanies evaporation. By placing water in a shallow vessel under the receiver of an air-pump,

and withdrawing the vapour as fast as it rises, evaporation may be made to go on so rapidly that the water becomes frozen.

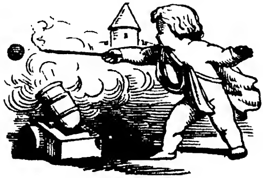
SOURCES OF HEAT.

255. Next to the sun (see ASTRONOMY), chemical combinations are the chief sources of heat. When two substances unite chemically, their temperature is almost always raised. When the heat is evolved so rapidly as to render the substances luminous — which most substances become when heated to a certain degree—the process is called *combustion* (see CHEMISTRY). *Fire* is a solid rendered luminous or incandescent by combustion; *flame* is gas at a white heat.

256. In ordinary combustion, one of the combining bodies is the oxygen of the atmosphere, which is called the *supporter* of combustion; the body with which the oxygen unites is the *combustible*. Most combustible substances are composed chiefly of two elements—carbon and hydrogen, for both of which oxygen has a strong affinity.

257. *Animal heat* has the same source as the heat of a fire or of a candle; it arises from a species of combustion. The oxygen taken into the body by the lungs, unites with the carbon and hydrogen of the waste parts of the blood and solids, and converts them into carbonic acid and vapour of water—burns them, in short, and thus produces heat. The average temperature of the human body is 98°·6; and it varies but little with race, climate, or state of health. The other mammalia have almost the same temperature as man; but that of birds is about 10° higher. The temperature of amphibia, fishes, &c., varies but little from the surrounding medium.

258. Heat is also produced by mechanical means, such as compression, percussion, and friction. A piece of iron may be rendered hot by hammering; and axles of carriages often ignite from friction. It is found that the amount of heat thus produced is always in proportion to the force expended in the process.



Edinburgh: Printed by W. and R. Chambers.

NATURAL PHILOSOPHY.

SECOND TREATISE.

MECHANICS—MACHINERY.

NOTICE.

The present Treatise—comprehending the *Elements of Mechanics, Theoretical and Practical*—forms the Second Department of Natural Philosophy, according to the arrangement described in the Prefatory Notice to the *Laws of Matter and Motion*.

A passage of that notice, as of general application, may appropriately be repeated here: 'The paragraphs under the different heads or chapters are meant to contain each some distinct truth, or some distinct step in the investigation of a truth, which should be thoroughly understood, before proceeding to the next. The examples given in illustration of the several natural laws laid down are necessarily few; but the pupil should be led to find others in the everyday concerns of life, and in the natural phenomena that come under his own observation. The readiness with which he can do this, and the clearness with which he can point out the exact way in which the general principle acts in each case, are the best tests of how far he has mastered the subject.' It is further suggested that pupils should, by the aid of small pieces of wood, cord, and other easily procured materials, endeavour to work out with their own hands the various principles of Mechanics that are demonstrated in the following pages. It is believed that by no other means could these principles be so well fixed in the memory, or have such a powerful effect in cultivating the understanding.

In the present, as in the preceding Treatise, great pains have been taken to render the language simple and intelligible, and

to introduce as few of the technicalities of mathematics as possible. Nevertheless, the nature of the subject is such, that in order to profit fully by a course like the following, the student ought, in addition to a knowledge of the ordinary rules of Arithmetic, to be familiar with certain notions of a geometrical kind. It is desirable, indeed, that he should have gone through a course of Euclid's *Elements*, and of Trigonometry and Algebra; and to master the more advanced parts of Mechanics, a training in these, and in the yet higher branches of mathematics, is essential. But for the profitable study of an elementary course like the present, nothing more is absolutely necessary than what ought to be taught, and is now generally taught, along with ordinary Arithmetic. For the guidance of teachers, as well as of those who may wish to engage in the study of this subject for themselves, it may be well to state more specifically what kind of preparatory mathematical knowledge we have supposed the learner to possess.

1. The ordinary rules of Arithmetic, embracing, of course, Fractions, vulgar and decimal.

2. Familiarity with the doctrine of Proportion. By this is meant, not merely the ability to do sums in the Rule of Three, as it is called; but a thorough understanding of what is meant by four numbers being in proportion, together with the more important properties of numbers so related. The illustrations of proportion should not be confined to mere abstract numbers; the principle should be exhibited in the lengths of lines, in the sizes and weights of bodies, &c., until it is thoroughly apprehended.

3. Acquaintance with the meaning of simple Formulas, such as are given in arts. 124 and 208. Formulas are of the greatest use even in ordinary arithmetic, and render a rule or direction, which is long and intricate in words, comprehensible at a single glance of the eye. The objection to them, as being hard and forbidding, is a prejudice which altogether vanishes on trial. Those once accustomed to this mode of expression, never feel that they have fully apprehended a truth or a rule in mathematics or in physics, until they see it in the shape of a formula. Nor is there anything more easily learned than their use. The youngest arithmetician can be taught to understand as readily, when he sees, for instance, $\frac{20 \times 5}{4}$, that it is a direction to multiply 20 by 5, and then divide the product by 4, as if the direction were given in words. And then the step from writing numbers in this way,

to the use of letters for expressing a general rule, is equally easy. The rule, for example, for calculating the interest (i) on any sum or principal (p), for a year, at a given rate (r), is not less simple, and suggests more readily the mode of procedure, when given in the form $\frac{p \times r}{100} = i$, than when the learner is told, in words, to 'multiply the principal by the rate per cent., and divide by 100.' To any one accustomed to this mode of expressing the usual arithmetical operations—and it is now becoming current both in books and in well-conducted schools—the formulas used in this treatise need present no difficulty.

4. Such a knowledge of lines, angles, triangles, parallelograms, and other common plane and solid figures as is meant by Practical Geometry, as distinguished from the scientific study of geometry after the manner of Euclid. One important point deserves special notice under this head; that, namely, of Similar Triangles. *When two triangles are similar, that is, have the several angles of the one equal to those of the other, then any two sides of the one are proportional to the corresponding two sides of the other.* This property of triangles is the foundation, so to speak, of Practical Trigonometry, and is of essential use in very many of the demonstrations of mechanics. It is one of those truths that are demonstrated or proved in Euclid's Elements and other scientific books; but, by drawing a number of triangles of varying size, yet all having their angles the same, and by placing such triangles concentrically, the one within the other, the proportionality of the sides will be rendered so palpable to the eye, that after seeing it under every variety of shape, the mind will receive it, without further demonstration, as almost a primary, self-evident fact.

The student, prepared as we have above indicated, would, it is believed, find no insuperable difficulty in following the reasoning in any part of this book. There are, however, a few of the demonstrations which it may be well, except for those having considerable familiarity with mathematical reasoning, to pass over. The truth or proposition laid down may be accepted on trust, as an established fact, without attempting to follow the process by which it is arrived at. Those demonstrations are inserted for the sake of students who may have enjoyed a more complete preliminary training. The paragraphs more particularly in view are 25, 64, 66, 119, 120, 206, 207, 208. It need hardly be added that besides these paragraphs, other portions, thought to be too difficult, may, with some verbal explanation, be also passed over

by less advanced pupils, according to the discretion of the teacher.

In order that the teacher, or the student for himself, may be able to test how far the principles explained have been clearly apprehended, a number of exercises have been added on such as admit of being applied to numerical calculation. This new feature of the present edition will, it is hoped, add not a little to the value of the treatise as an instrument of education.

CONTENTS.

MECHANICS.

1. MECHANICS, in its widest sense, is the science that investigates the effects of forces or pressures upon bodies.

2. Force is defined as 'any cause which moves or tends to move a body, or which changes or tends to change its motion.'

3. When several forces act upon a body at the same time, they may be so related to one another as to keep the body at rest. The forces are then said to *balance* one another, or to be in *equilibrium;* and the investigation of such cases forms the branch of mechanics called STATICS. DYNAMICS, on the other hand, treats of the effects of forces producing motion.

4. Bodies exist in three forms, solid, liquid, and aëriform; and the effects of forces on these several forms are in some respects different, and require separate consideration. Hence the equilibrium of liquids is treated of under the distinct head of HYDROSTATICS, and their motions under that of HYDRODYNAMICS. AËROSTATICS and AËRODYNAMICS are corresponding terms in respect of air and other gases. The simple terms Statics and Dynamics are thus confined to solid bodies.

5. Solid bodies have the power of *transmitting* force in a variety of ways. Holding a *rigid* solid, as a bar of iron, by one end, I can transmit the force of my hand through it so as to push a heavy body from me, or draw it towards me; or, by placing a prop below the bar, I may raise the body up. With a *flexible* solid, as a cord, a heavy body may be drawn along, but cannot be pushed or raised lever-wise. By passing the cord, however, over a fixed peg above, the body may be made to ascend by the hand pulling down.

6. It is seldom that a force is made to act directly on the body that is to be moved, or the resistance that is to be overcome. In raising coals from a coal-pit by steam-power, the expanding steam is not put below the load of coals, and made to blow it up the shaft, as it were. The steam first raises a piston, which then moves a beam, which in its turn moves a rod connected with a crank; and thus the motion is propagated from one piece of matter to another until it arrives at last at the load. During this transmission the force undergoes various changes which make it act more advantageously for the end in view.

7. Instruments thus interposed between the moving power and the resistance, with the view of changing the direction of the force or otherwise modifying it, are called *machines* (Gr. *mechanē*, contrivance); and to explain the laws of their action and their various applications forms the subject of the present treatise. Mechanics, in the narrower sense of the term as here used, is understood to treat of the abstract theory of machines, or those general principles, mathematical and physical, on which their action depends; the special application of these principles is the subject of Practical Mechanics or Mechanism.

8. Machines are of various degrees of complexity; but the simple parts or elements of which they are all composed are very few. These elementary machines, called the MECHANICAL POWERS, are usually reckoned six in number, three being primary, and three secondary, or derived from the others.

MECHANICAL POWERS.

PRIMARY.	SECONDARY.
1. The LEVER.	4. The WHEEL-AND-AXLE, derived from the Lever.
2. The PULLEY.	5. The WEDGE, derived from the Inclined Plane.
3. The INCLINED PLANE.	6. The SCREW, derived from the Inclined Plane.

THE LEVER.

9. When a box, or other heavy body, is lifted in the hands, the power is applied to the weight directly; it acts without the intervention of machinery. But when a bar is used, as represented in the figure, resting on a support F, we have an instance of a machine interposed between the power and the weight. The effect of this machine is to modify the power in more ways than one. When the hand lifts the weight directly,

it pulls upwards at the same point and with exactly the same force with which the weight pulls downwards. But with the

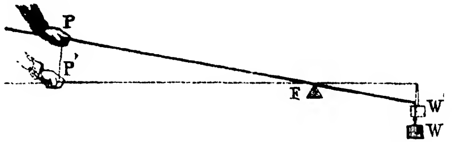


Fig. 1.

machine the hand pushes or pulls downwards as well as the weight, and one effect of the solidity of the rod and of the fixed prop is to convert the downward force at P into an upward force at the other end. And not only this, but when the prop is nearer to the weight than to the power, the power seems to be increased; a downward pressure of one pound at P, causes an upward pull of, it may be, two or more pounds at W.

10. A rod or bar used in this way is called a *lever*, from the French word for 'to raise.' The lever is the most important of the simple machines, or mechanical powers, and, in its various modifications, enters most extensively into the composition of complex machinery. The law by which it acts, therefore, deserves attentive consideration. To comprehend thoroughly the nature of the *advantage* conferred by the lever, is to comprehend the fundamental principle of all mechanics.

11. In treating of the *theory* of the lever and other mechanical powers, certain assumptions are made. Thus, the object of the lever is to *move* the weight; but the question really examined in the theory of the lever is, not what power is necessary to move a certain weight, but what power is necessary to balance it; what force at P will just keep W at rest, or suspended, if unsupported below. The subject is thus considered as belonging to Statics, or the doctrine of forces in equilibrium. It is obvious that when P and W once balance one another, the least additional force to P will suffice to begin motion.

12. Again, it is assumed that machines are themselves without weight; that the rod or lever, for instance, is a mere rigid line. This is done for simplicity; in practice, the weight of the bar itself has an effect on the resistance, and must be allowed for. An inflexible or *rigid* body, like the rod, is also supposed to be perfectly so; that is, not to yield in the least to any force, either in the way of bending, or of being compressed, or otherwise altering its form. Cords, again, are supposed to be perfectly flexible, or to require no force to bend them.

13. Another assumption is, that machines move without

friction. A certain amount of force is always necessary to turn the lever about its axis or point of support; but this we do not take into the account. The amount of power consumed in overcoming friction, and the means of diminishing it, are questions for practical mechanics.

14. Before proceeding to demonstrate the fundamental proposition of the lever, it is necessary to lay down a few definitions and axioms.*

DEFINITIONS AND AXIOMS.

15. *Definition* 1.—The fixed point about which the lever turns, as F in fig. 1, or C in figs. 2 and 3, is called the *fulcrum* or *centre of motion;* and the portions of the rod between the fulcrum and the points where the forces are applied, are called the *arms* of the lever.

16. *Def.* 2.—When the arms are portions of the same straight line, they form a *straight* lever; if they are not in the same straight line, it is a *bent* lever. Fig. 3 is an example of a bent lever, of which the fulcrum is the fixed point or axis C, and CA, CB are the two arms.

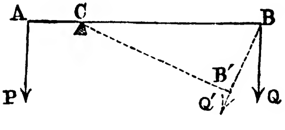


Fig. 2.

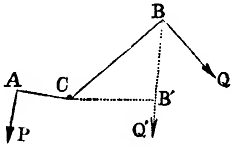


Fig. 3.

17. *Def.* 3.—The force P (figs. 2 and 3) acting at A in the direction AP, exerts an effort to turn the lever about C in its own direction; and the same is true of the force Q. If any force, as Q, so acting, is expressed by a number, say 6 (pounds), and the length of its arm also by a number, say 7 (feet), then the product of Q multiplied by CB, or $7 \times 6 = 42$, is called the *moment* of the force Q, and represents, as we shall see, the value of its effort to turn the lever about C. In like manner P, multiplied by AC, is the moment of the force P. This description of moment, however, applies only when the force acts at right angles to the arm of the lever. If a power Q′ act obliquely, as in the direction BQ′, the moment is expressed

* An axiom is a proposition so simple and self-evident as not to require or even admit of proof, there being no truth more plain and evident by which to prove it.

by multiplying Q′ by CB′, the perpendicular from C upon BQ′. Therefore, generally: *The* MOMENT *of a force is that force multiplied by the perpendicular drawn from the centre of motion upon the direction of the force.*

18. *Axiom* 1.—If two equal weights, as P, Q (fig. 4), are suspended from the extremities of equal arms AC, CB of a straight lever, they will hold each other in equilibrium. Besides the testimony of uniform experience, the mind assents at once that this must be so, on the abstract ground that no reason can be imagined why the lever should turn in the one direction rather than in the other. The axiom applies not only to weights, but to any equal forces acting at right angles to equal arms of a lever, and tending to turn it opposite ways.

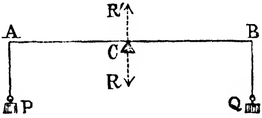


Fig. 4.

19. *Ax.* 2.—When two weights or other parallel forces thus balance each other on opposite arms of a straight lever, the pressure on the fulcrum is equal to the sum of the two weights or pressures, whatever be the length of the lever, and acts in a line parallel to the lines of direction of the two forces. The pressure sustained by C (fig. 4) is the same as if a force R, equal to the sum of P and Q, acted directly upon C in the direction CR. The reaction of the fulcrum being equal and contrary, would be represented by an equal force, R′, acting in the direction CR′.

20. *Ax.* 3.—So long as a force acts perpendicularly to the straight arm of a lever, its effort to turn the lever about the centre is the same whatever angle that arm makes with the other arm. If the equal forces P, Q, act perpendicularly to the equal arms CA, CB, of the bent lever ACB (fig. 5), they will keep each other in equilibrium on the same grounds as when the lever was supposed straight; namely, that there is no cause why the one should prevail over the other. And if we suppose the arm CA removed and again fixed in the position CA′, with an equal force P′ acting in the perpendicular line A′P′, the force Q will still be balanced as before; that is, the effort of the force P to turn the lever is the same in its new position as it was in its old.

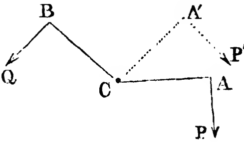


Fig. 5.

21. *Ax.* 4.—If a rigid body, as the plane ABD (fig. 6), is movable about a fixed centre or axis C, and if we conceive it

to be held in its present position by a cord pulling or a rod pushing in the direction of the line hp (the weight of the plane,

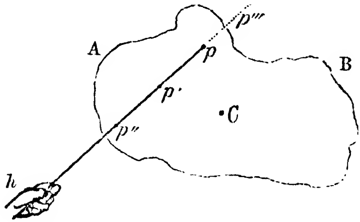


Fig. 6.

or some other force tending to make it turn on the centre C); then it will not matter at what point in the line the rod or cord is applied, whether at p, or p', or p'', &c., the same amount of force will hold the body in equilibrium at whatever point in the line of its direction it is applied.

PROPOSITIONS.

22. *Proposition.*—A prism or cylinder of solid matter AB (fig. 7), which is everywhere of the same density, will balance itself on its middle point, C. We might assume this as an axiom, on the self-evident ground that there is no cause to make one side preponderate over the other. But it may also be demonstrated by shewing it to be a consequence of Axiom 1; for if we conceive the prism composed of a number of small cross-sections of equal length, any two corresponding parts on the opposite sides of the centre, as the extreme sections a, b, or two intermediate ones d, e, will form a pair of equal weights at the ends of equal arms of a lever, and will therefore balance each other (Ax. 1); and as this is true of every separate pair, it is true of the whole mass. AB, therefore, if placed horizontal on C, will remain so.

Fig. 7.

23. *Corollary.**—It is clear that the cylinder acts by its weight, exactly as if the whole of its matter were collected in its middle point.

24. *Fundamental Proposition of the Lever.*—Every one is,

* A corollary is a truth immediately deducible from a previous train of reasoning.

by experience, familiar with the fact, that a force acting at the long arm of a lever, as in fig. 1, has an advantage over a force acting at the short arm. We are now to shew how this truth may be demonstrated—that is, proved to follow as a consequence from the simpler and self-evident truths already laid down. The demonstration will also determine the exact amount of the advantage gained by the length of the arm—a point which ordinary experience does not settle.

25. *When two weights balance each other on a straight lever on opposite sides of the fulcrum, the lengths of the two arms are inversely as the weights.*

Let P, Q (fig. 8) be two weights composed of matter of

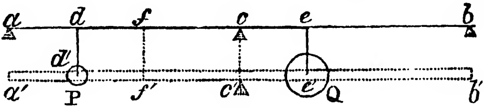


Fig. 8.

uniform density; and suppose this matter extended into a uniform cylinder $a'b'$, P making up the part $a'f'$, and Q the part $f'b'$, which parts are to be conceived as separate, though contiguous. Next, let ab be a rigid straight line equal to $a'b'$, supported horizontally on fulcrums at a and b; and let the two cylinders $a'f'$ and $f'b'$ be suspended by their middle points d' and e' to corresponding points d and e in the line ab. Each of the cylinders will balance itself on its centre (art. 22), and if the two are placed in the same straight line, they will remain so. Conceive, then, that the cords dd', ee' have become rigid, which does not affect the equilibrium when once established, and the whole system of cylinders and lines is now one rigid body; which—since the line ab and the attachments are without weight—will act as a uniform prism. It will therefore remain horizontal if the fulcrums at a and b are removed, and a fulcrum placed at c' or at c, the middle point of ab (22). But as the separate cylinders $a'f'$, $f'b'$, still hang balanced each on its middle point, the weight of each acts as if collected in that point (23). We may therefore consider the matter of these cylinders to be again collected into the weights P, Q, without disturbing the equilibrium; and these weights are now suspended and balancing each other on the lever de, resting on the fulcrum c.

Now, since the matter was uniformly distributed over the cylinder $a'b'$, the weight P is to Q, as the length of $a'f'$ to that of $f'b'$, or as af to fb. But the whole line ab being made up of af and fb, the half of it, or cb, must be equal to $\frac{1}{2}\,af + \frac{1}{2}$

fb. Now, *cb* is made up of *ce* and *eb*, and *eb* is $\frac{1}{2}$ *fb*; therefore *ce* is equal to $\frac{1}{2}$ *af*. In the same way it can be shewn that *cd* is equal to $\frac{1}{2}$ *fb*. Therefore, since P is to Q as *af* to *fb*, and since things are proportional to their halves, P is to Q as $\frac{1}{2}$ *af* to $\frac{1}{2}$ *fb*, or as *ce* to *cd*—that is, the arms of the lever are inversely as the weights.

26. *Cor.* 1.—It follows from this proposition, that $P \times cd = Q \times ce$; in other words, the *moments* of two weights that balance each other on a straight lever are equal.

27. *Cor.* 2. The pressure on the fulcrum *c* is equal to the sum of the two weights (23).

28. *Cor.* 3.—The proposition is true of the bent lever as well as the straight, so long as the forces act perpendicularly to the arms. If the forces P, Q (fig. 9), hold each other in equilibrium while acting on the bent lever *dce*, we may conceive the arm *ce* moved round to the position *ce′* without altering the effect of the force; then *de′* being a straight lever, P is to Q as *ce′* to *cd*; or as *ce* to *cd*.

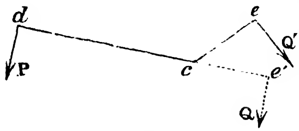


Fig. 9.

29. When a force acts obliquely to the arm to which it is attached, as Q′ in figs. 2 and 3, the real or effective length of the arm is the perpendicular CB′ drawn from the centre of motion upon the line of direction of the force; for we may conceive the lever ACB to be part of a rigid plane moving about the centre C. Now, a force Q′ acting in the direction BQ′ in that plane, will (art. 21) have the same effect at whatever point in the line Q′B it is attached; but if Q′ were attached at B′, and balanced P, then by art. 28, P is to Q′ as CB′ to AC; or $P \times AC = Q' \times CB'$. Generally, therefore, *when two forces balance each other on a lever, whether acting perpendicularly to the arms or obliquely, their moments about the centre of motion are equal.*

30. In speaking of the lever as a practical implement, used in raising weights, and otherwise moving heavy bodies, it is usual to call the moving force, P, the Power, and the resisting force, W, the Weight. If we suppose that a weight P (fig. 10) is used to raise up another weight W, if the arm FA is double the other, FB, it is found by experience that a weight of one pound at P will balance a weight of two pounds at W. And if the arms are in any other proportion—if, for example, AF were made seven feet long, while FB were only two

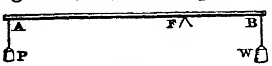


Fig. 10.

feet; then two pounds at P would balance seven pounds at W. In general, *the power and the weight are to one another inversely as their distances from the fulcrum.*

31. It looks at first sight like magic, that a weight of one pound should be made equal in effect to one of two pounds. It seems as if the lever gave us the means of multiplying force to any amount; as if the strength of a boy might be put on a footing with that of a man. It is necessary to get over this false impression. So far as the power of doing work is concerned, there is no creation or multiplication of force in the lever, or in any other machine, as will appear from the following considerations. If we suppose the power slightly increased beyond what is necessary to balance the weight, it will begin to descend; but in descending six inches, for example, it will raise W only three inches, as represented in fig. 1. What is thus gained in one way, is lost in another; if we make a small force raise a great weight, the force must be exerted through a proportionally great space. One pound will lift ten pounds; but to lift the ten pounds through one foot, it must descend ten feet. The two weights, when thus in motion, have equal momenta; the power multiplied into its velocity, is equal to the weight multiplied into its velocity. Since the velocities are in proportion to the distances from the centre of motion, this is the same as to say, the power multiplied into the length of its arm, is equal to the weight multiplied by its arm.

32. The comparative spaces through which P and W would move in the first instant of time if their equilibrium were disturbed, are called their *virtual* velocities; and the principle that when P and W balance each other, P multiplied by its virtual velocity is equal to W multiplied by its virtual velocity, is called the Golden Rule of Mechanics, and is true not only of the lever, but of all the other simple machines.

33. When the lever seems to multiply the power, it only accumulates or concentrates it. Suppose that a bale of goods of six hundredweights has to be raised to a platform a yard high, by a man who can lift only one hundredweight. If the bale can be taken to pieces, he may raise it by six lifts of a hundredweight each; but if it is impossible or inconvenient to divide it, he has recourse to a machine to accumulate his strength. By means of a lever with one arm six times the length of the other, he can raise the bale by a pull or a push of one hundredweight; but as the weight rises one yard, the long arm where the man's force acts must move over six yards. He thus makes six exertions of a yard each, exactly as if he lifted the bale in pieces; so that there is no actual saving of force.

The lever acts like the mill-dam, that, by treasuring up the flow of a small stream for twelve hours, gives it power to drive the wheel for one hour.

34. The power does not necessarily act at the long arm of the lever. The larger weight W (fig. 10) may be used to raise the less P, as well as P to raise W. When the power acts at the short arm, it requires, of course, to be greater in amount than the resistance it has to overcome; but a small motion of the power in this case makes the weight move over a great space; what is lost in power is gained in velocity. This is often as great an object as an increase of power.

SEVERAL KINDS OF LEVERS—EXAMPLES.

35. As yet we have represented the prop or fulcrum as situated between the power and the weight. This constitutes a *lever of the first kind*, as in fig. 11. But the power and

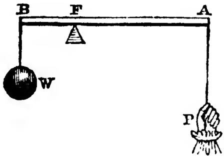


Fig. 11.

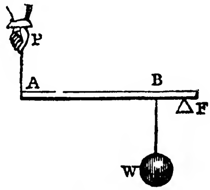


Fig. 12.

weight may be both on the same side of the fulcrum. In this case, when the weight is between the power and fulcrum, as in fig. 12, the lever is *of the second kind;* and when the power is between the weight and fulcrum, as in fig. 13, it is a lever *of the third kind.*

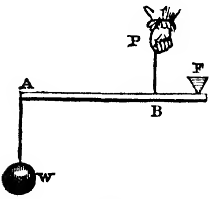


Fig. 13.

36. These different arrangements make no alteration in the principles already laid down. The effect of each of the forces, P and W, is still measured by its distance from the fulcrum. This distance forms its *leverage* or advantage; and the greater the leverage of either force, so much less does it require to be, in order to balance the other.

The following are examples of implements which act, more or less, on the principle of the lever, beginning with the lever of the first kind.

37. *Lever of First Kind.*—When the common spade is used

in delving, the blade is first forced into the soil by the foot, the handle is then pressed back, the edge of the undelved ground becomes a fulcrum, and the lever power of the long handle enables a moderate pressure to dislodge the earth from its place. Fig. 14 represents an iron lever or crow-bar used by stone-masons to move heavy blocks.

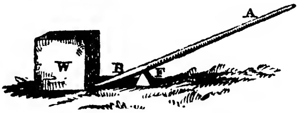


Fig. 14.

38. The power of the first kind of lever is frequently seen to operate in machines or instruments having two limbs. The most common examples of this nature are pincers, scissors, and similar instruments. The pin joining the two limbs forms the common fulcrum of both.

THE BALANCE.

39. The Common Balance, or *Scales*, for weighing, is an example of the first kind of lever, having two arms of equal length. The lever itself is called the *beam*, and the dishes suspended from the ends, for holding the substance to be weighed and the weights, are called the *scale-pans*.

40. If c, the centre of support of the beam AB (fig. 15), is also its centre of gravity, the beam, by itself, will rest in any position in which it is put. Also, if c is in the same straight line with A and B, the points of suspension, and makes $Ac = cB$, then with equal weights, represented in the figure by the equal forces, P and Q, the beam will still rest in any position indifferently. For, if we suppose it to be in the oblique position represented in the figure, the horizontal line dce drawn through c, will be perpendicular to the lines of direction of the forces P, Q, and cd being manifestly equal to ce, the moments of the forces are equal (29), and therefore they are in equilibrium. Such a beam would form an *indifferent* balance, and would be of no use in practice.

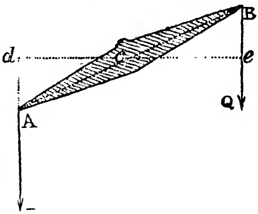


Fig. 15.

41. The conditions requisite for a good balance will be understood by reference to the accompanying figure, representing a section of a beam. C, the axis, on which the beam turns, is a prism passing through the beam at right angles, and resting with its lower edge, called the *knife-edge*, on two

supporting surfaces at the opposite sides of the beam. There is thus much less friction than if the axis were a cylinder with

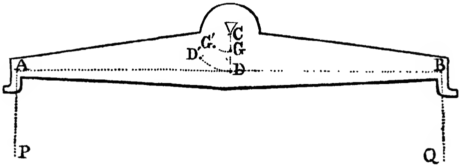


Fig. 16.

its ends moving in sockets. G is the centre of gravity of the beam; and AB is the straight line joining the points from which the scales are suspended. When the beam is at rest on C, the line CG will be vertical, and therefore, in order that the beam may be horizontal when at rest, the line AB must be perpendicular to CG. CG (produced to D) must also bisect AB, in order that equal weights may be in equilibrium in this position.

42. The *horizontality* of the beam with equal weights being thus secured, the other two points to be attended to are its *stability* and its *sensibility*. When a beam is disturbed, or driven from the position of rest, it is desirable that it should return and oscillate quickly about that position, so as soon to come to rest—this is called its stability. When the beam is made to turn through any angle, the points G and D describe arcs about C, and come into positions like G′ and D′. Now the whole weight of the beam may be considered as concentrated in G, and the two weights P, Q, in D (art. 19), and when moved to G′ and D′, they act as forces tending to turn the beam about C back to the horizontal position. The sum of the moments, then, of these forces—in other words, the weight of the beam multiplied by CG, plus the sum of P and Q multiplied by CD, expresses the stability of the balance.

43. When a very small weight added to one of the scales makes the beam turn, the balance is said to be *sensible*. This is essential for accurate weighing. The sensibility of a balance will, of course, be greater the less friction there is at the axis; and this is the object of making the axis a knife-edge. It is also evident that the longer the beam is, the additional weight will have the greater leverage in turning the beam about the centre. If we consider, again, that it is the forces of gravity concentrated at G and D that resist the moving of the beam from its horizontality, and that their efficacy depends upon their distance from C, the resistance will be diminished and

the sensibility of the balance increased, by constructing the beam so that G and D shall be near to C. We have seen, however, that the stability of the balance depends upon the distance of G and D from C; so that sensibility acquired in this way is acquired at the expense of stability.

44. It is clear from art. 42, that the more weight is concentrated at G and D—in other words, that the greater the weight of the beam, and also of the weights in the scales, the greater will be the stability, and, therefore, the less the sensibility. For delicate operations in weighing, therefore, the beam should be as light as possible, and the quantities weighed at a time should be small.

45. Most balances have a needle attached at right angles to the beam, and a graduated arc so fixed that, when the beam is horizontal, the needle points to zero. If, while the beam is oscillating, the needle is observed to describe equal arcs on both sides of zero, it is a sign that the beam, when it comes to rest, will be horizontal; if the arcs on one side are larger than on the other, that side will preponderate. This obviates the necessity of waiting till the beam comes completely to rest; and thus beams of considerable sensibility can be used in ordinary weighing operations without too great a loss of time.

46. Balances for scientific purposes require to be of the utmost delicacy. One made by Ramsden for the Royal Society, turned with the thousandth part of a grain.

47. There is another kind of balance, called the *steelyard*, which consists of a lever with arms of unequal length. Fig. 17 is a representation of the steelyard balance. C is the fulcrum or pivot on which the beam is suspended; CA is the short arm, and the opposite end is the long arm; W is the

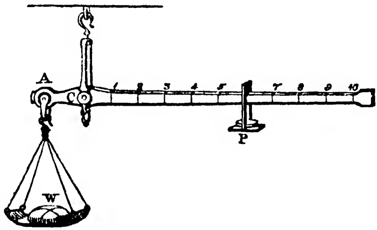


Fig. 17.

scale for the reception of the article to be weighed. On the long arm, a number of divisions are set off, marked 1, 2, 3, &c.; C1 being equal to CA, C2 equal to twice CA, &c. P is a

weight of a certain heaviness, and being movable by a ring, it can be slipped along the bar to any required point. The same weight is always used, which constitutes one of the principal conveniences of this kind of balance. In proportion as the article to be weighed in the scale W is heavy, so is the weight P slipped along to a greater distance from the fulcrum; and when it is brought to a point where it balances the article, the figure on the bar at that point indicates the amount of the weight. If P be one pound, and if, when suspended from the division at 6, it balance the weight at W, it is evident that the weight will be six times P, or six pounds; and so on with all the other divisions.

48. The steelyard, though not so ancient as the common balance, is of considerable antiquity. It was used by the Romans, and has long been in use among the Chinese. Neither the common balance nor the steelyard is suitable for shewing the varying weight or heaviness of an article at different latitudes of the earth's surface, because the weights employed are equally affected by the attraction of gravitation and centrifugal force, as the article to be weighed. For this purpose a balance formed of a spring of elastic metal is used.

49. *Lever of Second Kind.*—Examples of the second kind of lever-power are also common. One of the most familiar is that of a man pushing or lifting forward a bale of goods, as represented in fig. 18, in which the bale or weight W presses against the lever between the power P and the fulcrum F.

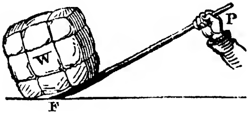


Fig. 18.

50. Another example of this lever is that of a man using a wheel-barrow, as represented in fig. 19. The axle of the wheel is the fulcrum; the body of the barrow, with its load, is the weight; and the two handles form the lever, the man's muscular strength being the power, and acting at the extremity, while the weight is situated between the extremity and the fulcrum. In proportion as the man shortens or lengthens the leverage of the handles by changing the position of his hands, so does he increase or diminish the weight he has to sustain; but when he eases the downward pull upon his arms by taking hold of the handles nearer the ends, he


Fig. 19.

throws more of the weight upon the wheel, and thus makes it more difficult to push the barrow over an obstacle.

51. Two men carrying a load between them on a pole is also an example of the second kind of lever. Each man acts as the power in lifting the weight, and at the same time each man becomes a fulcrum in respect to the other. If the weight hang fairly from the middle of the pole, each man will bear just a half of the burden; but if the weight be slipped along, so as to be nearer one end of the lever than the other, then the man who bears the shorter end of the pole supports a greater load than the man who is at the long end. The weight increases precisely in proportion as it advances towards him. In yoking horses to the extremities of cross-bars in ploughs, coaches, or other vehicles, if the cross-bar is not attached to the load by its middle point, one horse will have to pull more than the other.

Fig. 20.

52. The instrument used for cracking nuts (fig. 21) is an example of the second kind of lever with two arms or limbs. The fulcrum is the joint which connects the two limbs; the nut between them is the weight or resistance; and the hand which presses the limbs together, in order to break the nut, is the power. As each limb is a lever, a double lever-action takes place in the operation. The oar of a boat in rowing is also a lever of this kind. The hand of the rower is the power, the water against which the blade of the oar pushes is the fulcrum, and the resistance to be moved forward is the pin fixed into the side of the boat.

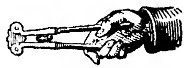


Fig. 21.

53. *Lever of Third Kind.*—In a lever of the third kind, the power must, from its position, be always greater than the weight; and from this circumstance it has sometimes been called the *losing* lever. But this gives a wrong impression; for what is lost in the amount of the power is gained in the velocity, or in the space over which the resisting weight is made to move. Levers of this kind are used where the object is not to gain power and overcome great resistance, but to produce rapid motion where the resistance is comparatively small.

54. An example is found in the foot-board of the turning-lathe (fig. 22). The foot of the workman presses on the board

or plank near the end which rests on the ground, or fulcrum, and, at the cost of a short movement of its own, causes the opposite extremity of the board to move in a downward direction over a considerable space.

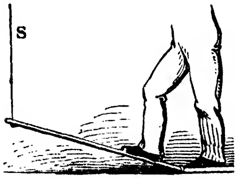


Fig. 22.

55. When earth is lifted on a spade, if the hand at the extremity of the handle is held fixed while the other makes the spade turn upon it as a centre, we have a lever of the third kind. If the hand that grasps the handle at some distance from the end is held fixed, and the other communicates the motion, it is a lever of the first kind. In practice, both hands move, acting reciprocally as power and fulcrum to each other. In using a pitchfork, a pencil, a pen, &c., the first and third levers are similarly combined.

ANIMAL LEVERS.

56. The movements in the limbs of animals are mostly produced by the action of levers of the third kind. The tendons or ropes which move the bones are attached near the joints, which are the pivots or fulcrums of the bone-levers. A striking instance is exhibited in the human arm. A strong muscle, arising in the shoulder, passes down in front over the joint of the elbow, and is inserted into one of the two parallel bones which compose the framework of the fore-arm. On being contracted in the slightest degree, at the impulse of the will, this muscle (the *power*) elevates instantaneously the hand (the *weight* or resistance) to the shoulder, bending the arm upon the elbow-joint (the *fulcrum*). Fig. 23 illustrates this mechanism of the arm. F represents the elbow-joint; P the power or will acting over, or from, the shoulder S, through the contracting muscle M; and B the arm from the hand to the elbow; while the weight W is an object supposed to be placed in the hand.

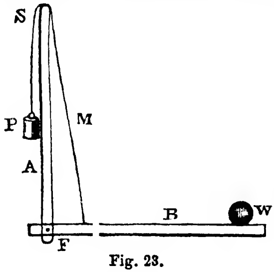


Fig. 23.

57. Nothing can better illustrate the characteristic advantages of levers of the third kind than this action in the human arm. The contraction of the muscle, to the extent of only one inch, raises the hand, even with a very considerable weight in

it, through a semicircle of twenty-one inches; and by relaxing the muscle only a little, the hand, as represented in fig. 24, is allowed to drop over a similarly wide range. No doubt other muscles in the arm assist in this action, but the principal part is performed by the one described, and here indicated by the letter M.

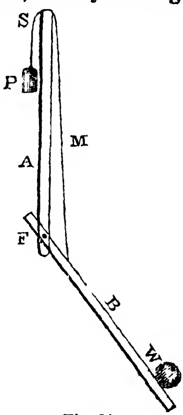


Fig. 24.

58. It is worthy of observation, that in the second of these figures, illustrating the mechanism of the arm, the lever-power acts under increased disadvantage, from the greater inclination of the limb. We can raise or sustain a much larger weight with the hand when the arm is bent at right angles, than when it is extended nearly in a straight line; and the more the arm is stretched out, the more is the power diminished. What is gained, however, in force, is necessarily lost in extent of motion.

COMBINATIONS OF LEVERS.

59. By disposing a number of simple levers, so that the one shall act on the other, the effect of the power may be increased to any desired degree. Fig. 25 represents a combination of three levers of the first kind. They are supposed, for simplicity, to be of equal length, the long arms being six inches

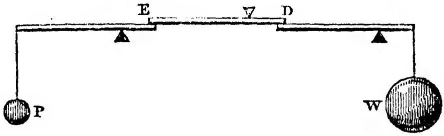


Fig. 25.

each, and the short ones two inches each; required—the weight which a moving power of 1 pound at P will balance at W. In the first place, 1 pound at P would balance 3 pounds at E, according to the rule of calculation for the simple lever; for $1 \times 6 = 3 \times 2$. This upward pressure of 3 pounds at E is next converted, by the second lever, into a downward pressure at D three times as great, or equal to 9 pounds; and, lastly, the downward pressure of 9 pounds at D is converted, by the third lever, into an upward

pull at W of 3 times 9, or 27 pounds. Thus, 1 pound at P will balance 27 pounds at W.

60. In this instance, the increase of power is comparatively small, because the proportion between the long and short arms is only as 2 to 6, or 1 to 3. If we make the proportions more dissimilar, the increase of force becomes very great. For example, let the long arms be 18 inches each, and the short ones 1 inch each; 1 pound at P will then balance 18 pounds at E, and the second lever will be pushed up with a power of 18 pounds. This 18 being multiplied by 18, the ratio of the second lever, gives 324 pounds as the power which will press down the third lever. Lastly, multiply this 324 by the ratio of the arms of the third lever, 18, and the product is 5832 pounds, which will be the final weight at W, which 1 pound at P would balance.

61. The general rule for combinations of levers, whether equal or not, is, that the power multiplied successively by all the arms next it in the system, is equal to the weight multiplied by all the other arms. From this equation we may find what power will be necessary to raise any given weight, or what weight any given power will raise. *Ex.*—Suppose, in a combination of four levers, the arms next the power are 3, 2, 7, and 9 feet, and the others 1, 2, 5, and 3 feet; what power will balance a weight of 1000 pounds? Here the product on the side of W is $1000 \times 3 \times 5 \times 2 \times 1 = 30{,}000$; and the product on the side of P is the unknown number $P \times 3 \times 2 \times 7 \times 9$, or $P \times 378$. Therefore P is equal to 30,000 divided by 378, or 79·36 pounds.

62. The same advantage might be gained by one lever as by many, provided the one were made long enough. But a long rod, in order to be inflexible, must be thick and heavy, and is, besides, inconvenient from the space it occupies; while a combination of short levers, producing the same effect, can be made comparatively light, and can be packed into little room. Machines for weighing loaded wagons, frequently seen at toll-bars, consist of a system of levers arranged below a table or platform on the level of the road. The wagon being wheeled upon the platform, is balanced by a comparatively small weight attached to the extremity of the system. The Balance of Quintenz, much used for weighing luggage at railway-stations, is an ingeniously arranged combination of levers. The principle of the steel-yard is frequently superadded to a combination of levers in these weighing-machines, so that one counterpoise serves for all cases.

COMPOSITION AND RESOLUTION OF FORCES AT A POINT.

63. In the treatise on *Matter and Motion*, a general notion was given of the Composition and Resolution of Forces, but without any attempt at demonstration. There are two ways of treating this subject. Some writers on mechanics begin by demonstrating the proposition of the *Parallelogram of Forces*, as it is called, and then deduce the proposition of the Lever from it as a consequence. Others reverse the order and begin with the Lever; and this method seems preferable in an elementary treatise. Having, therefore, established the proposition of the lever on independent grounds (25), we now proceed to shew that the proposition of the parallelogram of forces follows from it.

64. *Composition of Forces.*—Let two forces P, Q, acting at right angles to the arms of the bent lever BCD (fig. 26), be in equilibrium. If we suppose the lever to be part of a rigid plane, and produce the lines of direction of the two forces to meet in the point A in that plane, then the points of application of the forces may be both transferred to A, and the forces will still hold the plane in equilibrium (21). Now, when two forces act on the same point, it accords with universal experience that they urge that point to move in a line intermediate between the lines in which they themselves act, and that a single pressure of a certain magnitude can counterbalance their effect. The single force or pressure which thus arises out of the joint action of the two, is called the *resultant*, of which they are the *components*. The two pressures P and Q, therefore acting upon the point A, will produce a single pressure or resultant, and the direction of that resultant force must pass through the fixed point or axis C, and be supported by it; otherwise, the equilibrium would be destroyed. Through C, draw Cp parallel to DA, and Cq parallel to BA, making the figure ApCq a parallelogram; so that the opposite sides Cp and Aq are equal, and also Ap and Cq. Now, since the forces P, Q are in equilibrium on the lever BCD, P : Q : : CD : CB (28). But the triangles CBp and CDq are similar, or have all their angles equal.

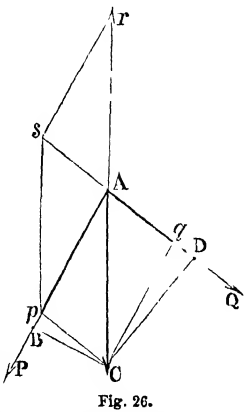


Fig. 26.

For the angles at B and D are right angles, and the opposite angles of the parallelogram at p and q being equal, the adjacent angles CpB and CqD are also equal; two angles in the one triangle are thus equal to two in the other, and therefore their third angles must be equal. Since the triangles are thus similar, their corresponding sides are proportional; therefore, CD : CB :: Cq : Cp, or :: Ap : Aq; therefore, P : Q :: Ap : Aq.

65. The lines Ap and Aq are thus proportional to the forces P and Q, and may be taken to represent them in direction and magnitude. It therefore appears that *when the two sides of a parallelogram represent two forces in direction and magnitude, the diagonal represents the direction of their resultant.*

66. But does it also represent the *magnitude* of the resultant? Suppose the effect of the forces, Ap, Aq, upon the point A, to be counteracted by a force pulling it the opposite way, the direction of which must be in the same line with AC, and let Ar represent the magnitude of that force, and complete the parallelogram Apsr. Then Ap and Ar, representing two forces acting on the point A, the diagonal As will represent the direction of their resultant; and as that resultant is counteracted by the force Aq, As must be in the same straight line with Aq, and equal to it. The two triangles, Asr and AqC, may now be easily shewn to be similar, and having one side equal to one side, Ar is also equal to AC. Therefore, *the diagonal* AC *represents the magnitude of the resultant as well as its direction.*

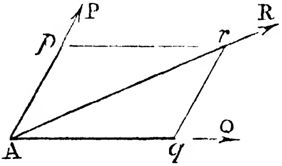


Fig. 27.

67. It is thus easy to find a single force that shall be the equivalent or the resultant of two or more given forces acting upon the same point. Let two forces, P, Q, fig. 27, acting upon the point A, be represented in direction and magnitude by Ap, Aq; through p and q draw lines making a parallelogram, and draw the diagonal Ar; that diagonal is the resultant required. If there are three forces, P, Q, S, fig. 28; find Ar', the resultant of two of them, as before; and then find Ar, the resultant of Ar' and As—and so on, however many forces are to be compounded.

68. *Resolution of Forces.*—The reverse process of *resolution* is equally simple. To resolve a given force R, whose direction and magnitude is Ar (fig. 27), into two forces acting in any directions that may be chosen, as AP, AQ, we have only to

draw parallels through r, which determine the lines Ap, Aq, representing the magnitude of the forces required.

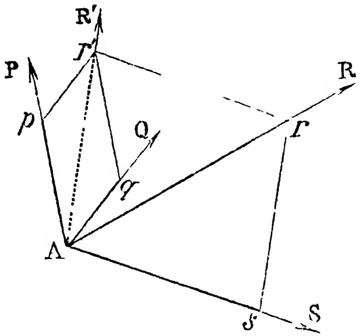


Fig. 28.

69. It is evident that there are an infinite number of pairs of forces into which Ar might be resolved, according to the direction in which the new forces are to act. It is also evident that the resultant of two diverging forces acting at a point is always less than the sum of the forces; for Ar, one side of a triangle, is always less than $Aq + qr$ ($= Ap$), the sum of the other two sides. The greater the angle pAq, at which the directions of the forces are inclined to each other, the more of their effect is lost in pulling against each other. As the angle is diminished, this loss becomes less; and when the two forces come together and act in the same straight line, their whole effect goes to cause the motion of the point A; in other words, their resultant becomes equal to their sum.

THE CENTRE OF GRAVITY OF A SYSTEM OF BODIES.

70. 'The centre of gravity of any body or system of bodies, is a point upon which the body or system will balance itself in all positions.' The determination of this point depends upon the principle of the lever. The centre of gravity in single bodies is considered in *Matter and Motion*, p. 39; the common centre of gravity of a number of bodies is found as follows. Let A and B, fig. 29, be two spheres, and AB a rigid line joining their centres. The spheres being supposed of uniform density, will act by their weight as if the whole matter of each were collected in its centre. If, then, the line AB be divided in C, so that the weight of A is to that of B as

BC to CA, or A × CA = B × CB; the two spheres will balance each other on a fulcrum at C, according to art. 25. In

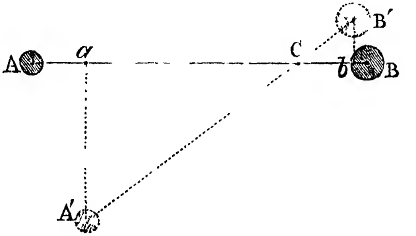


Fig. 29.

that article it is only the horizontal position that is considered; but the equilibrium is not disturbed by turning the system into any other position, as A′CB′; for the effective arms of the lever are now C*a*, C*b* (arts. 29 and 40), and, owing to similar triangles, these new arms have the same proportion to each other as the original arms, so that A × C*a* = B × C*b*, or the moments on both sides are still equal. The system, therefore, balances itself on the point C in all positions, and thus C is its centre of gravity.

71. If there is a third sphere D (fig. 30), join its centre with the point C, determined as above. The weight of the two spheres, A and B, may now be considered as collected in the point C; if, therefore, the line DC is so

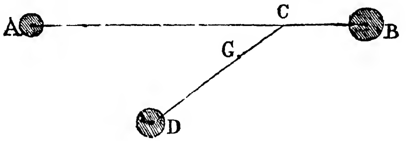


Fig. 30.

divided in G, that D × DG = (A + B) × CG, the point G will be the common centre of gravity of the three spheres. In like manner, we may proceed with four or any number of bodies.

72. *Ex.* Let the three bodies, A, B, D, weigh 4, 12, and 8 oz. respectively, and let the line AB be 20 inches. The position of C is thus found:

$$\begin{array}{rcccccc} A+B & : & A & :: & AB & : & CB \\ 16 & : & 4 & :: & 20 & & \\ & & & & 4 & & \\ & & & & 16\overline{)80} & & \end{array}$$

The point C is, therefore, 5 inches from B. Let CD be now joined and measured, and suppose it to be 9 inches. We then find the position of G by stating thus:

$$\begin{array}{rcccccc} A+B+D & : & D & :: & CD & : & CG \\ 24 & : & 8 & :: & 9 & & \\ & & & & 8 & & \\ & & & & 24\overline{)72} & & \\ & & & & 3 & & \end{array}$$

G, the centre of gravity, is, therefore, 3 inches from C.

THE WHEEL AND AXLE.

73. When a lever is movable upon an axis, and is susceptible of being turned completely round, it assumes the character of the diameter of a wheel. AB (fig. 31) is an equal-armed lever, fixed on an axis F. A force acting at A or B, if not held in equilibrium, would continue to turn the lever round and round.

Fig. 31.

Let another lever CD (fig. 32), with arms of the same length, cross AB, turning on the same axis and fixed to AB, so as to form a solid system; it will be indifferent whether a force act at A, C, B, or D; so long as it acts at right angles to the arm, it will have the same effect in turning the system on the common axis. The same is true of any number of levers crossing one another; and if a ring is made to unite their ends, like a wheel, a force acting at any point on the ring, will have the same effect as if it acted at the end of one of the levers. A wheel is thus a system of equal-armed levers crossing one another in the centre.

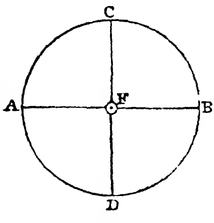


Fig. 32.

74. The way in which such a system is turned to account as a moving power, is represented in fig. 33. A wheel is

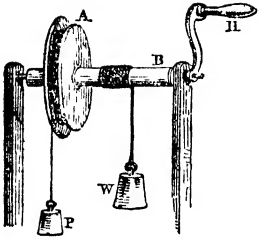


Fig. 33.

fixed on an axle, the ends of which turn in upright supports. A cord is wound round the wheel, and another round the axis in the opposite direction. If a weight, P, is then attached to the cord of the wheel, it will by its descent turn the wheel, and the axle with it; and the winding of the cord on the axle will lift a weight, W, attached to it. This is the machine called the *wheel and axle*, which may be characterised as a *perpetual lever*.

75. Fig. 34, presenting a section of the wheel and axle, shews the lever-power, with which the two forces act. F, the centre of both wheel and axle, is the fulcrum; AF, half the diameter of the wheel, is the arm at which P acts; and FB, half the diameter of the axle, is the arm at which W acts. Thus, the larger the wheel, and the smaller the axle, the greater advantage the power has over the weight. But this advantage in power is gained at the sacrifice of speed, for every turn of a small axle raises the weight but a short way.

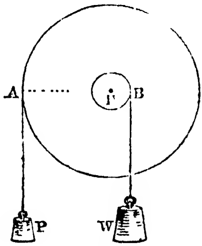


Fig 34.

Instead of a wheel and weight, the axle may be turned by a handle called a *winch*, as at H (fig. 33). The handle is just part of a wheel, and the force of the hand that turns it acts at an advantage proportioned to the wideness of the circuit it makes.

The proportion of the power to the weight in the wheel and axle is calculated precisely as in the case of the simple lever; $P \times AF = W \times FB$. If the diameter of the wheel is six times that of the axle, or, which is the same thing, if the circumference of the wheel is six times that of the axle, a power of one pound will sustain a load of six pounds.

PRACTICAL APPLICATIONS.

76. The principle of the wheel and axle, or perpetual lever, is introduced into various mechanical contrivances, which are of great use in many of the ordinary occupations of life. One of the simplest machines constructed on this principle is the common *windlass* for drawing water by a rope and bucket

from wells. Fig. 33 without the wheel A would represent such a machine. Coal is lifted from the pits in which it is dug by a similar contrivance, wrought by horse or steam power.

The *capstan* in general use on board of ships for drawing up anchors, and for other operations, is an example of the wheel and axle; it is often constructed in an upright or vertical, instead of a horizontal, position (fig. 35). The moving power is applied by the sailors pushing against the ends of spokes stuck in the head.

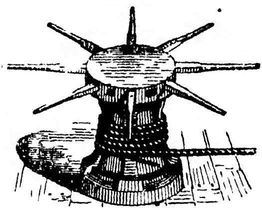


Fig. 35.

COMBINATIONS OF THE WHEEL AND AXLE.

77. There are combinations of the wheel and axle, as there are combinations of the simple lever. Cranes, watch and clock work, and wheel-machinery in general, mostly consist of such combinations. The parts are usually made to act upon one another by means of projections called *teeth* or *cogs;* or, when at a distance, by straps. In any case, the advantage is calculated in the same way as in combinations of the lever. For illustration, let us take the combination of three wheels and axles represented in fig. 36; and suppose that the radii of the wheels A, B, and C are 12, 15, and 20 inches respectively; those of the pinions F and G, 3 inches each, and that of the axle E 2 inches—the radii of the toothed-wheels and pinions being measured from the centre to the point of contact of the teeth. By the principle of the lever, the downward force of P becomes an upward pressure on the teeth of wheel B, and is increased as 12 to 3, or made fourfold. Similarly, the upward pressure on the teeth of wheel B becomes a downward pressure on those of C, and is increased as 15 to 3, or five times; that is, it is equal to 20 times P. Again, this downward pressure of 20 times P upon the teeth of C becomes an upward force at E, increased as 20 to 2, or 10 times; so that W is sustained by a force equal to 200 times P.

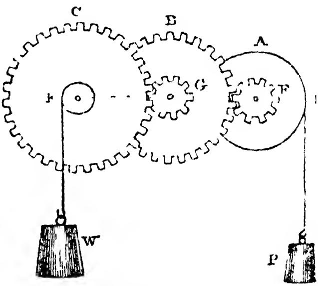


Fig. 36.

78. The general rule for such combinations is, that P multiplied by the product of the radii of all the wheels, is equal to W multiplied by the radii of all the axles (or pinions). In the example just given, $P \times 12 \times 15 \times 20 = W \times 3 \times 3 \times 2$; that is, $P \times 3600 = W \times 18$; and dividing by 18, $P \times 200 = W$.

79. If instead of the wheel A there were a winch, having the radius CD, attached to the axis of the pinion F, the arrangement would be similar to that used in cranes. A man exerting a pressure of 1 cwt. on the handle at D, would, on the former supposition, sustain a block of stone of 200 cwt. or 10 tons. But on the principle of virtual velocities, the ascent of W would be extremely slow; the handle, with a pressure of 1 cwt. upon it all the while, must move over 200 feet, or make more than thirty revolutions, in order to lift W one foot.

80. The object of wheel-work is often, not to gain power, but to gain velocity at the expense of power. In this case the moving force is applied as at W, and the gain in velocity in the revolution of the last wheel in the system, might be calculated from the radii, in much the same way as the gain in power is calculated. It may also be calculated from the number of teeth in the several wheels and pinions (see art. 164).

THE PULLEY.

81. The pulley consists of a small wheel, called a sheaf, turning on an axis in a block, with a flexible cord resting in a groove in the circumference of the wheel. There are two kinds of pulleys—the *fixed* and the *movable*.

The annexed cut represents a pulley, A, fixed by its block or frame to a beam or roof, B, and having two weights attached to the ends of the cord. In order that P and W may be in equilibrium in this case, it is evident that they must be equal. This appears from the wheel acting as a lever with equal arms, so that neither P nor W has any advantage. It is also plain that the cord, being free to move either way, must be equally stretched, or have the same *tension* throughout its whole length. A fixed pulley, therefore, does not increase the power; it only serves to change the direction in which the power acts. This is often as great a gain as increase of power would be. A force, for instance, at P, pulling downward, can raise W upward.

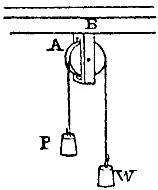


Fig. 37.

82. The wheel is not an essential part of the pulley, in theory at least. The same object might be gained by bending the

cord over the axis of the wheel, or over any bar, and making it slide on it; but in this case the friction of the cord on the surface of the bar would cause great resistance, and would chafe the cord; and it is to obviate this that the wheel is used. Theoretically, then, the whole virtue of the pulley resides in the flexible cord.

83. It is only with the movable pulley that there is a gain of power. The movable pulley is generally attached to the weight or resistance, and moves along with it. In the annexed cut, a cord is carried from a fixed point at A round a pulley B, from which the weight is suspended; it is then made to pass over a fixed pulley at C, and the power is represented by a hand drawing downwards. The parts of the cord BA and BC being equally stretched, each sustains evidently *half* the weight; but the part of the cord PC has the same tension as the rest, therefore P pulls down with a force equal to half the weight. A single movable pulley, then, *doubles* the effect of the power. The only effect of the fixed pulley C is to change the direction; if the hand were at C pulling upwards, it would sustain just half the weight.

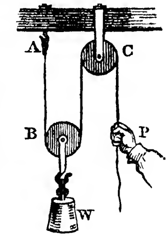


Fig 38.

84. When the two cords, BA, BC, are not parallel, besides sustaining the whole weight, they pull against each other to a greater or less degree, and are therefore stretched with a tension greater than that due to half the weight. Part of the advantage, then, is lost when the strings attached to the movable pulley are not parallel.

85. The secret of the saving of power in the movable pulley lies in making the hook in the beam at A support the half of the weight. It is as if there were two hands, one at A, and another at C, sustaining the weight between them; but, although the hook A can thus act as a substitute for a hand to sustain W at rest, it can take no share in *lifting* W. In order to lift W one foot, the hand at C must rise through two feet, or—which is the same thing—the hand at P must descend through two feet; and as it is bearing up a weight equal to half W all the time, it makes in effect two lifts of a foot, with half the weight of W each lift, which amounts to lifting the whole of W one foot. We thus find the same principle in the pulley that has been illustrated regarding the lever in arts. 31 and 33.

86. As represented in fig. 39, a man may seat himself in a loop or seat attached to one end of a cord, and passing the cord

Fig. 39.

over a fixed pulley above, may pull himself upwards by drawing at the other end of the cord. He requires to support with his arms only a little more than half the weight of his body, in order to make the seat ascend. By adding a movable pulley and another fixed pulley to the apparatus, the exertion of pulling would be diminished one half. An apparatus of this nature, having two fixed pulleys and one movable pulley, is used by masons and other artisans in making repairs on the fronts of buildings.

87. Technically, the wheel of a pulley is called a *sheaf;* for protection and convenience, this sheaf is ordinarily fixed with pivots in a mass of wood called a *block;* and the ropes or cords are called a *tackle.* The whole machine, fully mounted for working, is termed a *block and tackle.* By causing a wheel and axle to wind up the cord of a block and tackle, the power of the lever is combined with that of the pulley in the operation.

SYSTEMS OF PULLEYS.

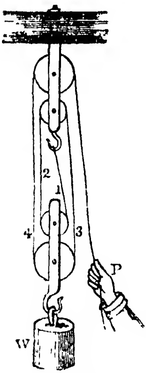


Fig. 40.

88. Several pulleys are often used in combination, forming a *system of pulleys.* There are various modes of arrangement. One of the most common is that represented in the figure, and called the 'First System,' in which the same cord passes over all the pulleys, and the pulleys are divided into two sets—one set working in the same fixed frame or block, and the other set in a movable block to which the weight is attached. The cord being the same throughout, the tension of all its parts is the same; and, therefore, neglecting any slight want of parallelism among the cords, the weight is equally distributed between the cords 1, 2, 3, and 4. As the cord at P has the same tension as the others, being, in fact, a combination of the same cord, P thus sustains a fraction of W, depending upon the number of cords that join the movable block. In the present case, P is one-fourth of W, or $P = \frac{W}{4}$, or $4P = W$; so that a weight of a cwt. at W would be sustained by a weight of 28 lbs. at P.

89. Fig. 41 represents another mode, commonly used in practical operations, of constructing a system of pulleys on the principle of the same cord passing over all the sheaves. As seven cords join the lower block, $P = \frac{W}{7}$.

90. In the 'Second System' of pulleys, each movable pulley is suspended in a separate cord, as represented in fig. 42. To find what fraction P is of W, we have only to consider that the

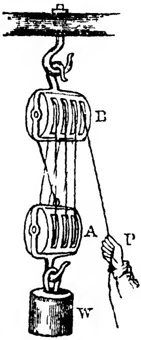


Fig. 41.

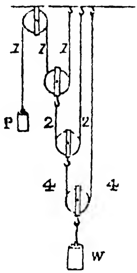


Fig. 42.

tension of each of the cords, or rather, of each of the parts of the one cord, marked 4, 4, is $\frac{1}{2}$ W; that the tension of cord 2, 2, is $\frac{1}{2}$ of cord 4—that is, $\frac{1}{2}$ of $\frac{1}{2}$ W; and that the tension of cord 1, 1, or P, is $\frac{1}{2}$ of cord 2, or $\frac{1}{2}$ of $\frac{1}{2}$ of $\frac{1}{2}$ W. Hence $P = \frac{W}{2 \times 2 \times 2} = \frac{W}{2^3}$, or $W = 2^3$ P. Expressed generally, P is equal to W divided as often by 2 as there are movable pulleys. *Ex.* With 4 movable pulleys, what weight would a power of 12 lbs. sustain? *Ans.* $W = 12 \times 2^4 = 12 \times 16 = 192$ lbs.

91. The 'Third System' of pulleys is represented in fig. 43. It is evident that the tension of the cord passing over pulley a_1 is equal to P; but pulley a_1 is sustained by the cord 2, 2, therefore the tension of the cord 2, 2, is equal to $2 \times$ P. In like manner, the tension of cord 3, 3, is double that of 2, 2, or is equal to $2 \times 2 \times$ P; and that of 4, 4, is equal to $2 \times 2 \times 2 \times$ P. But W is equal to the sum of the tensions of the cords 1, 2, 3, 4; that is, W is equal to $P + 2 \times P + 2 \times 2 \times P + 2 \times 2 \times 2 \times P$; or $W = P + 2P + 4P$

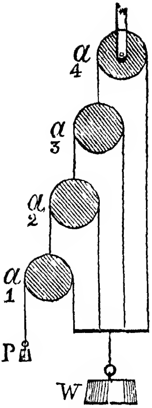


Fig. 43.

$+ 8P = 15P$. In general, $W = P + 2P + 2^2 P + 2^3 P +$ &c., according to the number of pulleys.

92. The weight of the pulleys has, for the sake of simplicity, been left out of the consideration. When it is desirable to take it into account, it is evident that in the First System, the whole weight of the movable block and its sheaves must be added to W. In the Second System, P has to sustain one-half the weight of the uppermost movable pulley, one-fourth the weight of the next, and one-eighth of the next. The whole value of P, then, with three movable pulleys, is $\frac{W}{2^3} + \frac{1}{8}$ the weight of the pulley next to W, $+ \frac{1}{4}$ the next, $+ \frac{1}{2}$ the next. In the Third System, the weight of the pulleys helps to sustain W, so that less weight at P is necessary. Each of the pulleys, a_1, a_2, a_3, is, in fact, by its weight, an additional power, whose effect is to be calculated, like that of P itself, by the number of pulleys above it. Thus, a_3, stretching the string 4, 4, by its weight, sustains a part of W equal to its own weight, which we may call p; a_2, supposing it of the same weight, sustains $3p$, and a_1, $7p$. The whole value, then, of W is $15P + 11p$.

THE INCLINED PLANE.

93. If AB, fig. 44, represent a horizontal plane, AC will represent a sloping or *inclined* plane. If we suppose AC to be a plank resting on the ground at A, and with its other end at C on the edge of a platform, it is a familiar fact, that a man can roll a heavy cask from A to C, which it would be far beyond his strength to lift perpendicularly from B to C. In such a case, the inclined plane is used as a mechanical power.

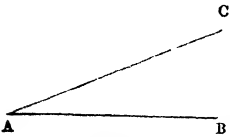


Fig. 44.

94. In investigating the advantage afforded by the inclined plane, we consider, as in all the other mechanical powers, what force is necessary to hold the weight in equilibrium—in other words, what force is necessary to keep the cask, when once on the plane, from rolling down. When it is once balanced in

this way, a little additional force will make it move up. In practice, if the heavy body to be moved up is of a square shape, as a box, its own friction is sufficient to keep it from sliding down, unless the elevation is very great; and to push it up requires force to overcome this friction, in addition to what would be necessary to push up a round body of the same weight. But for the theory of the inclined plane, we consider bodies as moving without friction; and this is most easily conceived by representing them either as round, or as resting on wheels, because a rolling motion is attended with comparatively little friction.

95. When a cylinder rests on a level plank, its whole weight is supported by the reaction of the plank, and it has no tendency to roll either way; but if one end of the plank be raised, however slightly, the cylinder begins to roll, and a certain force is required to keep it at rest. As the end is more and more elevated, the restraining force requires to be increased, and at the same time the pressure on the plane becomes less, until, when the plank comes into the upright position, it ceases to sustain any pressure, and a force equal to the whole weight of the cylinder is required to keep the latter in its position.

96. In order to find the relation of P to W in a given inclined plane, we have recourse to the resolution of forces. In fig. 45, which represents a cylinder resting on a plane, and kept from rolling down by a weight suspended over a pulley at *p*, so placed that *cp* is parallel to AB; let *cf*, drawn from the centre of gravity of the cylinder in a vertical direction, represent the whole weight of the cylinder. This force may be resolved into two others, *ce* perpendicular to the plane, and *cg* parallel to it (see art. 68). But *ce* is counteracted by the resistance of the plane, and can produce no effect; therefore *cg* or *ef* represents the force with which the cylinder is urged in the direction of BA; *fe* will therefore represent the force of P which keeps W in equilibrium. Now, it can be shewn that the triangle *cfe* is similar to the triangle ABD; therefore *fe* is to *cf* as AB to BD—that is, *W is to P as the length of the plane to its height.* If AB is twice the length of BD, one hundredweight at P will sustain two hundredweights at W; and if AB is six times the length of BD, W will be six times P.

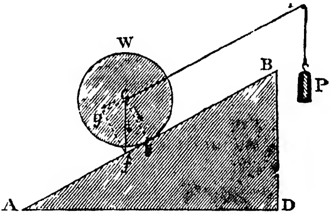


Fig. 45.

97. The proportion of P to W now stated is true only when the direction of the force P is parallel to the plane; in any other direction, part of the effect of P is lost.

98. In speaking of sloping roads, they are said to have an inclination or rise of one foot in ten, one in thirty, one in a hundred, and so on. The degree of slope in a railway is called the *gradient*, and seldom exceeds one in 50 or 60. The annexed cut represents a cart drawn by a horse up a common road with

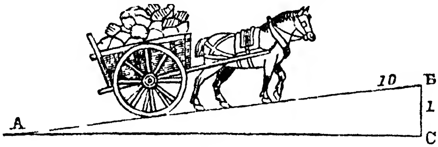


Fig 46.

a rise of one in ten. On a perfectly level road, a certain force of draught would be required to overcome friction (see art. 194); on the incline there is required, in addition to this, a constant pull to counteract the tendency of the cart to run down the slope. That tendency is, in this case, equal to a tenth of the weight of the load; and if the load is a ton, the horse has to pull with a force of one-tenth of a ton, or 224 pounds, above what would have sufficed to draw it along a level.

99. In going over ten feet of the road, the horse has raised the cart one foot perpendicularly, but he has done it by instalments; the exertion has been spread over a movement of ten feet. Intensity of exertion is saved at the expense of time.

100. Fig. 47 represents two inclined planes of the same height, but different slopes, meeting together at the top, with a weight resting on each, P and Q, hanging by a string, which passes over the pulley M. If the length of the longest plane from A to M be two feet, and that of the shorter from B to M be one foot, then two pounds at Q, on the short side, will balance four pounds at P, on the long side; and so for other proportional lengths. In general, the weights that will thus balance each other, are inversely as the lengths of the planes.

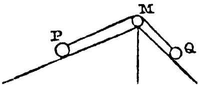


Fig. 47.

101. When roads have necessarily to be carried to the summits of heights, they are made either to wind round the ascent, or to take a zigzag direction, so as to make the ascent gradual. In ascending a steep part of a road, horses, no less than their drivers, are aware of the saving of violent strain that results

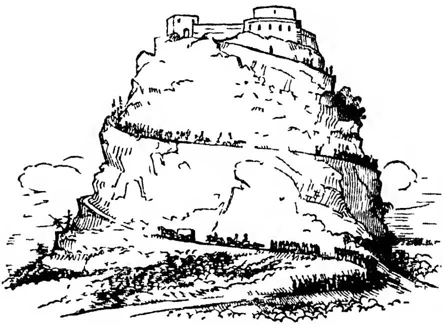


Fig. 48.

from going zigzag, from side to side of the road, instead of straight forward.

THE WEDGE.

102. A common form of the wedge is represented in the figure. By forcing such a body below the bottom of an upright post, for instance, which can move only vertically, we may raise it a few inches. It obviously acts on the principle of the inclined plane; it is, in fact, the inclined plane made movable. A wedge of the ordinary shape is really two inclined planes placed base to base.

Fig. 49.

103. The proportion of the power to the resistance in the wedge is calculated theoretically in the same way as in the inclined plane. But the theory is of little or no value. The power employed being percussion or blows, cannot be rightly compared with the resistance; it cannot be estimated in pounds, like other forces. The friction, too, is so great as to render any precise statement of proportion impossible. In a general way, however, the wedge is more powerful as the angle is more acute; just as the advantage of the inclined plane increases with the smallness of its height.

The wedge is used in splitting timber, stones, &c. Ships are raised in docks by driving wedges under them. In expressing oil from seeds, the seeds are placed in bags between solid pieces of wood, and these are forced together by means of wedges, till the seeds become a mass as compact as wood.

Fig. 50.

Cutting and piercing instruments all act on the principle of the wedge. The plough is also an instance.

THE SCREW.

104. The screw is an inclined plane wound round a cylinder, resembling the spiral road winding round the hill in fig. 48. Take a cylindrical ruler AB, and cut a slip of paper in the form of a right-angled triangle *abf*, having *ab* equal to the length of the cylinder, and *bf* equal, say, to four times its circumference. If the edge *ab* is applied to the cylinder

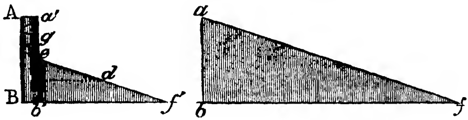


Fig. 51.

lengthwise, and the triangle is then wrapped round and round, the slanting side, *af*, will form a spiral going four times round the cylinder before it reach the bottom. In the fig. to the left of the cut, it is represented as half wrapt on.

105. If a ridge or projection were raised on the cylinder along the spiral line, it would form a screw. The several turns or coils of the spiral are called the *threads* of the screw; *ed* represents the *length* of one thread, and *ec* the *distance between the threads*. In moving over a complete round of the spiral, a perpendicular ascent or descent is made equal to *ec*. As in the inclined plane, therefore, the mechanical advantage of the screw depends on the proportion between the length of a thread and the distance between two contiguous threads.

106. In using the screw as a machine, the resistance is not applied directly to the spiral surface, as in the wedge or the straight inclined plane; the screw is made to work in a hollow cylinder with spiral grooves cut out to correspond to the projecting threads. This concave screw is technically called a *nut*, as represented at M, fig. 52. The threads and corresponding grooves are sometimes of a triangular form, as in fig. 52; and sometimes flat, as in fig. 53. To produce pressure with the screw, either the screw or the nut must be fixed. Whichever is free, is then turned, and made to press against the resistance.

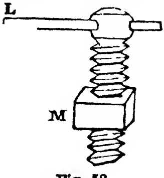


Fig. 52.

107. Practically, the screw is never used as a simple

machine, the power being always applied by means of a lever, passing either through the head of the screw or through the nut. The screw, therefore, acts with the combined power of the lever and inclined plane; and in investigating the effects, we must take into account both these simple mechanical powers, so that the screw now becomes really a compound machine.

Fig. 53.

108. To arrive at the proportion of the power to the weight in this compound machine, we may apply the principle of virtual velocities. The power acts at the end of the lever L (fig. 52), and in turning the screw once round, it describes a circle, of which the lever is the radius. The circumference of this circle represents the velocity of the power. In the meantime, the weight has moved over a space equal to the distance between two threads, which is its velocity. Now, the power multiplied by its velocity is always equal to the weight multiplied by its velocity; therefore, *in the screw, the power multiplied by the circumference it describes, is equal to the weight multiplied by the distance between two contiguous threads.*

109. *Ex.*—The length of the lever of a screw is forty inches—counting from the axis of the screw—and the distance between the threads is one inch; what power will produce a pressure of 8000 pounds? The diameter of the circle described by the power is here eighty inches, and the circumference may be taken, in round numbers, at three times that, or 240 inches. The product of the weight into the distance between two threads is $8000 \times 1 = 8000$; the power must therefore be $8000 \div 240 = 33\frac{1}{3}$ pounds. This, however, is only in theory; in practice, the power must be increased by a third, for the purpose of overcoming the friction; it would in this case, then, be about forty-four pounds.

110. It is evident that the power of a screw increases with the closeness of its threads—that is, with its fineness; but this way of augmenting the force soon comes to a limit, because the threads, if too fine, bend or break off—the screw overdraws. To obviate this is the object of a contrivance called, from its inventor, the Hunterian Screw. It consists of two screws whose threads differ slightly in breadth. The smaller of the two is made to work into the other as a nut, and one turn of the outer screw in its nut advances the end of the inner screw by a space equal to the difference between the breadths of the two sets of threads. Thus, if there are six threads in the inch of the outer screw, and seven in the inner, the advance caused by one revolution is $\frac{1}{6} - \frac{1}{7}$, or $\frac{1}{42}$ of an inch. The power is thus the same as if the screw had been single, with forty-two

threads in the inch; while at the same time the threads are large and strong.

APPLICATIONS OF THE SCREW.

111. The most common purpose for which the screw is applied in mechanical operations, is to produce great pressure accompanied with constancy of action, or retention of the pressure; and this quality of constancy is always procurable from the great friction which takes place in the pressure of the threads on the nut, or on any substance, such as wood, through which the screw penetrates. The common standing-press used by bookbinders for pressing their books, affords one of the best examples of the application of the screw to produce great pressure (fig. 54). The screw A has a thick round lower extremity B, into holes in which the lever is inserted. This extremity B is attached by a socket-joint to the pressing-table C, so that when the screw is turned in one direction, the table sinks, and when turned in another, the table rises. The books, D, lie upon a fixed sole S, below the table. H is a cross-beam above, in which is the box or overlapping screw, to give the necessary resistance.

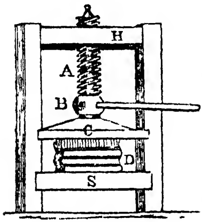


Fig. 54

112. *Ex.*—Suppose that the length of the lever in this machine is two feet, the thickness of the threads $\frac{1}{2}$ inch, and that the pressure applied to the lever is 100 pounds; required the pressure upon the table. Here the circumference described by P is $4 \times 3{\cdot}1416 = 12{\cdot}5664$ feet $= 150{\cdot}7968$ inches; and $W \times \frac{1}{2} = 100 \times 150{\cdot}7968 = 15079{\cdot}68$; therefore $W = 15079{\cdot}68 \times 2 = 30159{\cdot}36$ pounds.

113. The force of the screw is sometimes employed to turn a wheel by acting on its teeth, by which means there is a combination of two mechanical powers—the screw, and the wheel and axle (fig. 55). The screw is upon the horizontal spindle, and by turning it by the thread handle, each turn of the thread receives a tooth of the wheel, and brings it forward, so as to produce a perpetual revolution of the wheel. This is called

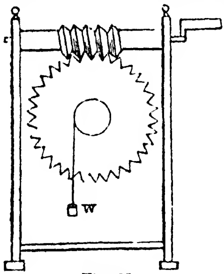


Fig. 55.

an endless screw, because it never stops in its action; no sooner is one turn of the thread disengaged, than another has come into operation. W represents a weight to be raised hanging from the circumference of the axle.

PRACTICAL MECHANICS.

STRENGTH OF MATERIALS AND OF STRUCTURES.

114. A knowledge of the simple principles of mechanics above considered, together with that of the general properties of matter, enable us to determine what forms of bodies and what positions are best calculated to resist forces that tend to break, crush, or overthrow them. This constitutes engineering. A few of the more important general truths thus arrived at may be here noticed.

115. The strains to which the parts of machines and edifices are subjected, are chiefly of four kinds: 1. Tension, as when a rope is pulled in the direction of its length; 2. Compression or crushing; 3. Transverse or Cross Strain; and 4. Torsion or twisting.

TENSION.

116. When a rod of wood is suspended vertically, and a weight attached to it tending to tear it asunder, all its fibres act equally, and its strength evidently depends on the strength of the individual fibres and their number, that is, the area of the cross-section of the rod. If there are two rods, then, of the same material, the one having an area of one square inch in its cross-section, the other an area of three square inches, the last will sustain three times the weight that the other will sustain, and this whatever be the shapes of the sections. The same reasoning applies to a rope; and in the case of a rod of iron or other metal, we may consider the atoms as arranged in rows in the direction of the length of the rod, and by the force of their cohesion forming virtual fibres.

117. The power of bodies to resist tension or tearing force, is called their Tenacity, Direct Strength, Absolute Strength, or Tensive Strength. It depends upon the force with which their atoms cohere; and differs in different substances. The following are the absolute or tensive strengths of some of the more

important materials, as determined by experiments: A rod of ash, 1 square inch in section, sustains a weight of 17,000 lbs.; a similar rod of English oak, from 8000 to 12,000; of red pine, 12,000; cast-iron, 18,000; wrought-iron, 29 tons or 65,000 lbs.; iron wire, 41½ tons or 93,000 lbs.; cast-steel, 134,000 lbs.; copper, 21 tons or 47,000 lbs.; hempen rope, 6000 lbs. From these numbers the tensive strength of a rod or beam of any given dimensions may be found, by multiplying by the number of square inches in the section of the rod or beam. *Ex.* What weight will a cylindrical rod of wrought-iron, 3 inches in diameter, sustain? Here area of section is $3^2 \times \cdot 7854 = 7\cdot 0686$ square inches; and $29 \times 7\cdot 068 = 205$ tons, nearly.

COMPRESSION.

118. Numerous experiments have been made on the power of different materials, to resist crushing force; but no law has been found by which we can deduce with any accuracy the strength of bodies of dimensions different from those experimented upon.

TRANSVERSE STRAIN.

119. Let *ab* be a beam fixed at one end into a wall *ag*, and loaded with a weight W at the other end. If the beam is

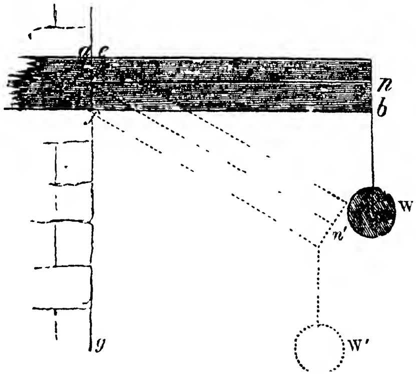


Fig. 56.

to break (supposing it of uniform dimensions throughout), the rupture will evidently be at the wall, because there W acts with the greatest leverage. Before actual rupture, a

portion of the fibres in the upper part of the section from a to c are drawn out or stretched, and a portion in the lower part, from c to f, compressed; and there must therefore be a line of fibres somewhere, as at c, which are neither stretched nor compressed. This line is called the ***neutral axis of rupture;*** and as W tends to bring the beam into the position marked by the dotted lines, we may conceive c to be the fulcrum of a bent lever with three arms, of which W acts on the long arm cn', ce and cd being two short arms at right angles to cn'. The fibres of the upper part of the section (which we suppose not yet ruptured, but only extended) are acting all along the arm ce, as at ik, ae, drawing it towards the wall; while those in the lower part are resisting compression, and pushing the arm cd away from the wall. Both these sets of forces resist the tendency of W to turn the lever about the axis c, and thus together constitute the strength of the beam. The effects or moments of the two sets must evidently be equal to each other; for the neutral axis so adjusts itself that the amount of resistance on the one side, balances that on the other. If the tensive strength of the fibres is equal to their compressive strength, the neutral axis will be in the middle of a rectangular beam; if the tensive strength is greater, the neutral axis will be nearer the upper side; and *vice versâ*.

120. But the most important point to observe in this investigation is that the fibres are not all equally well situated for offering resistance. A fibre at a, for instance, acts with twice the leverage of one at i, supposing i in the middle of ca. If we call the tensive strength of a fibre of certain area 1, its moment or efficacy at a will be expressed by $ca \times 1$, or simply ca; while the moment of a similar fibre at i will be $ci \times 1$, or ci. The sum of the moments of a row of fibres from c to a would thus be expressed by the number of such fibres multiplied by their *mean distance* from c, which is $\frac{1}{2}\ ca$. Since the number of fibres along the line ca increases with the length of the line, we may take that length to represent the number; and therefore, the sum of the moments will be $ca \times \frac{1}{2}\ ca$, or $\frac{1}{2}\ ca^2$. But the resistance to compression made on the arm cd, is equal to the resistance on ca; therefore, the whole resistance to fracture made by a line of fibres from a to f, is expressed by (2 times $\frac{1}{2}\ ca^2$ or) ca^2. And if we put b for the breadth of the beam, the whole strength of the beam is expressed by $b \times ca^2$.

121. Strictly speaking, the fibres, besides having different leverages, do not all pull equally; ae being further stretched than ik, is pulling with greater force. When this is taken into account, the formula has a somewhat different absolute value. It continues, however, to give the same *comparative* strength

for beams of different dimensions; and as this is its chief use, we have, for the sake of simplicity, left the varying tension of the fibres out of account.

122. From a variety of causes, the formula above given does not enable us to calculate at once the transverse strength of a beam from merely knowing the tensive strength of a rod of its fibres. One of these causes is that we have no means of determining where the neutral axis is situated. The formula, however, serves this valuable purpose, that when the actual transverse strength of one beam of certain dimensions has once been determined by experiment, it enables us, from this data, to calculate the strength of another beam of the same general form, but of different dimensions. For, if we take, for instance, two rectangular beams of fir, and suppose $ac = \frac{1}{2}\ af$ in the one, it will be so in the other; the same will be the case if $ac = \frac{2}{3}\ af$, or any other fraction. Thus, whatever fraction ac is of af, in a beam of any material, it is the same fraction in all beams of that material, or the length of ac varies with the length of af; and, therefore, to express the relative strengths of beams, we may put af in the formula instead of ac. If, then, we put d for the depth, or af, the formula becomes $b \times d^2$, which expresses the leading proposition in the strength of materials, that THE TRANSVERSE STRENGTH OF A BEAM INCREASES AS THE BREADTH MULTIPLIED BY THE SQUARE OF THE DEPTH.

123. The truth of this important proposition is fully confirmed by experience, and is constantly acted upon in all kinds of constructions. Rafters, joists, &c., are never made square in the section; but as deep as possible, and no thicker than is necessary to prevent them from bending laterally, or *buckling*, as it is called. Strength is thus gained, and material saved. *Ex.* Let A (fig. 57) be the section of a square wooden beam of 10 inches in the side, and B that of another beam of the same wood, 5 inches broad by 20 deep. The areas of the two sections are equal, being 100 square inches each; but the transverse strength of A is as $10 \times 10^2 = 1000$, and that of B is as $5 \times 20^2 = 2000$. B has thus double the strength with the same amount of material. When the breadth or thickness of a beam is doubled, retaining the same depth, the strength is merely doubled; when the depth is doubled, retaining the same breadth, the strength is quadrupled.

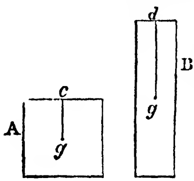


Fig. 57.

124. There is still another point to be taken into account—

namely, the length of the beam. If *cn*, fig. 56, were twice as long as it is, W would act at twice the leverage, and the beam would, in this case, be able to sustain only half the weight; if *cn* were three times as long, the weight sustained would be only one-third. The transverse strength of a beam, therefore, diminishes with its length; and taking this into account, and calling the length l, the formula becomes $\frac{b \times d^2}{l}$.

125. In order to make this expression available for finding the actual weight that any beam will bear, small beams or prisms are taken, of the most usual materials, having a section of one square inch, and a length of 12 inches, and loaded with weights till they break. The breaking weights of these small bars are then used as *constants* or co-efficients, for calculating the breaking weights of other beams of the same materials. The following are a few of the results of experiments made on small bars fixed and loaded as in fig. 56:

Ash,	2030	Oak, English from	1200
Elm,	1030	Oak, English to	2260
Fir, Scotch,	1140	Cast-iron,	8100
Red Pine,	1340	Wrought-iron,	9000

To find the weight that will just break a beam of given dimensions, we have only to multiply the constant for the material in the table by the breadth and square of the depth, and divide by the length—the dimensions being all reduced to inches; or, employing the formula, W = Constant $\times\ b \times d^2 \div l$. *Ex.* What weight will break a beam of Scotch fir, fixed at one end, and loaded at the other, its breadth being 3 inches, depth 11 inches, and length 9 feet? *Ans.* W = $1140 \times 3 \times 11^2 \div 108 = 7663$ lbs.

When the beam is cylindrical, find the strength of a square beam, whose side is equal to the diameter of the cylinder, and take two-thirds of the result. *Ex.* What is the strength of a round beam of ash, diameter 10 inches, and length 12 feet? Here $2030 \times 10 \times 10^2 \div 144 = 14097$ lbs., the strength of a square beam of 10 inches in the side. But a cylindrical beam of 10 inches diameter, has less area by about one-third, and therefore its strength will be $14097 \times \frac{2}{3} = 9398$ lbs. More exactly, the square is to the circle as 1 to ·7854; and $14097 \times {\cdot}7854 = 11071$ lbs.

126. If the weight, instead of being suspended from the extremity of the beam, is distributed uniformly along its length, its effect is as if it were collected in its centre of gravity—that is, in the middle point of the beam. Its leverage is thus only half the length of the beam; and, therefore, *a beam*

loaded uniformly sustains twice the weight that it does when loaded at the extremity. If, in the former example, the load were uniformly distributed, the answer would be $7663 \times 2 = 15326$.

127. The weight of the beam itself constitutes a part of the strain it has to resist. In a uniform rectangular beam, as in fig. 56, that weight is uniformly distributed, and therefore acts at the leverage of $\frac{1}{2}$ *cn*; and calling it w, its moment or strain will be $w \times \frac{1}{2}\, cn$ or $\frac{1}{2}\, w \times cn$. Now, the strain of W is $W \times cn$; therefore, the whole strain on the beam is $W \times cn + \frac{1}{2}\, w \times cn$; or $(W + \frac{1}{2}\, w) \times cn$. The rule applied to the example in art. 125 gives the weight that might be suspended from the end of the beam, supposing it had no weight of its own; therefore, to find the weight that in practice could be attached at the end of the beam, we must subtract from the answer found as above, half the weight of the beam itself. The weight of the beam in art. 125 would be about 88 lbs.; so that the weight that the beam would bear in addition to this is $7663 - 44$, or 7619 lbs.

128. The most frequent case of transverse strain is that of a beam resting freely on supports at the two ends, with a weight or weights pressing somewhere between. If the weight rests on the middle point, as in fig. 58, each of the supports sustains a pressure equal to $\frac{1}{2}$ W. We may therefore consider the reaction of either of the supports, B, as a force acting upwards, and tending to break the beam at C, while the two forces at A and C merely hold the end of the beam fixed, as the wall does in fig. 56. The moment of the force of rupture is thus $\frac{1}{2}\, W \times CB = \frac{1}{2}\, W \times \frac{1}{2}\, AB = \frac{1}{4}\, W \times AB$. But if the beam were fixed in a wall at A, with a force W acting at B, the moment would be $W \times AB$. That is, *a beam supported freely at both ends will support four times as much weight at its middle point, as it would if fixed at one end, with the weight resting on the other.*

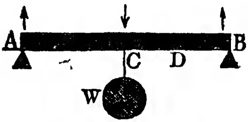


Fig. 58.

Ex. A bar of cast-iron, 2 inches square and 15 feet long, is supported at both ends, what weight applied at its middle will break it? Such a bar, if fixed and loaded as in fig. 56, would support, by the former rule, $8100 \times 2 \times 2^2 \div 180 = 360$ lbs. The breaking-weight in the present case is, therefore, $360 \times 4 = 1440$ lbs.

129. If a beam supported at both ends is loaded uniformly, the weight supported is twice as great as when the load rests on the middle (see art. 126).

130. The strain of W is greater when applied at C, the middle point, than when applied at any other point, D. For if we suppose DB = $\frac{1}{4}$ AB, by the principle of the lever, B will support $\frac{3}{4}$ W, or will press up the end of the beam with a force equal to $\frac{3}{4}$ W; and as the leverage of this force to break the beam at D is DB, or $\frac{1}{4}$ AB, its moment is $\frac{3}{4}$ W $\times$ $\frac{1}{4}$ AB = $\frac{3}{16}$ W $\times$ AB. But when W rests at C, the moment is by art. 128, $\frac{1}{4}$ W $\times$ AB, which is equal to $\frac{4}{16}$ W $\times$ AB, or greater than the former by $\frac{1}{16}$. In general, the strain of a weight at the middle point C, is greater than at any other point D, in proportion as AC $\times$ CB, or CB^2, is greater than AD $\times$ DB. *Ex.*—If a beam 18 feet long, and supported at both ends, sustains 1000 lbs. applied at the middle, what weight will it sustain at 3 feet from one end? Here 15 $\times$ 3 : 9 $\times$ 9 : : 1000 : 1800 lbs., the answer.

131. A beam resting obliquely, as AB (fig. 59), is stronger than the same beam resting horizontally; for the leverage of the forces of rupture are measured, not by AD or DB, but by AE or EC (see art. 29).

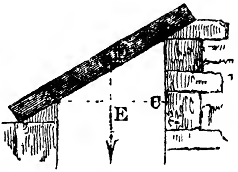


Fig. 59.

132. *Form of Greatest Strength.*—It is evident (see fig. 56) that the fibres of a beam near the neutral axis on both sides, have little efficacy in resisting rupture, and might be removed without much affecting its strength; and this principle is largely applied in constructing metal beams or girders, in order to produce the greatest strength with the least material. For girders of cast-iron, resting on both ends, and loaded in the middle, the form of section adopted is that of an inverted T (fig. 60). Cast-iron has much less power of resisting extension than compression, and therefore the lower *flange* is made much greater than the upper, so as to throw the neutral axis as nearly as possible in the middle of the beam; for the sum of the moments of the two sets of resistances is greater when the axis is in the middle than when it is anywhere else. With wrought-iron, which resists extension better than compression, the upper flange is made the larger. The advantage of the T-form is, that the great mass of the material being collected on the two flanges, acts at the greatest possible leverage. In a series of experiments with cast-iron beams, the strongest form

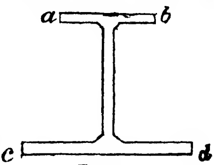


Fig. 60.

was found to be that represented in fig. 60, in which the lower flange, *cd*, is six times the area of the upper, *ab*. With this proportion, the strength per square inch of section of the whole beam was 4075 pounds, whereas 'in the best form of girder used before these experiments, there was never attained a strength of more than 2885 pounds. There was, therefore, by this form a gain of 1190 pounds per square inch of the section, or of $\frac{2}{5}$ the strength of the beam.'

133. *Hollow or Tubular Structure*.—Another way of throwing the great body of the material at a distance from the neutral axis is, to make it into the shape of a tube or hollow cylinder. Let B be the section of a hollow cylinder, the thickness of whose walls is represented by the shaded ring; and A be the section of a solid cylinder of the same material. If the area of A is equal to that of the ring in B, the two cylinders will contain the same quantity of matter, but B will be stronger than A, in proportion as *cg* is longer than *dg*.

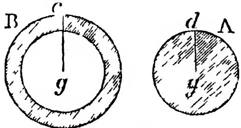


Fig. 61.

The principle of hollow structure prevails both in nature and art, wherever strength and lightness have to be combined. It is seen in the stems of plants, especially of the grasses; the bones of animals are also hollow, and those of birds, where great lightness is required, are most so. A feather, with its hollow stem, is perhaps the best instance of the union of strength and lightness that could be given. In art, again, we have hollow metal pillars; and sheet-iron for roofing and other purposes is *corrugated*, or bent into ridges and furrows, to give it depth. Each ridge or furrow is, as it were, half a tube, and resists bending with twice or thrice the energy it would if flat.

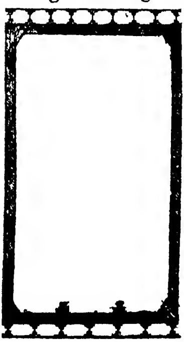


Fig. 62.

134. The most striking application of the principle of hollow structure is seen in Tubular Bridges. Fig. 62 represents a section of the tube of the Conway Bridge. The object being to resist a vertical strain, the form is made rectangular, and the chief mass of the material is thrown into the top and bottom. The tube may, in fact, be considered as an immense beam or girder constructed on the principle of fig. 60, the top and bottom being the two flanges, and the

two sides serving to connect them, instead of the one rib in the middle. As it is constructed of plate-iron, the top requires more metal than the bottom, in order to resist the compression; but instead of putting the metal into one thick plate, or into several plates laid the one on the other, it is made to form a set of minor tubes or cells, which gives additional stiffness and strength to the whole tube. The floor, in like manner, contains cells. Each of the tubes over the Conway is 24 feet high, 14 feet wide (outside), and 420 feet long, and weighs 1300 tons; yet these enormous hollow beams sustain not only their own weight, but the heaviest railway-trains without sensible deflection.

135. Fig. 63 represents an ingenious contrivance for strengthening the wooden beams supporting a bridge. An iron rod, fixed to the beam AB at the two ends, is kept at a distance by struts c, c'. The beam cannot now be bent downwards without stretching the rod; which thus has to bear the tensive strain, while the beam itself sustains only the compressive strain.

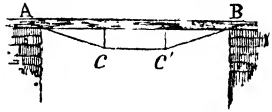


Fig. 63.

136. Another way of removing part of the strain from a girder, is to fix a king-post and two oblique pieces on its upper side, as in fig. 64. The whole is now one composite girder; and when any weight bears upon it, the whole of the compressive strain is thrown upon the pieces a, b, and only the tensive strain is left for the beam to sustain.

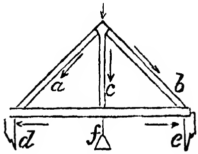


Fig. 64.

137. When a beam AB is fixed at one end, and loaded at the other, the strain is greatest at B, and is less at other points, as c, c', in proportion as Ac, Ac', the levers at which it acts, are less than AB. The beam may therefore be made to taper off towards the end, and we may determine the exact form the beam should have, in order to be equally strong at every point. For, supposing the breadth uniform, the strength increases as the squares of the depths $c'd'$, cd, while the strain increases as the levers Ac', Ac; and thus, if $Ac : Ac' :: cd^2$ to $c'd'^2$, the strengths are

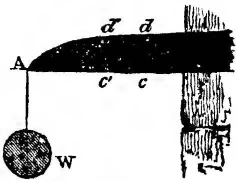


Fig. 65.

equal at those points. This proportion will always hold good, if the curve of the beam is that of a parabola; and, accordingly, this is the shape given to the beams of steam-engines.

138. In beams supported at both ends, the strain is greatest in the middle (130); girders are therefore made strongest in the middle, and taper towards the ends.

139. For beams of other forms of section than rectangular and circular, no general rule can be given. With cast-iron beams of the form and proportions described in art. 132, Mr Hodgkinson finds that the strength increases as the area of the lower flange, and as the depth of the beam.

140. In experimenting on transverse strains of materials, it is found that a certain amount of straining force produces a *set*, or permanent change of form. This state must result from an entire destruction of the elasticity by the overstraining or *setting* force; and, as it cannot take place suddenly, it follows that the elasticity must gradually diminish as the force increases, till the material attains this state. This setting effect of a straining force, assists in explaining the rather anomalous results obtained by different experimentalists on the strength of materials.

TORSION.

141. If one end of the axle or shaft of a wheel is immovably fixed, and a power acts at the circumference of the wheel (or at the end of a lever or winch), the power may be so increased as to twist the shaft asunder at its weakest point. If a shaft A has twice the diameter of another shaft B, there will be four times as many fibres in the section of fracture of A, to resist the twist, as in that of B. But as the separation takes place by the one end of the fracture turning round upon the axis of the shaft, making the ends of the separating fibres describe circles, those fibres that are furthest from the centre will have the greatest power of resistance, and the sum of their moments, or their united effect, will be in proportion to their mean distance from the centre. This mean distance in A is twice that in B; therefore, the resistance in A is 2×4 or 8 times the resistance in B. Generally, *the strength of shafts to resist torsion is as the cubes of their diameters.* On the other hand, the twisting force that may be applied to the wheel or lever, will be less as that lever is longer. The torsive strengths of shafts 1 inch diameter, and with weights acting at 1 foot leverage being found by experiment for different materials; the strength of shafts of other dimensions are found from these 'constants' by multiplying by the cube of the diameter, and dividing by the length of the lever. It is evident that the

torsive strength of a hollow shaft will be greater than that of a solid one of the same quantity of material, on the same principle that its transverse strength is greater (see art. 133).

TRANSMISSION OF MOTION.

142. The object of all machinery is to transmit and modify motive-power. It is not, as already explained, to create or multiply power; for no machine can give off more working-power at one part than has been applied to it at another. The most usual and convenient form of motive-power is rotary or circular motion; having once a motion of this kind, we can transmit it to any point where it is required, and can alter its velocity, and change it into another kind of motion, at pleasure. The more important elements of machinery by which these purposes are effected, we now proceed to notice.

ELEMENTS OF MACHINERY.

143. *Shafts.*—A rigid bar of metal or wood made to revolve on its axis by any motive-power applied at one part, is capable of conveying and giving off that power at any part of its length, to machinery connected with it. A large bar or beam of this kind is called a *shaft;* a smaller one, a *spindle.*

144. In fig. 66, *abc* represents a portion of a horizontal shaft; *hi*, of a vertical one; *de* and *f* are portions of a small shaft or spindle. The part of a shaft on which it rests, while turning, is called the *journal*, or *gudgeon*, as *a*, *f*, *i*. The *boss* is the part of a shaft on which a wheel is fixed, as *b* and *oo*. The

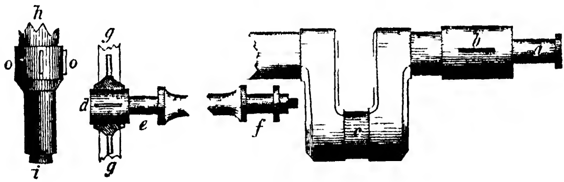


Fig. 66.

boss has two or more grooves cut in it, and the eye of the wheel has corresponding grooves; when the wheel is passed on to the boss, wedges, called *keys*, are driven into these *key-ways*, which prevent the wheel from slipping round on the shaft. *gg* is part of a wheel keyed on to the boss. The journal is usually turned smaller than the rest of the shaft, with projections at the sides, called *ruffs* or *collars*, as at *f*.

145. *Bearings.*—The *bearings* of a shaft are the rests in which its journals or ends turn. For horizontal shafts the form is that called a *plummer-block*, represented in fig. 67. It consists of two parts; a base *aa*, with bolts *b*, *b* for fastening it to the beam; and a cap *cc*, which is fastened to the base by bolts *d*, *d*. The block itself is of cast-iron; but the cavity or eye is lined with a cylinder of brass, gun-metal, or other alloy. The lining is made in two parts or half-cylinders, which are called *bushes* or *brasses*. To prevent them from turning round with the shaft, the outer surface forms a hexagon, as seen in the left-hand side of the cut, which gives a section of the block, the right-hand side being a front view. The ends of the bushes project in the form of a circular flange, seen at *f*; this prevents them from sliding outwards or inwards. On the top of the cap at *o*, is an oil-cup, with a hole passing down to the shaft, for lubricating it. As the brasses wear away, and the journal works loose, the cap is screwed down by the nuts *d*, *d*. Fig. 68 gives the plan of the block, with the shaft *ee* in its place.

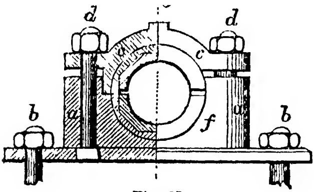


Fig. 67.

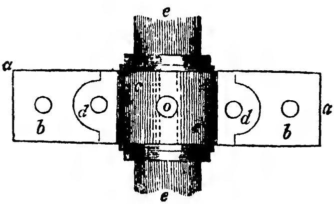


Fig. 68.

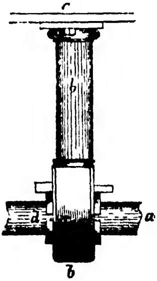


Fig. 69.

146. Horizontal shafts are sometimes supported by *Hangers* or *Suspension-bearings*, *bb* (fig. 69), bolted to a beam *c* above. The shaft *a* turns in brasses, the flange of which is seen projecting, as at *d*.

147. The lower bearing of a vertical shaft is called a *step*.

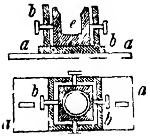


Fig. 70.

Fig. 70 gives a vertical section and a plan of a step. Inside a cast-iron box *bb* is placed a brass block, with a cavity *e*, in which the end of the shaft (see *i*, fig. 66) turns. The screws are for adjusting the brass, so as to make the shaft perfectly vertical. The iron box is bolted down by its flanges *a*, *a*.

148. *Couplings for Shafts.*—To get a shaft of sufficient length, it is sometimes necessary to join two or more pieces. Fig. 71 illustrates the ***square-box coupling***. The two ends to be joined, *b*, *a*, are faced up square, and inserted in a square box *cc*; corresponding grooves having been made in the shaft and box, wedges or keys are driven in, to tighten the grasp of the box. The *round coupling* is

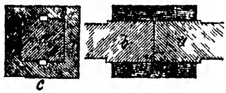


Fig. 71.

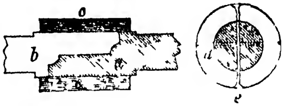


Fig. 72.

similar to that now described, except that the ends of the shafts and the box are cylindrical. The ends of the shafts are sometimes made to overlap, as in fig. 72, called the *half-lap coupling*.

149. *Bands or Straps.*—Bands or straps are used to convey motion from one shaft to another parallel and distant shaft. In fig. 73, A is a *drum* or *pulley* with a flat surface, supposed to be fixed on a shaft put in motion by the source of power, B is another pulley fixed on the shaft S; and over the surfaces of the two a broad leather belt, L, is stretched tight. As A turns, the friction between its surface and that of the belt, carries the latter along with it, and thus B is also made to revolve. The shaft S thus set in motion, may be made to communicate rotation to any number of pulleys and spindles, C, E, each of which may drive a separate apparatus. The main stream of driving-power is thus distributed into a number of small rills.

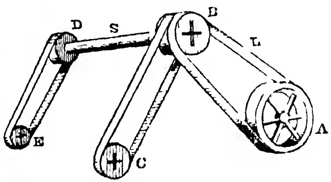
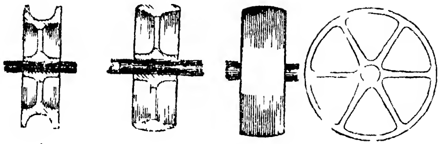


Fig. 73.

150. Fig. 74 presents front-view, edge-view, and section of a pulley used with a flat band; the surface is made convex, as at *b*, which prevents the band from coming off, as it has a

tendency to keep to the highest part. *d* is the section of a pulley driven by a chain or rope.



d c b a

Fig. 74.

151. In order to economise the power, the band should have no more tightness than is just sufficient to prevent its slipping; for it is evident that the greater its tension, it will cause the axes of the pulleys to press the more against their bearings, and thus increase the friction of the machine. When the two parts of the band are made to cross each other, as in fig. 75, it works with less tension, owing to its embracing a larger arc of each drum. There is thus some economy of power in this arrangement; besides that it serves another purpose—namely, that of reversing the direction of the rotation.

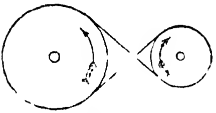


Fig. 75.

152. *Toothed Wheels.*—If the two pulleys or drums in fig. 75 were near enough to touch each other, by making the one revolve, the other would be made to revolve with it. This method of communicating motion serves only when the resistance to the driven pulley is very slight. The usual and sure method is to raise projections or *teeth* on the pulleys; this contrivance is called *toothed gearing*.

153. Let C and C′ (fig. 76) be the centres of two toothed wheels in gearing with each other (the centres are not represented in their actual positions, in order to save space). The line CC′ is called *the line of centres;* and the two circles, *gg*, *hh*, which touch each other in T, midway between the extreme projections of the teeth, are the *pitch circles*. The *pitch* of the teeth of a wheel is the distance, AB, from the centre of one tooth to the centre of the next, measured upon the pitch circle; and it is evident that for two wheels to work together, the pitch must be the same in both—that is, AB must be equal to TD. The breadth of the teeth is made a little less than the intervals between them, that they may have room to engage and separate without becoming locked in consequence of any slight irregularity.

154. The actual size of the wheels is not indicated by their solid rims or bosses, *ee*, *ff*, but by the pitch circles, *gg*, *hh*; and

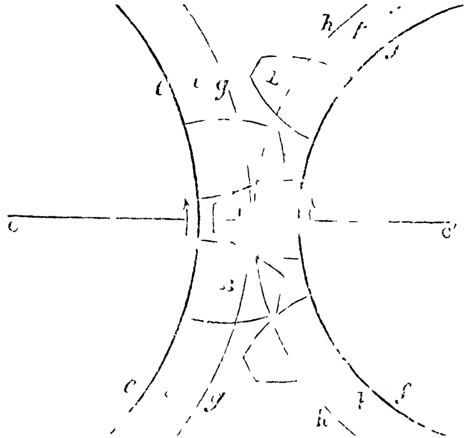


Fig. 76.

we shall best understand what is necessary to the true working of toothed wheels, if we conceive these circles to be, in the first instance, two drums touching at T, and the one driving the other by friction. It is obvious that while any portion, as a foot, of the drum *hh*—supposing it to be the driver—passes through the point of contact T, it causes an equal portion of the driven drum *gg* to pass through the same point; in other words, the driven circumference moves with the same velocity at every instant as the driving one. Now, when a part of the width of each wheel is removed, and projecting teeth substituted, these teeth must be so contrived that the primitive or pitch circles shall still move exactly as if they were in contact. Mere straight pegs or blades, like the paddle-boards of a steamboat, would not answer the purpose. In the first place, the corners of one tooth would have to slide along the face of another, thus grinding away the teeth, and wasting the driving power. Secondly, the motion would not be equal; during the first portion of the time that two teeth were in contact the driven wheel would move slower than the driver, and during the last portion, faster. The faces or edges of the teeth must, therefore, be curved, so that they shall roll on one another

rather than slide; and the curves must be such that the driven wheel shall move at all times uniformly with the driver.

155. Mathematicians have discovered a great variety of ways of satisfying this condition. One way, frequently adopted, is to give the teeth of one wheel the form of the curve called the epicycloid, and those of the other that of the hypocycloid. But the form of teeth that possesses the greatest number of advantages for most purposes is that of the curve called the *involute*.

156. Let the circle, fig. 77, represent a disc or pulley lying on the plane of the paper, and $m9$ a thread fastened to the disc

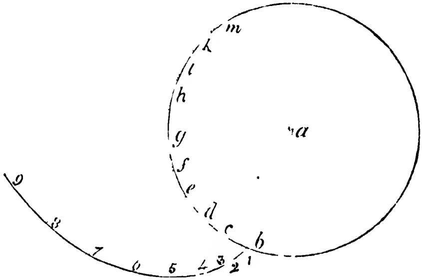


Fig. 77.

at m; if a pencil-point at 9 keeps the thread stretched while winding it on the circumference, from m to b, the curve 9, 8, &c., b, which it traces, is called an Involute (wound on). The unwinding of the thread will evidently trace the same curve. The circle on which the thread is wound is called the *base* of the involute.

157. In striking a pattern for the involute teeth of a wheel, a circle ii, fig. 76, is drawn with a radius a certain small fraction less than that of the pitch circle gg; care being taken that the proportion of the corresponding circles, kk and hh, in the other wheel, is made the same. A small portion of the involute raised on the base, ii, or kk, will give the outline of the edges proper for the teeth of the wheel. A practical way of tracing the curve is to take a piece of thin board, with a circular edge similar to an arc of the circle ii. A piece of watch-spring is then applied to the edge of the board in the manner of a hoop, and has one of its ends fastened with a tack; while, at the other end, a fine point projects from the edge of the spring. The edge of the board being now applied to the circle ii, with the

projecting point of the spring downwards, and the free end of the spring being allowed to uncoil, the point will trace the involute.

158. Since any small portion of an involute, as the arc from 2 to 4, fig. 77, is very nearly an arc of a circle described from *e* with radius *e*3, an approximation may be made to the shape of the teeth by circular arcs described from centres in the bases *ii*, *kk*.

CHANGES IN THE PLANE AND IN THE DIRECTION OF ROTATION.

159. When the two shafts to be connected are not parallel to each other, conical or *bevel* wheels are employed. Fig. 78 represents a horizontal shaft driving a vertical one (or the converse), by means of bevel gearing. The two shafts may have any inclination to each other and to the horizon. When the two wheels are equal and the axes at right angles, they are called *mitre* wheels. The outline of the edges of bevel teeth are struck on the same principles as in plain gearing; only that the dimensions both in depth and width decrease inwards according to the inclination of the axes. A *crown wheel* A, and *trundle* B, as represented in fig. 79, is another contrivance for effecting the same purpose.

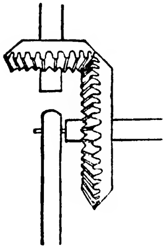


Fig. 78.

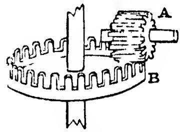


Fig. 79.

160. In the case of two toothed wheels, the direction of the rotation is always reversed, as shewn by the arrows in fig. 76. In order to preserve the same direction in both, wheels are sometimes geared internally, fig. 80. By interposing a third wheel between the driving and driven wheels, the original direction is also retained; thus, the pinion F, fig. 81, revolves in the same direction as the wheel C. A crossed band (see fig. 75), reverses the direction.

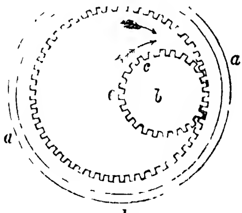


Fig. 80.

CHANGES IN SPEED.

161. If the wheel C, fig. 81, had three times as many

teeth as the pinion G, one revolution of C would evidently cause three revolutions of G. If, instead of being an exact multiple, the teeth of C were, say 210, and those of G 90, which is in the proportion of 7 to 3, then 3 revolutions of C would cause 7 of G. Suppose, again, that C with 101 teeth were driving B with 100 teeth. These numbers being prime to one another, the proportion is not expressible in any smaller numbers than the numbers of the teeth themselves; in other words, if we note the two teeth that are at any instant in contact, C must make 100 revolutions, causing B to make 101, before the same two teeth will be again in contact —that is, before the two will finish together each an exact number of revolutions.

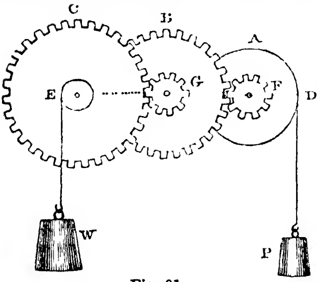


Fig. 81.

162. In general, then, the rates of rotation of two toothed wheels are *inversely* as the numbers of the teeth. Since the teeth of wheels in gearing with one another must be of the same pitch, the numbers of the teeth will be in proportion to the circumferences; which are again in proportion to the diameters or to the radii. The rates of rotation are, therefore, inversely as the circumferences, or as the diameters, or as the radii—meaning, of course, the circumferences, diameters, or radii of the pitch circles. The same is true of wheels or pulleys connected by bands.

163. By this proportion the size of wheels is so adjusted, as to give the driven wheels or *followers* any desired speed, as compared with the drivers.

Ex. 1. It is required to communicate motion from one shaft to another, so that the driven shaft shall make 7 revolutions for 2 of the driver. Suppose, first, that the shafts are at a distance from each other, and are to be driven by a band. The size of one of the pulleys being fixed upon, say the driver at 15 inches radius, the radius of the driven pulley is found by the following proportion:

$$7 \ : \ 2 \ :: \ 15 \ : \ \frac{2 \times 15}{7} = 4\tfrac{2}{7} \text{ inches.}$$

Next, suppose that toothed gearing is to be used, and that the distance between the centres of the wheels is fixed at 25 inches. This line must be divided between the radii of their pitch

circles (see fig. 76) in the proportion of 7 to 2; which is done as follows:

$$(7 + 2, \text{or})\ 9 \ : \ 7 \ :: \ 25 \ : \ \frac{7 \times 25}{9} = 19\tfrac{4}{9}.$$

This gives $19\frac{4}{9}$ in. for the radius of the larger wheel; and, therefore, $5\frac{5}{9}$ (or $25 - 19\frac{4}{9}$) in. for that of the smaller. The pitch or thickness to be given to the teeth will depend upon the strain to be sustained; but the numbers of the teeth in the two wheels will be in proportion to their diameters—namely, as 7 to 2, and, therefore, their relative speed will be as 2 to 7.

Ex. 2. A driving-wheel with 200 teeth of $\frac{3}{4}$ in. pitch revolves 5 times a minute; required the radius of a wheel to gear with it revolving 50 times a minute. Since the pinion has to move 10 times faster than the driver, it must have $\frac{1}{10}$ the number of teeth, or 20; and to hold 20 teeth of $\frac{3}{4}$ in. pitch, its circumference must be $20 \times \frac{3}{4} = 15$ inches. The circumference of a circle being to its diameter as 3·1416 to 1, the diameter of the pinion is found thus:

$$3{\cdot}1416 \ : \ 1 \ :: \ 15 \ : \ 4{\cdot}77.$$

The radius is therefore 2·39 inches.

164. In a train of wheels connected as in fig. 81, the velocity of the last wheel is found by the following rule: Multiply the velocity (or number of revolutions in a given time) of the first driver by the product of the numbers of teeth in all the drivers, and divide the result by the product of the numbers of teeth in all the followers. In this rule, the diameters, or the radii, or the circumferences, of the wheels may be substituted for the numbers of the teeth.

Ex. Let the wheels C and B, fig. 81, have 80 and 60 teeth respectively, and the pinions G, F, 12 teeth each; and let the speed of C be 30 revolutions a minute; required the speed of the pinion F.

$\frac{30 \times 80 \times 60}{12 \times 12} = 1000$, the number of revolutions made by F in a minute.

165. In the motion of wheels there are two kinds of velocity, which it is necessary carefully to distinguish. While a tooth of the pinion G makes, say, one-sixth of a revolution, or describes one-sixth of the circumference of the pinion, a tooth of B also makes one-sixth of a revolution, or describes one-sixth of the circumference of B. The two teeth thus describe equal angles about the common centre, and are therefore said to have the same *angular* velocity. But if we consider the absolute spaces over which the teeth of the pinion and of the wheel pass as measured in inches, and not in degrees, it is

evident that a tooth of B describes a line longer than that described by a tooth of G, in proportion as the one circle is larger than the other. This may be called the *linear* velocities of the teeth. If we compare G with C, the linear velocity of the teeth is the same in both; for their pitch is the same, and the breadth of a tooth of each must pass through their point of contact in the same time; but their angular velocities or rates of motion, each round its centre, are very different.

166. In a train of wheels, the strain upon the teeth is different at different parts of the train, according to the greater or less linear velocity with which they are moving. Suppose that the teeth of C are moving at the rate of a foot a second, and exerting on those of the pinion G a pressure of 1 cwt.: the teeth of G move at the same rate, and have to sustain the same strain; but those of B, being five times as far from the centre, move 5 feet in a second; and as the working-power of a machine can at no point be greater than the power that sets it in motion, the pressure that the teeth of B can exert is only $\frac{1}{5}$ cwt.; for a pressure of $\frac{1}{5}$ cwt. moved through 5 feet is as much work done as 1 cwt. moved through 1 foot. Without the consideration of work done, the principle of the lever shews, that if the motive pressure on the teeth of G is 1 cwt., any greater resistance on those of B than $\frac{1}{5}$ cwt. would stop the motion. The strain on the teeth thus diminishes with their linear velocity, so that the teeth of C and G require to have three times the strength that would suffice for those of B and F; while the teeth of A, if we suppose it a driver in turn, might be still weaker than those of F.

167. Sometimes it is desirable to vary the speed of a part of a machine at different parts of a process, while the driving-power continues the same. One way of effecting this is by conical pulleys or drums, as in the fig. (82). According as the band is shifted along from one end to the other—and this the machine is made to do itself—the relative speed of the two pulleys will vary to the desired degree. A slightly different contrivance is represented in fig. 83, in which the difference of diameter proceeds by steps. With the belt over *b* and *c*, the velocities of

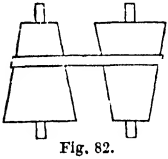


Fig. 82.

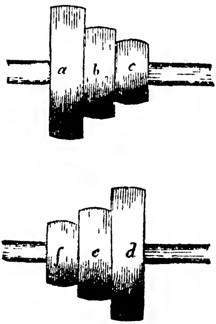


Fig. 83.

the two shafts will be equal; over *a* and *f*, the velocity of the lower shaft will be greater than that of the upper; over *c* and *d*, the reverse.

CHANGE OF ONE KIND OF MOTION INTO ANOTHER.

168. By means of a rack, *b*, and pinion, *a* (fig. 84), a rotary motion is converted into motion in a straight line. Both motions in this arrangement are necessarily alternating.

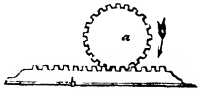


Fig. 84.

169. Fig. 85 represents a continuous rotary motion converted into an alternate rising and falling. The projections, called *cams* or *wipers*, of the wheel W, depress the end of the lever bearing the hammer H, so that it is made to rise and descend on the anvil A thrice during each revolution of the wheel. The stampers of fulling-mills, and those used for crushing oil-seeds and other substances, are in a similar way lifted and dropped by the wipers of a wheel coming in contact with a pin projecting from the stamper.

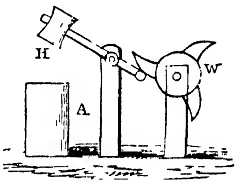


Fig. 85.

170. For converting an alternating straight motion into a continuous revolution, the most common means is the *crank*. A crank is an arm or bend on an axle or shaft (see *c*, fig. 66). The winch (fig. 33) is a crank on the end of an axis, intended to be driven by the hand; but more usually a crank is driven by a connecting-rod, as in fig. 102, where *g*, the rod, is connected with the crank-arm, *h*, by a pin turning in an eye in the end of the rod. The pin thus corresponds to the handle of the winch. A crank may have part of the shaft on both sides, so that one rod, S, may drive two wheels, W, W, as in fig. 86.

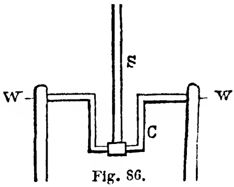


Fig. 86.

171. In turning a machine with a winch, the hand may exert more or less pressure at every point of the circuit; but it is only about the middle of each of the two semicircles, while the handle is being pulled towards the body, and again while it is being pushed away, that the full strength of the arms is effective. In a crank worked by a rod, there are two positions in which the effect of the pressure of the rod is reduced to nothing—namely, in fig. 102, when the crank-arm *h* is in the

same straight line, upwards or downwards, with the rod *g*. A push or pull of the rod in these positions can only press the shaft against its bearings. The effect is greatest when the rod and crank-arm are at right angles, and it decreases gradually on both sides of that position, until at the top and bottom it is reduced to nothing. In order to carry the crank over these *dead points*, as they are called, a heavy wheel, called a *fly-wheel*, is fixed on the shaft; this receives part of the force of the rod while at its best, acts as a reservoir, and by its stored-up momentum carries the shaft round when the rod is powerless.

172. In marine-engines and in locomotives, a fly-wheel is unnecessary, and would be objectionable, owing to its weight and the room it requires. In these cases, the power is generally applied to the shaft at two points by two connecting-rods, each proceeding from a separate cylinder; and the two cranks or bends of the shaft are made at right angles to one another, so that while the one is at one of its dead points, the other is at one of its best positions.

173. The crank can effect the converse of the change now described—that is, the conversion of a rotary motion into an alternating rectilinear one. But it is more usual, when an alternating motion of only small power is required, to take it from a revolving shaft by means of an *eccentric;* thus, in fig. 100, the force-pump *h′*, which supplies the boiler with water, is worked by the rod *nn* attached to an eccentric wheel on the revolving shaft. An eccentric is a circular disc or pulley, fixed on a shaft or axis which does *not* pass through the centre of the disc. The right-hand figure of cut 87 represents a side-plan of an eccentric; *aa*

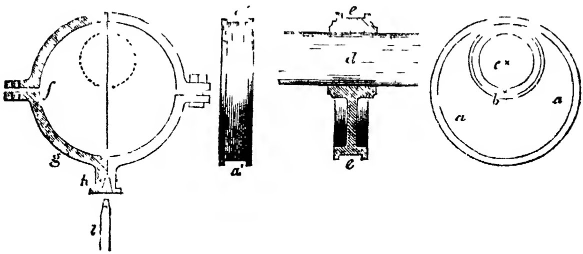


Fig. 87.

the disc, the centre of which is at *b*; the inner circle is the shaft, with its centre at *c*; *ce* is a section along the axis of

the shaft d; and $a'a'$ an edge-view. A hoop, gf, embraces the groove $a'a'$, allowing the disc to turn within it. As the eccentric revolves with the axis, the hoop is alternately raised and lowered, and with it the rod l, which is keyed into it at h. The extent of the rise and fall of the rod is equal to twice cb, the distance between the centres.

174. A great variety of fluctuating movements are effected by means of cams or wheels of irregular outline. Fig. 88 represents a heart-shaped cam, on the upper part of whose circumference rests a pulley a, supporting the end of a rod c. As the cam turns on the centre of the shaft b, the rod c, which is prevented from moving laterally, will be alternately raised and let down, and that with ever-varying velocity.

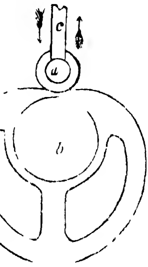


Fig. 88.

175. One of the most simple and elegant contrivances for converting motion in a curve into motion in a straight line, is the *Parallel Motion*, invented by Watt. In the usual form of the condensing-engine, the piston-rod, instead of directly turning the crank of the shaft, as in fig. 102, is made to give an alternating motion to a lever-beam, as in fig. 104. As the end of the beam moves in a curve, while the piston-rod must move straight up and down, it is evident that the two cannot be directly jointed to one another. Nor will a simple connecting-rod do, for the piston-rod would be pushed and pulled from side to side against the air-tight stuffing-box of the cylinder. One way of obviating this is to give the piston-rod a *cross-head, ee* (fig. 102), and *guide, ff*. But Watt's parallel motion effects the same purpose, with less friction. In fig. 89, b is the end of the beam of an engine, and e a point midway between b and the centre of motion of the beam. To b and e are jointed

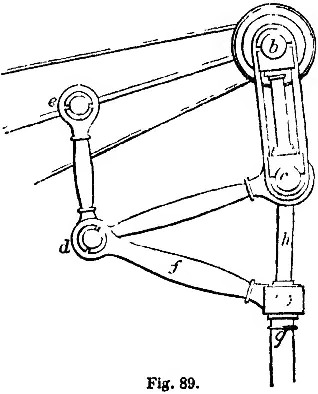


Fig. 89.

two equal rods or links *bc*, *ed*, which are connected by another *dc*, forming a parallelogram. Another rod *f*, called the *radius-rod*, equal to *eb*, is jointed to *d*, and to a fixed fulcrum *g*. Now, as the beam oscillates, the points *b* and *e* describe curves turned in one way, and the point *d* a curve turned the opposite way; but the point *c* is so controlled by *b* and *d*, that it describes a path which is very nearly a straight line. The piston-rod *h* is therefore jointed to *c*, and thus moves the beam without itself suffering lateral thrusts. (In the figure, *h* is to be conceived as working behind, and unconnected with the fixed support *g*.) The middle point of the rod *cd* also moves in a straight line, and usually has the rod of the air-pump *g* (fig. 104) attached to it.

ACCUMULATION AND REGULATION OF POWER.

176. When a boy has been for a time giving successive pushes to a swing with another boy seated in it—if he were to attempt to stop the swing all at once, he would likely be thrown down. And yet there is no more force in the swing than he himself put into it—not even so much, for part has been lost by friction. The force of each push is not spent in one oscillation, but is carried on, with slight loss, through all succeeding oscillations; and the resulting momentum is the sum of all the separate impulses, and makes up a force too great for any one effort that he can make to arrest it. It was on this principle that the battering-ram of the ancients was constructed. A heavy beam or tree, with an iron head—usually in shape like that of a *ram*—was suspended horizontally in ropes from a high frame; and a number of men stationed at the sides made it swing backwards and forwards by cords, and give successive blows to the wall to be demolished. Powerful as the blows were, they could never exceed the accumulated impulses given by the men in drawing it back through a considerable curve, and then pulling it forward again; the ram only served to concentrate into one instant the efforts of the men continued over a sensible length of time. In like manner, the use of a sling is to accumulate in the stone the successive impulses given by the hand to the string while it is swung round and round. The short race taken before a leap is another illustration of the same thing.

177. The stamping of coins and medals furnishes an example of the application of this principle in the arts. The annexed figure represents a coining-press in its simplest form. A thick screw, S, works freely in a strong iron frame by means of a horizontal lever loaded with heavy weights, L, L, at the

ends. A steel die with the device for one side of the coin or medal is fixed at D, and the die for the other side is fixed on the end of the screw. A disc of blank metal of the required thickness and weight is laid upon the lower die, rapid motion is given to the lever, and the screw descends, expending in one rapid blow the force accumulated in the whirling weights, L, L. The press is fed with blank discs, and the lever put in motion, by machinery; and one press can stamp from 60 to 70 pieces per minute.

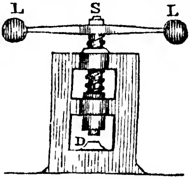


Fig. 90.

178. The fly-wheel furnishes an instance both of the accumulation and equalising of power. A fly-wheel is made of cast-iron, as large in diameter as convenient, and with the greater part of the material in the rim, and is placed on the shaft near the driving crank. In most operations, the resistance offered to the working parts of a machine fluctuates from one part of the process to another; and were it not for the fly-wheel, the motion would be constantly changing, with sudden jerks, from slow to rapid, and from rapid to slow. But the fly-wheel, whenever the resistance of the work diminishes, acts, by its inertia, as a reservoir of resistance, and makes the increase of velocity gradual; and when the resistance of the work becomes, on the other hand, excessive, the superfluous power previously absorbed and treasured up, is now expended, and prevents a sudden check of the speed.

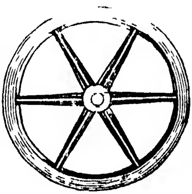


Fig. 91.

179. In the *fusee* of a watch, we have an ingenious contrivance for making a constantly diminishing motive-power produce an equable motion. The motive-power is a steel spring in the form of a coil, fig. 92. It is placed in a box, B,

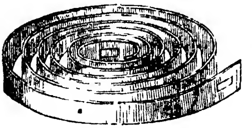


Fig. 92.

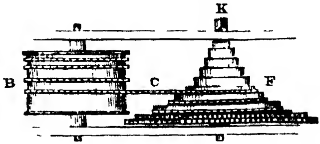


Fig. 93.

fig. 93, the outer end being fixed to the circumference of the box, and the inner to the axis on which the box turns, which axis

is fixed in the frame. The turning of the box in the direction BC, thus winds the spring on the axis, and the force of recoil of the spring makes the box again turn in the direction CB. The fusee F is a conical-shaped wheel with a spiral groove, on which the chain C is wound, which is also attached by its other end to the barrel B. A key applied to the end of the axis of F winds the chain from the box B on to the groove of F, and in doing so coils up the spring in the box. Being now left to itself, the recoil of the spring makes the box pull the chain, and thus turn the fusee; and the lower edge of the fusee being provided with teeth, sets the train of wheels in motion. But the recoil of the spring is a varying force, being greatest when it is furthest compressed or wound up—that is, when the whole chain is wound from the box on to the fusee. We might, therefore, expect it to cause the wheels to move with less and less rapidity as it became unwound. But now the regulating effect of the fusee comes into operation; for while the force of the spring is at its best, the chain acts on a part of the fusee of small radius or leverage; and as that force diminishes, the radius of the fusee increases in the same proportion, so that the axis of the fusee is always turned with a constant force. The shape of the fusee that makes this compensation perfect is the curve of the hyperbola.

180. The *governor* of the steam-engine is another contrivance for regulating motion, which will be described afterwards (242).

ENGAGING AND DISENGAGING MACHINERY.

181. Where numerous machines are propelled by a common power, it is important to possess the means of stopping any one of them at pleasure, and of restoring its motion without interfering with the rest. One of the simplest devices, in the case of motion by a belt, is that called *fast and loose pulleys*, and consists in having two pulleys side by side on the driven shaft, one fixed to the shaft, the other loose. When the shaft is in motion, the belt passes over the fixed pulley; when it is wished to stop the shaft, the belt is shifted on to the loose pulley, which then turns with the belt, but without turning the shaft. The shaft is again put in connection with the moving power by shifting the belt to the fast pulley. The strap is turned off and on by a forked lever, which answers readily to the hand of the workman.

182. Another method of throwing a shaft in and out of connection is the *sliding-clutch*, or coupling. *a*, fig. 94, is the end of the shaft in connection with the driving-power; *b*, with the machine to be driven. On *a*, a coupling-box, *d*, with

indentations, is fixed; on *b* is another box *g*, with corresponding indentations; but while *d* is immovable on the shaft *a*, *g* can

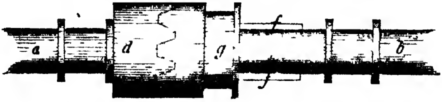


Fig. 94.

slide backwards and forwards on *b*, and is yet prevented from turning round on *b* by means of two projections or feathers, *f*, *f*. When *b* is to be disengaged from *a*, a lever whose fork embraces the sliding-clutch at *g*, draws it back along the shaft until the projections of the two boxes are clear, when the motion of *b* ceases. The reverse motion of the lever again engages it.

FRICTION.

183. When one body rubs against another as it moves, a certain force is felt to resist the motion. This resistance is called *friction*. As a considerable proportion of the motive-power in all operations is spent in overcoming the friction of the parts of the machine upon one another, and is thus lost for the useful work, it is of great importance to understand the nature of this obstructive force, with a view to reduce it to the least possible amount. Accordingly, a great many careful experiments have been made on this subject, and the result is a number of precise and valuable facts or laws regarding friction, which are now considered certain and reliable. The more important may be thus stated and illustrated.

184. When a block of oak—say, a cubic foot, which weighs about 60 lbs.—is placed on a horizontal table of cast-iron, the two surfaces being flat and smooth, it requires a force of nearly $\frac{2}{5}$ the weight of the block, or 24 lbs., pulling horizontally, to make it slide along the table. This measures the friction between the two surfaces. Another block of the same size and shape laid on the same table, would require the same force to draw it; and if the two were laid side by side, and fastened together so as to become one block, it would evidently require double the force, or 48 lbs., to draw the double block; the amount of the friction being thus still $\frac{2}{5}$ of the weight, or of the pressure between the two surfaces. But suppose that, instead of being laid side by side, the second block were laid on the top of the first, what is to be expected? Here the weight is doubled as before, but the extent of rubbing surface remains unaltered; it would be natural, therefore, to expect that this

would make a difference, and that, though the friction would, of course, be increased, the increase would be less than in the former case. Experiment, however, shews that there is no difference, and that the friction is just double in both cases. In short, the unexpected and important fact is established, that, *within certain limits, the friction of any two surfaces increases in proportion to the force with which they are pressed together, and is wholly independent of the extent of the surfaces in contact.*

185. The amount of friction between two bodies is thus a constant fraction or proportion of the force with which they are pressed against each other. This fraction differs for the different kinds of surfaces. Thus, between oak and cast-iron, it is, as already stated, about $\frac{2}{5}$, or more exactly, ·38; for wrought-iron on wrought-iron (we speak at present of dry surfaces, without grease or unguent of any kind), it is ·44; for brass upon cast-iron, ·22. This constant fraction (expressing the proportion between the pressure of two surfaces and their friction) is called the *coefficient of friction* for these two surfaces.

186. Another way of illustrating this law of friction is the following, which has an important bearing on the erection of structures, and on mechanics in general. Suppose a slab AB, in contact with another slab CD, of the same or of different material; and that a force PQ presses on AB obliquely. Let QR be the perpendicular to the two surfaces, and draw PR, PS parallel to AB and QR, thus resolving the force PQ into two forces, one, PS, pressing AB against CD, the other, PR or SQ, tending to make AB slide towards C. It will clearly depend upon the strength of friction between AB and CD, how far the force PQ may be made to decline from the perpendicular without actually causing the one body to slide on the other. Suppose that when the pushing force is brought into the position P′Q, AB is just ready to slip on CD, and that it is a case of oak upon iron; then, since P′S′ or R′Q is the force pressing the surfaces together, and P R′ or S′Q the force tending to produce motion, P′R′ will be $\frac{2}{5}$ of R′Q (by 185). The angle P′QR′ is called *the limiting angle of resistance* of the two surfaces AB, CD; for so long as the direction of the pressure PQ is within that angle, the friction of the surfaces will sustain

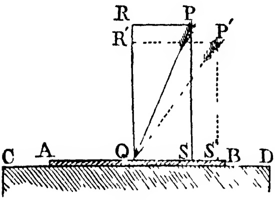


Fig. 95.

it; but if the obliquity is greater, the surfaces will slip. This is true, independently of the extent of the surfaces in contact; and also of the amount of the pressure; for the stability depends upon the proportion of PR to RQ, and that is the same, whatever is the length of PQ, so long as its inclination is the same.

187. If the slab CD were tilted up, so as to form an inclined plane, until AB were on the point of sliding, the angle of inclination would be found to be equal to the limiting angle of resistance RQP′ (see art. 96).

188. Knowing the coefficient of friction of any two substances, their limiting angle of resistance is easily found. *Ex.* The coefficient of brick upon hard limestone is ·60; required the limiting angle. Take a line QR′ of any convenient length, raise a perpendicular R′P′ equal to $\frac{6}{10}$ of QR′, and join QP′; R′QP′ is the angle required: if measured, it would be found to be about 31°. In any structure, then, the obliquity of the thrust between two surfaces of these materials must always be considerably within this limit, in order to be safe.

189. The *friction of quiescence*, that is, the resistance to the commencement of motion, is greater than the resistance to its continuance; and the more so if the surfaces have been a considerable time in contact. But the slightest shock or jar is sufficient to destroy this cohesion, or whatever it is that constitutes the peculiar initial resistance; and as machinery is almost constantly undergoing shocks and jars, it is only the constant and regular friction of motion that is of much consequence in practice.

190. Friction is very much diminished by the use of grease or unguents. The coefficient of wrought-iron upon oak, which, in the dry state, is ·49, is reduced by the application of water to ·26, and by dry soap to ·21. The result of experiments on this subject is stated to be, 'that with the unguents, hog's-lard and olive-oil, interposed in a continuous stratum between them, surfaces of wood on metal, wood on wood, metal on wood, and metal on metal (when in motion), have all of them very nearly the same coefficient of friction, the value of that coefficient being in all cases included between ·07 and ·08.' Tallow gives the same coefficient as the other unguents, except in the case of metals upon metals, in which the coefficient rises to ·10.

191. The most important fact, perhaps, and one that could hardly have been anticipated before experiment, is, *that the friction of motion is wholly independent of the velocity of the motion.*

192. The resistance to the motion of a wheeled carriage proceeds from two sources; the friction of the axle, and the inequalities of the road. The resistance of friction to the turning of a shaft in its bearings, or of an axle in its box, has

evidently the greater leverage, the thicker the journal or the axle is; the axles of wheels are accordingly made as small as is consistent with the required strength. The resistance that occurs between the circumference of the wheel and the road, constitutes what is called *rolling* friction. There are on all roads, to a greater or less extent, visible rigid prominences, such as small stones, in passing over which the wheel and the load resting on it have to be lifted up against gravity. But even were these wanting, the hardest road yields, and allows the wheel to sink to a certain depth below its surface; so that in front of the wheel there is always an eminence or obstacle, which it is at every instant surmounting and crushing down. This is the case even on iron rails, though of course to a much less extent than on any other road. Now, for overcoming this resistance, it can be shewn on the principle of the lever that a large wheel has the advantage over a small one; and by numerous experiments, the fact has been fully established, that on horizontal roads of uniform quality and material, *the traction varies directly as the load, and inversely as the radius of the wheel.*

193. The best direction of traction in a two-wheeled carriage, is not parallel to the road, but at a slight inclination upward, in proportion to the depth to which the wheel sinks in the road.

194. On a perfectly good and level macadamised road, the traction of a cart is found to be $\frac{1}{30}$ of the load; that is, to draw a ton, the horse requires to pull with a force equal to 75 lbs. On a railway, the traction is reduced to $\frac{1}{280}$ of the load, or to 8 lbs. per ton. For the addition to traction occasioned by an incline in the road, see art. 98.

195. While friction thus acts as an obstruction to motion, and wastes a portion of the motive-power, it has also important uses. It is, in fact, an indispensable condition, no less than gravity, in the stability of every structure, and in every mechanical motion on the earth's surface. How essential it is to our own movements we experience when we try to walk on ice. Even on ice there is still considerable friction, so that one foot can be slightly advanced before the other; were it altogether annihilated, we could not stir a fraction of an inch, even supposing we could stand upright. Without friction, a ladder could not be planted against a wall, unless there were a hole in the ground to retain the foot. In short, no oblique pressure of any kind could be sustained. The virtue of railways consists chiefly in the diminution of friction; but were this diminution carried much further, there could be no motion whatever, at least by means of locomotives. Without considerable friction, the driving-wheels of the locomotive would slide round on the rails without advancing; and this sometimes happens, when

particular states of the weather render the rails as if they were greased.

196. The force of friction is often directly employed in mechanics. It is used, for instance, to communicate motion by means of belts, chains, &c. (149). It is the force that holds a knot. It is specially useful when a machine, with great momentum, has to be checked or arrested in its motion. The best example of this is the *break* used on railways. By means of a system of levers, blocks of wood are made to press against the circumferences of a number of the carriage-wheels; and thus the momentum of a train weighing hundreds of tons, and moving with a velocity of perhaps 50 miles an hour, is gradually destroyed in a wonderfully short space of time.

WORK.

197. Work, in the mechanical sense, implies two things; it implies that something is moved, and that force or pressure is required to move it. In other words, all mechanical work involves the exertion of a continuous pressure over more or less space. To bear or resist a pressure without motion is not work. A man standing still with a burden on his back is not working, any more than a post that sustains the end of a beam. The hand, fig. 1, does no work while it merely holds the weight suspended, although it requires force to do so; for a fixed catch or pin above the lever would do the same. But when the upward pressure of the lever is overcome by the hand through the space PP′, a certain amount of work is done. Nor does motion, where there is no resistance, constitute work. While a cannon-ball is flying through the air, it is doing no work (if we neglect the resistance of the air that is overcome); it only works when it forces its way through an obstacle, as the planks of a ship's sides.

198. *The Unit of Work.*—When one man lifts six gallons of water from a well twenty feet deep, while another lifts eighteen gallons from the same well; it is easy to see that the one has done three times as much work as the other. But in order that we may compare exactly the quantities of work in operations of different kinds, we must have some common standard to refer to. Just as in comparing the height of a mountain with a distance on a road, we take a standard or unit of length, say a foot, and, applying it to both, find that the height of the mountain contains 800 of the unit, and the length of the road 900, and thus get a precise knowledge of their relative magnitudes; so it is necessary to have a *unit of work*, and thus be able to say of different operations, that they contain each so many of such units.

199. *The unit of work adopted in this country is the amount of exertion necessary to overcome a pressure of one pound through a space of a foot.* Thus a hand does a unit of work when it lifts a pound-weight a foot high; or when it pulls in a foot of the cord of a pulley, the tension of which is equal to a pound-weight; or when it pushes with the force of a pound against the lever of a capstan (fig. 35), while the point against which it presses moves through a foot.

200. If to lift one pound-weight one foot high makes one unit of work; to lift one pound two feet, is twice as much work, or two units; and to lift a pound three feet, is three units. Also, to lift two pounds one foot, is two units of work; and three pounds lifted one foot is three units. It follows, that three pounds lifted two feet is twice three, or six units; that four pounds lifted through three feet, is three times four, or twelve units; and that, in general, the amount of units of work expended in any exertion of lifting, pushing, or pulling, is found by multiplying the number of pounds in the pressure by the number of feet through which it is exerted.

201. *Ex.* 1.—How many units of work are contained in lifting a ton of coals from a pit 300 feet deep? *Answer*—2240 (lbs. in a ton) $\times$ 300 = 672,000 units of work.

Ex. 2.—What is the work expended in drawing a cart with a load of 2400 lbs. along a mile of a level macadamised road? The traction or resistance to the draught of the horse is here, by art. 194, 2400 $\div$ 30 = 80 lbs.; and 80 $\times$ 5280 (feet in a mile) = 422,400, the amount of work.

202. 10 lbs. of coal lifted 10 feet high is the same amount of work, namely 100 units, whether it is done in five seconds or in fifty. But when we consider the capacity of the agent for work, we require to take the time into account. An agent or worker that does 100 units of work in one second, has four times the working-power of an agent that requires four seconds to do 100 units. As a standard of capacity for work, the power of a horse is taken. From a number of experiments, Watt, the improver of the steam-engine, estimated that a horse, on an average, could do 33,000 units of work in one minute; that is, could lift 33,000 lbs. 1 foot high in 1 minute; and this has been generally assumed as the standard of working-power in all mechanical agents. When a steam-engine, for example, is said to be of 60 horse-power, the meaning is that it is capable of lifting 60 times 33,000 lbs. or 1,980,000 lbs. 1 foot high in 1 minute; or, which is the same thing, 19,800 lbs. 100 feet high in 1 minute.

203. *Ex.* How many horse-power does it require to lift 1 ton of coals every five minutes from a pit 1000 feet deep?

$2240 \times 1000 = 2{,}240{,}000$, the work to be done in 5 minutes.
$2{,}240{,}000 \div 5 = 448{,}000$, the work to be done in 1 minute.
$\therefore 448{,}000 \div 33{,}000$ (the work in one H. P.) $= 13{\cdot}6$, the number of H. P. required.

WORK ACCUMULATED IN MOVING BODIES.

204. A heavy body in motion is capable of moving other bodies. The force exerted to put it in motion is all treasured up in it, and ready to be exerted on any obstacle that may oppose it (see art. 176). In other words, a moving body has accumulated in it a power of doing the same amount of work as was done upon it, abating what may have been lost by friction or other obstacles; and if we know the velocity of a moving body and its weight, we are able to calculate how many units of work have been done upon it, and, therefore, how much work it is capable of doing.

205. The velocity being given in feet per second, and the weight in pounds, the rule of calculation is as follows: *Square the velocity, multiply by the weight, and divide by* $2 \times 32\frac{1}{6}$, *or* $64\frac{2}{6}$.

206. The reason of the rule may be thus shewn: When a body begins to fall from a state of rest (see *Matter and Motion*, p. 35), it descends $16\frac{1}{12}$ feet in the first second, and it has then acquired a velocity which, were gravity to cease, would carry it on uniformly over $2 \times 16\frac{1}{12}$, or $32\frac{1}{6}$ feet per second. The spaces descended increase as the squares of the times—that is, in 2 seconds the fall is 4 times $16\frac{1}{12}$ feet, in 3 seconds, 9 times $16\frac{1}{12}$. The velocities, again, increase simply as the times—that is, at the end of 2 seconds the velocity is 2 times $32\frac{1}{6}$ feet, of 3 seconds, 3 times $32\frac{1}{6}$. Putting h for the height or space fallen through in any number of seconds, v for the velocity at the end of that time, t the number of seconds, and g for $32\frac{1}{6}$, the velocity caused by gravity in one second, the relations of these quantities are expressed in the following formulæ, which enable us to calculate any one of them from the others.

$h = 16\frac{1}{12} \times t^2$, or (1.) $h = \frac{1}{2}g \times t^2$; (2.) $v = g \times t$, and $\therefore$ $t = \frac{v}{g}$, and (3.) $t^2 = \frac{v^2}{g^2}$.

If, in formula (1.) we substitute for t^2 its value in (3.)—namely, $\frac{v^2}{g^2}$, we get $h = \frac{1}{2}g \times \frac{v^2}{g^2}$, or (4.) $h = \frac{v^2}{2g}$.

Knowing the velocity of a falling body, then, we can find the space through which it must have fallen in order to acquire it, by squaring the velocity and dividing by $2 \times 32\frac{1}{6}$. The height thus found is said to be the height *due* to that velocity.

207. A falling body is urged by a pressure equal to its own weight; and, during a second, that pressure acts upon it over $16\frac{1}{12}$ feet. Therefore, if the body is a pound in weight, gravity has done upon it in that time $16\frac{1}{12}$ units of work; and if it is 10 lbs., the work done is $160\frac{10}{12}$ units. In other words, the work done upon a falling body, and, therefore, the work accumulated in it, is equal to the space through which it has fallen multiplied by its weight. We may not know that space, but if we know the velocity, we can substitute for it its value, $\frac{v^2}{2g}$ (206).

208. Now, a cannon ball, moving with a given velocity, has the same force, whether it acquired that velocity by falling from a certain height, or by being shot from a cannon; and the same rule that enables us to calculate the units of work in it in the former case, applies also in the latter. Calling u the accumulated work, and w the weight of the body, the rule given in art. 205 may be expressed in a formula, thus: $u = \frac{v^2 \times w}{2g}, = \frac{1}{2} \times \frac{v^2 \times w}{g}$.

The expression $\frac{v^2 \times w}{g}$ is called the *vis viva* (living force) of a moving body; the accumulated work, therefore, is equal to one-half the vis viva.

209. *Ex.* 1.—A ram, weighing 5 cwt., or 560 lbs., as it strikes the top of a pile, has a velocity of 70 feet; what is the work accumulated in it?

$$\frac{70^2 \times 560}{2 \times 32\frac{1}{6}} = 42,653 \text{ units of work.}$$

Or we might proceed thus: The height from which the ram must have fallen to acquire the velocity of 70 feet, is, by art. 206, $\frac{70^2}{2 \times 32\frac{1}{6}} = 76{\cdot}166$ feet. To raise the ram to this height required $76{\cdot}166 \times 560 = 42,653$ units of work done upon it, and this work it is capable of reproducing by its fall.

Ex. 2.—A train weighing 100 tons has a velocity of 50 feet a second, how far will it run before stopping after the steam is shut off, supposing the friction to be 8 lbs. per ton, and that the train is ascending an incline of 1 in 100?

$$\frac{50^2 \times 224000}{2 \times 32\frac{1}{6}} = 8,704,663, \text{ the work accumulated in the train.}$$

Now, the resistance of friction is $8 \times 100 = 800$ lbs., and the resistance of the incline is $\frac{1}{100}$ of the whole weight, or 2240 lbs., making together 3040 lbs. To overcome this resistance over 1 foot makes 3040 units of work, which is to be taken out of the store accumulated in the train; for how many feet, then, will that store last? Evidently for $\frac{8,704,663}{3040} = 2863$ feet.

MODULUS OF MACHINES.

210. Of the moving-power applied to a machine, only part goes to do useful work; another part is expended uselessly in overcoming the friction of the machine, and other resistances to its transmission. For every 100 units of work applied to the handle of a crane, possibly only 70 are done upon the load. In this case, the fraction $\frac{70}{100}$, or ·7, expresses the *efficiency* of the machine, and is sometimes called its *modulus*. The working values of different machines may thus be exactly compared. Of machines, for instance, for raising water, the modulus of the bucket-wheel is ·6, and that of the Archimedian screw, ·7.

211. *Ex.* How many cubic feet of water per hour will an engine of 5 H. P. raise 10 feet high by means of a bucket-wheel?

33,000 × 5 = 165,000, the work applied per minute.

165,000 × ·6 = 99,000, the work done, or pounds lifted 1 foot, per minute.

99,000 ÷ 10 = 9900 pounds lifted 10 feet per minute.

But 1 cubic foot of water weighs 62·5 lbs.,

∴ 9900 ÷ 62·5 = 158·4, the cubic feet lifted per minute; and 158·4 × 60 = 9504, the cubic feet lifted per hour.

MOVING FORCES.—STEAM-POWER.

212. The principal moving forces are, the muscular power of men and animals; the weight of bodies, or gravity; wind, and the flow of streams; the expansive force of steam; electricity; and magnetism.

213. The labouring-power of animals varies greatly with the way or position in which they exert their muscular strength. In turning a handle or winch, the pressure exerted at different parts of the circuit is very different; but it has been found that a man can exert for a considerable time a mean pressure of 30 lbs., while moving the handle through 120 feet per minute. This would give 3600 units of work a minute, or about $\frac{1}{9}$ of a horse-power. Another estimate of labour of this kind, when continued for eight hours a day, makes the work per minute only 2600, or $\frac{1}{12}$ of

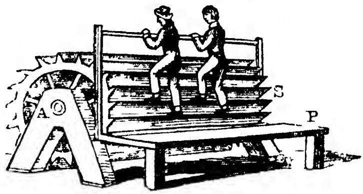


Fig. 96.

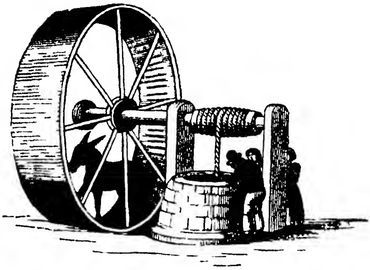


Fig 97

a horse-power. Rowing is one of the most advantageous ways of exerting muscular strength, the effective work being 4000. In working at the tread-mill (fig. 96) the man exerts his strength to raise his own body, and the weight of the body in descending turns the wheel. The work got in this manner is 3900. Fig. 97 represents an ass labouring in this way.

214. The consideration of water and wind as moving powers falls more conveniently under Hydrodynamics and Pneumatics. We conclude the present treatise with a brief account of

PROPERTIES OF STEAM.

215. If we suppose the piston, *pp* (fig. 98), to have been close at the bottom of the cylinder, and then drawn up to its present position, the space *a* below it is a vacuum, and the piston has to be held up against the whole pressure of the atmosphere, which is equal to 15 lbs. on the square inch. If, now, a little water could be introduced into the bottom of the cylinder, without admitting any air, a quantity of vapour would rise from it, and press with more or less force on the lower side of the piston, so as to sustain a portion of the weight of the atmosphere. How much vapour would rise, and how much elastic force or pressure it would exert, would depend upon the temperature of the water and cylinder. But before explaining this, it is necessary to advert to the way in which such pressures are measured.

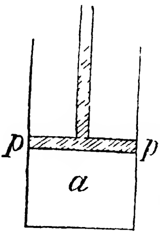


Fig. 98.

216. The elastic force of the atmosphere is expressed either in pounds of pressure on the square inch, or by the height of the mercury column it sustains in the barometer. The usual height of the barometer is about 30 inches, and in that condition the pressure is nearly 15 lbs. on the square inch. An inch of height of mercury, then, corresponds to half a pound of pressure on the inch; so that when the barometer falls an inch, the atmospheric pressure is less by half a pound.

217. Now, it is found that the vapour that rises at a low

temperature has little elastic force. At 32° the vapour in the space *a* would only sustain ·2 inches of mercury, corresponding to a pressure of $\frac{1}{10}$ of a pound on the square inch; if the apparatus were heated to 80°, more vapour would rise until the tension or force would sustain 1 inch of mercury, shewing a pressure of ½ lb. on the square inch. This would relieve part of the pressure of the external air on the piston, just as a portion of rarefied air would do. But there is this difference between vapour and air: if the space *a* were occupied by rarefied air, by lowering the piston and compressing it into less space, its elastic force would increase, until it became equal to that of the external air, or greater. Not so with vapour; for it the piston were made to descend on it, instead of being compressed and made stronger, a portion of it would be condensed into water, and what remained would have the same force as before. This is on the supposition that the temperature remains the same; by raising the temperature of vapour sufficiently, it may be made to sustain any amount of compression, and acquire any degree of force, as we shall see.

218. The force of the vapour in *a* increases rapidly, as more and more heat is applied. At 100°, its pressure is equal to 1·86 inches of mercury; at 180°, it is 15 inches, or half the pressure of the atmosphere; so that if the piston is sustained by a hand, the hand will now feel itself relieved of half its load. When at last the heat comes to 212°, the pressure of the vapour is 30 inches of mercury, or 15 lbs. on the square inch, and at this point the piston would be sustained without a hand, the pressures on its two sides being equal.

219. It is at 212° that water in an open vessel begins to boil; that is, the vapour rises rapidly and in volumes, being able to displace the atmosphere. In this state, it is usually called *steam;* but there is no essential difference between steam at 212°, and steam or vapour at 60°. The steam rising from boiling water in an open vessel is of the same temperature as the water—namely, 212°; but while passing into steam it is found to absorb a great quantity of heat which does not affect the thermometer, and is, therefore, called *latent* heat (see *Matter and Motion*).

220. When a cubic inch of water is converted into steam at the ordinary pressure of the atmosphere, its volume is increased to 1600 or 1700 cubic inches—that is, a cubic inch of water becomes nearly a cubic foot of steam. If produced under a pressure of two atmospheres, the steam of an inch of water will occupy only half the space, and so of other pressures.

221. When water is boiled in an open vessel, neither the temperature of the water, nor that of the steam rising from it,

ever rises higher than 212°, however hot the fire; the heat as it enters is carried off in a latent state in the steam. But under pressure, the temperature of both can be raised to any degree. If when the water and steam in *a*, fig. 98, came to 212°, the application of heat were still continued, more steam would continue to rise, and the pressure on the under side of the piston being now greater than that of the air above it, the piston would begin to ascend; but, suppose it held in the same position by force, the upward pressure of the steam would be found rapidly to increase, until it would soon require a weight of 15 lbs. on the square inch to keep it down, shewing that the elastic force of the steam was now equal to twice that of the atmosphere, or to 30 lbs. on the square inch. If at this point the temperature of the water and steam were examined, it would be found to be 250°.

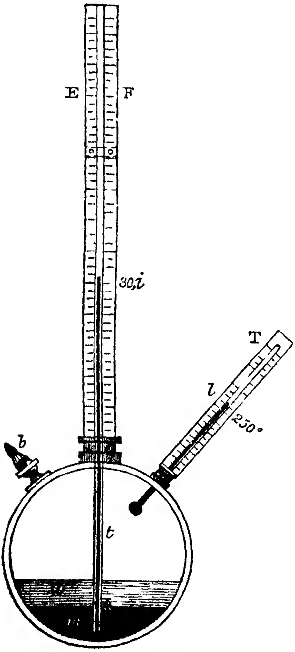


Fig. 99.

222. Fig. 99 represents an apparatus which shews the elastic force of steam raised at higher temperatures than 212°. A strong copper sphere has in the bottom of it a quantity of mercury *m*, and above that a quantity of water *w*. A strong glass tube, open at both ends, passes air-tight into the sphere, with its lower end dipping into the mercury, and has a scale of inches, EF, attached outside. T is a thermometer, with its bulb inside the sphere; and *b* is a tube opening into the sphere, with a stop-cock that may be opened and shut at pleasure.

223. The stop-cock *b* being open, let heat be applied to the bottom of the vessel until the water boils. The rising steam will soon expel the air, and occupy its place, being equal to it in elastic force. Before the application of heat, the mercury within the tube *t* does not rise above the level of the rest at *o*, for the tube differs from a common barometer in being

open at top; nor does it rise even when the water boils, for while *b* is open, the steam has no more pressure on the surfaces of the water and mercury than the air had. But if *b* is now shut, the pressure on the surface of the mercury within the sphere increases, and the mercury within the tube begins to rise and appear without the vessel. When it has risen to the point marked 30 inches on the scale (counted from *o*, the level of the surface within the vessel), it shews that the elasticity of the steam is able to sustain a column of mercury of 30 inches over and above the pressure of the atmosphere, which is resting on the top of that column, and is equal to another column of 30 inches. Its force is therefore equal to 60 inches of mercury, or 30 lbs. on the square inch; which is shortly expressed by saying that it has a pressure of two atmospheres. A further rise of 30 inches in the tube would indicate an additional pressure of 15 lbs., or of three atmospheres in all.

224. Along with the mercury in the barometer tube *t*, the thermometer also rises; but by no means at an equal rate. When the water boils it has mounted to 212°, and it continues steadily at that point so long as the tube *b* is open; but, on shutting *b*, it immediately begins to rise; and when the barometer tube is at 30 inches, the thermometer is found at about 250°. An addition of 38° of heat has thus doubled the elastic force of the steam. Another addition of 38° to the temperature would not simply add another atmosphere to the pressure, but a great deal more; as will be seen from the following table:

Temperature, Fahrenheit.	Pressure in Atmospheres.	Pressure in lbs. on the Sq. Inch.
212°	1	15
250°	2	30
275°	3	45
290°	4	60
372°	12	180
432°	20	300

The extreme danger of continuing to apply heat to a vessel while the steam is not allowed to escape, is thus evident; the bursting force soon becomes such as no vessel can resist.

225. Steam, like air and other gases, follows what is known as 'the Law of Mariotte' (see Pneumatics); its elastic force is always inversely as the space into which it is compressed. If the steam space in the vessel fig. 99 is a cubic foot, it contains, at 212°, the steam of a cubic inch of water; when the temperature comes to 250°, another cubic inch of water has risen in steam, and has thus compressed the former quantity into one-half the space, but at the same time made its elastic force twice

as great. If a cubic foot of this compressed steam is allowed to escape into a vessel of two cubic feet, its elastic force immediately falls from two to one atmosphere.

226. It is necessary here to bear in mind what was explained in art. 217 with regard to vapour at low temperatures; namely, that steam cannot exist at a certain tension or pressure, if it is below a certain temperature. If a vessel were filled with steam from the apparatus fig. 99, while at the temperature 250°, and pressure of 30 lbs. to the square inch; as it cooled, portions of it would be condensed into water, and the pressure would diminish, until, at 212° of temperature, the pressure would be only 15 lbs. This fact in the constitution of steam causes a considerable loss of steam-power, through the cooling of the tubes and cylinders of engines.

227. Another fact regarding the constitution of steam deserves attention, from its importance in point of economy. We should naturally expect that it would take more heat or fuel to convert a pound of water into steam at 250°, or 290°, than at 212°. In reality, however, it does not; and the circumstance is thus explained. Steam rising from water at a low temperature, absorbs more latent heat than it does at a higher temperature. Thus, steam rising at 212°, absorbs 1000° of latent heat; at 312°, it absorbs only 900°. Now, supposing the water put into the boiler at 60°, in the former case, 152° of heat have to be supplied to it to make the water boil, and then 1000° in the shape of latent heat—sum 1152°; in the latter, 252° are supplied to bring the water to boiling, and 900° to convert it into steam—sum, as before, 1150°. The expenditure of fuel is thus the same in both. With boiler and furnace of proper construction, one pound of coal or of coke is found to evaporate from 8 to 10 lbs. of water.

THE STEAM-ENGINE.

228. Such being the properties and force of steam, it remains to explain how this force is made to move machinery. Steam-engines are of two kinds—*condensing* and *non-condensing;* the non-condensing engine being the more simple of the two, may be first considered.

The Non-condensing Engine.—Fig. 100 represents a section of the form known as the *crank-overhead* engine; fig. 102 is a front-view of the same. The *cylinder, aa,* is made of cast-iron, and bored in a lathe; the *piston, vv,* is made to ply up and down in it, air or steam tight; and the *piston-rod, d,* passes through a hole in the cylinder cover and a *stuffing-box* above it, in which it is surrounded by packing of hemp and tallow, so as to prevent the escape of steam. From near the top and bottom of the

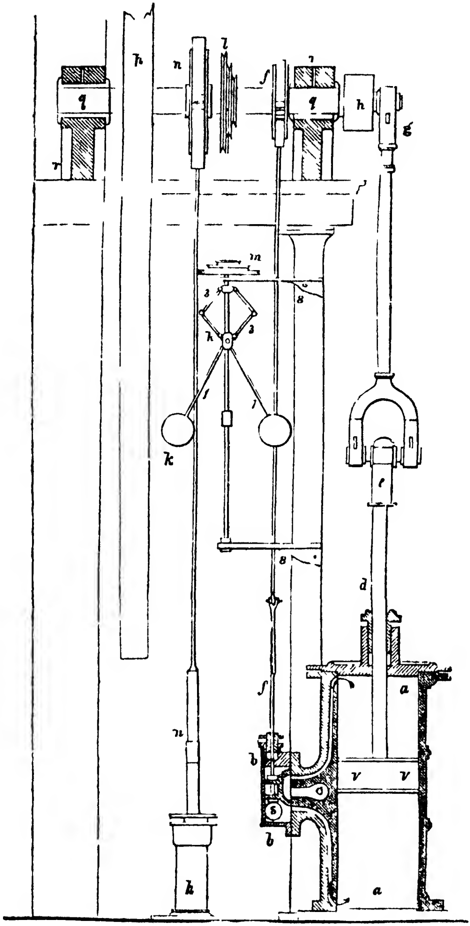


Fig. 100

cylinder proceed two tubes or passages, and open outwards near each other into a cavity *bb*, called the valve-chest, or slide-

jacket. Between the two openings or *steam-ports* that lead to the interior of the cylinder, there is another opening or port, *o*, which does not enter the cylinder, but leads to the waste-pipe, by which the steam escapes when it has done its work. The surface or facing where these three ports open, is made smooth, so that the valve, which is a hollow metal plate, may slide upon it steam-tight. Fig. 101 shews the three steam-ports, on a larger scale, with the valve *a* in two positions; in the one, it connects the upper duct *cc* with the waste-pipe *b*; in the other, the lower duct *d′* is connected with *b′*. The cavity behind the valve in the valve-chest *bb*, fig. 100, is always filled with steam, which enters from the boiler by the steam-pipe *s*.

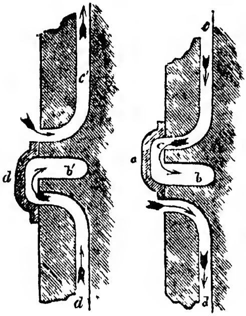


Fig 101.

When the valve covers the upper port, the lower is left open to the steam-chest, so that steam rushes into the cylinder below the piston, and presses it up, while the steam which is above the piston escapes by the waste *b* (or *o* in fig. 100). By the time the piston has reached the top, the valve is in the position *a′*, fig. 101, and the motion of the piston is reversed. The valve is made to slide up and down at the right time by the motion of the shaft *qq*, through means of an eccentric and rod *jj* (see art. 173). How the alternating motion of the piston-rod *d* is converted into the rotary motion of the shaft *gg*, is explained in art. 170.

229. The amount of working-power in a steam-engine depends on the tension or pressure of the steam, the size of the piston, and the rate at which it travels. Suppose that the area of the piston *vv*, fig. 100, is 1000 square inches; that the pressure of the steam entering the cylinder is 3 atmospheres, or 45 lbs. on the square inch; that the length of a *stroke*, that is, an ascent or a descent of the piston, is 5 feet; and that 20 strokes (10 up and 10 down, corresponding to 10 revolutions of the crank and shaft) are made in the minute. While the piston is ascending, the steam above it is open to the atmosphere through the waste-pipe, and, therefore, its pressure is equal to that of the atmosphere, or 15 lbs. to the inch; and the pressure below the piston being 45 lbs., it moves under a pressure equal to the difference, or 30 lbs. to the inch; which, for the whole surface, gives a force of 30,000 lbs. Now, a pressure of 30,000 lbs. over 5 feet, makes 30,000 × 5 or 150,000 units of

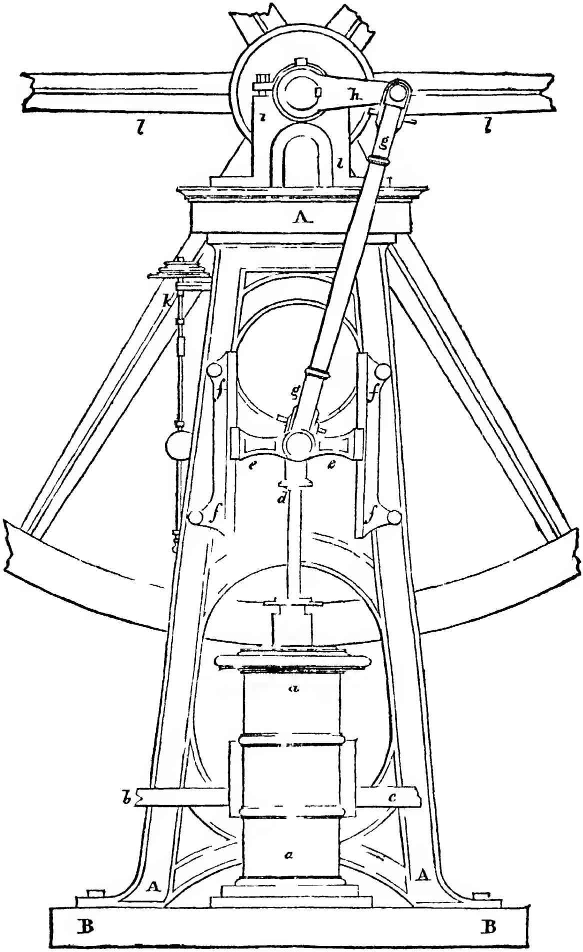


Fig 102.

work for one stroke; and $150{,}000 \times 20 = 3{,}000{,}000$ will be the work done in one minute. Dividing this by 33,000, the units of work in a horse-power, we get 90·9 (91 nearly) as the theoretical horse-power of such an engine.

230. The steam in the cylinder has always less pressure than that in the boiler, partly from being cooled, and partly from being obstructed by the comparative narrowness of the passages on its way. The pressure is also unequal at different parts of the stroke, owing to the opening and closing of the ports being only gradual. In calculations of work, then, a certain mean pressure has to be taken as the multiplier. In addition to this, allowance has to be made for losses by friction—the friction of the piston, of the piston-rod, and its cross-head *ee* (fig. 102), of the connecting-rod *gg*, of the shaft *qq* in its bearings, of the eccentric and its rod *jj*, of the feed-pump *nnh*, &c. (fig. 100). On an average, from $\frac{1}{4}$ to $\frac{1}{5}$ of the calculated power of an engine has to be deducted for these losses, in order to get the effective power. When all the parts work perfectly smooth and tight, and are well lubricated, the loss may sometimes not exceed $\frac{1}{10}$.

231. *Steam used expansively.*—A great saving of steam arises from working it expansively. Suppose that the steam enters below the piston, fig. 103, at a pressure of 4 atmospheres, or 60 lbs. to the square inch. Set off $ba = 4 \times 15 = 60$, to represent the amount of the pressure, and complete the parallelogram *bf*, *f* being the highest point to which the bottom of the piston reaches; *af* thus represents the length of a stroke, which we shall suppose to be 5 feet. Were there no pressure on the upper side, and did the pressure below continue uniformly 60 lbs. to the top, the work due to 1 square inch of the piston for each stroke of 5 feet, would be $60 \times 5 = 300$; and this may be represented by the rectangle *abef*, the area of which is found by multiplying *ba* by *af*. But the piston is resisted all the way by a force of 15 lbs. to the inch, or 1 atmosphere; and if *ak* is made $\frac{1}{4}$ of *ab*, the figure *akif* will represent the deduction to be made on

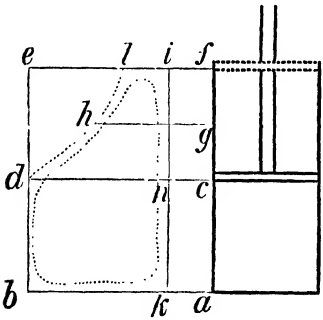


Fig. 103.

this account; so that the work really due to a square inch of the piston is represented by the rectangle *kbei*, or $45 \times 5 = 225$.

232. But instead of allowing the steam to flow into the cylinder during the whole time of the stroke, let it be cut off when the piston is at *c*, or one-half the stroke; the steam already in the cylinder will expand and follow the piston, still urging it on, although with a diminishing force. When the piston has reached the top, the steam will have expanded to twice its former bulk, and by the law of Mariotte (see art. 225), its pressure will be reduced to a half of what it was, or 2 atmospheres, which will be expressed by *fl*, taken equal $\frac{1}{2}$ *fe*, or 30. At any intermediate point, as *g*, the line *gh* representing the pressure is found by this proportion $ga : ca :: cd$ (or ab) $: gh$; that is, the pressures are inversely as the spaces occupied by the steam. If, for example, $ga = 3\frac{3}{4}$ feet, and $ca = 2\frac{1}{2}$ feet, the proportion will be $3\frac{3}{4} : 2\frac{1}{2}$, or $15 : 10 :: 60 : 40 = gh$.

When a number of pressure lines or ordinates are thus found, a line *dhl* traced through their extremities, forms a curve (part of a hyperbola); and the area of the figure *ndhli* will represent the work done on a square inch of the piston after the steam is cut off, still allowing for the resistance on the other side of the piston, which remains constant to the top. All this work is got without any additional expenditure of steam, and is thus clear gain.

233. The amount of the gain, in the case now supposed, may be thus calculated approximately. Find a number of ordinates, as *gh* was found (232), at points equally distant from one another, from *c* to *f*; from these find a mean pressure or ordinate, and multiply by *cf*, and the product will be the area of the figure *cdhlf* nearly. The more ordinates are taken, the result will be the nearer the truth. If we take the three ordinates *cd*, *gh*, *fl*, the mean of which is 43, the result will not be very far from the truth; this gives for the area $43 \times 2\frac{1}{2} = 107{\cdot}5$. If to this is added the area of $abdc = 60 \times 2\frac{1}{2} = 150$, we get the area of the whole figure $abdlf = 257{\cdot}5$; and deducting the area of $akif = 5 \times 15 = 75$, for the resisting pressure above the piston, we have 182·5 as the work done on each square inch of the piston by the expansive stroke. Now, the work of a stroke at full pressure the whole way is 225; but the same quantity of steam that does one stroke at full pressure, will do two strokes expansively, or 365, shewing a gain of 140 units of work to the square inch of the piston, on one cylinder-full of steam. The work done by the same quantity of fuel in the two methods is as 225 to 365, or 1 : 1·6.

234. The figure *kbdli* represents the theoretical work of a stroke; but owing to the causes mentioned in art. 230, and to other imperfections such as leakage, the actual work is more accurately represented by the dotted figure within. By means of an ingenious apparatus called an *Indicator*, the steam in the cylinder is made to record its own state of tension. The piston of a small attached cylinder carries a pencil, and, as it protrudes more or less according to the pressure, traces on a revolving piece of paper a figure like the dotted one in cut 103.

235. The higher the pressure of the steam, the greater is the saving resulting from the expansive method. The steam is generally cut off much earlier than half-way—at $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, &c., of the stroke; and the economy is thus much greater than in the case we have supposed. At $\frac{1}{7}$, the economy is stated by writers on the steam-engine to be as much as 1 to 3. The limit to this process lies in the approach of the curve at l to i, when the moving pressure on the piston would be nothing.

236. *The Condensing Engine.*—Condensing engines are generally at the same time beam-engines, but not necessarily so. Condensing engines may be worked with steam at the pressure of the atmosphere, or even under; hence they were at one time called *low-pressure* engines, and the non-condensing kind, *high-pressure* engines; but condensing engines are now as often worked with steam at a pressure of several atmospheres, as at low pressure; so that the old designations have ceased to be distinctive.

237. Fig. 104 is a section of a condensing engine with a beam *bb*, the action of which has been explained (art. 175). The fundamental difference between the condensing and non-condensing engine is this: that in the former the steam, instead of escaping into the air, is conducted by the waste-pipe into a *condenser*, *f*. This is a vessel exhausted of air and surrounded by cold water, so that when the steam rushes in, it is deprived of its latent heat, and turned to water. To make the process more rapid, a jet of cold water, which can be regulated by a handle *b′*, is injected in a shower through the vessel. This injected water and that of the condensed steam, together with any air that may enter with the water and steam, are pumped out by the air-pump *g*; the water is delivered, still hot, into the cistern *h*, from which it is sent to feed the boiler by the hot-water pump *jpp*. *k* is a cold-water pump for supplying the cistern surrounding the condenser.

238. The steam enters and leaves the cylinder in much the same way as in the non-condensing engine; but the amount of power developed is calculated somewhat differently. First, let us suppose that steam at the pressure of 1 atmosphere is entering

below the piston, then, the upper port being in connection with the condenser, which is theoretically a perfect vacuum, there ought, if the instantaneous escape of the steam were not

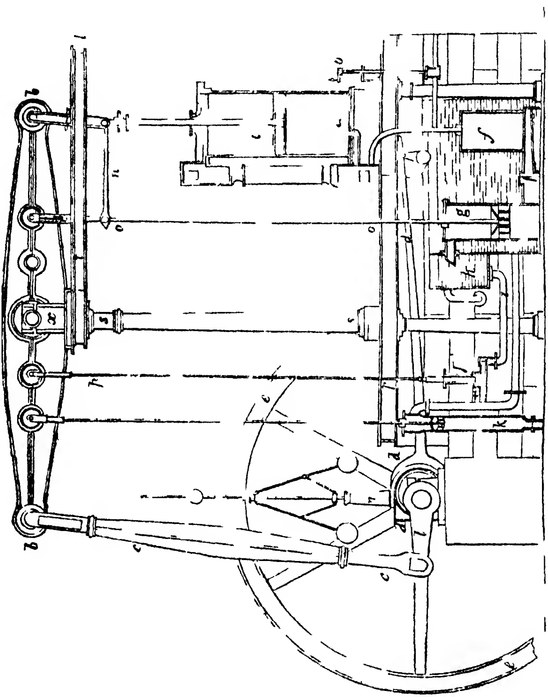


Fig. 104.

obstructed by the narrowness of the passage, to be a vacuum above the piston, so that it would move under a pressure of 1 atmosphere, or 15 lbs. to the square inch, but owing to the warmth of the water in the condenser, there must always be a portion of uncondensed vapour in the condenser (see arts. 217 and 218), having a pressure of several pounds, and from this and other causes, the tension of the steam before the moving piston can seldom be less than 4 or 5 lbs. The real moving-force is thus reduced to perhaps 10 lbs., which, with a piston

of 1000 inches in area, and a stroke of 5 feet, gives 50,000 as the work done on the piston in one stroke. Knowing the number of strokes in a minute, the horse-power of the engine may be calculated as before, allowance being made for losses by friction and other causes. In the condensing-engine, the air-pump occasions an additional loss of about $\frac{1}{10}$ the total power.

239. *Boilers.*—Boilers are usually in the shape of a long cylinder, with hemispherical ends. The fire-bars are placed below one end, so that the flames and the direct heat of the fuel act on the bottom; and the products of combustion are made to circulate over as much as possible of the surface, so as to send their heat into the water before they escape. In what is called the *Cornish* boiler, a flue or large tube passes through the water-space of the boiler, and the fire-bars are placed in the entrance. The fire is thus surrounded with evaporating surface, and the products of combustion, after passing through the heart of the boiler, are made to circulate round its outer surface before escaping. Sometimes boilers are made with two such central flues as *a*, *b*, in the end-view, fig. 105, each with its grate *c*, *c*. Fig. 106 is a plan of this arrangement. The two flues unite in the space *c*, called the mixing-chamber, and are again separated by a partition containing water. One advantage of this arrangement, besides presenting a large surface to the action of the heat, is, that it allows of *alternate firing*. While the fuel in one furnace, as *a*, is in a state of high combustion, evolving little or no smoke, but only intensely hot gases; the other, *b*,

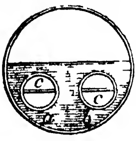


Fig. 105.

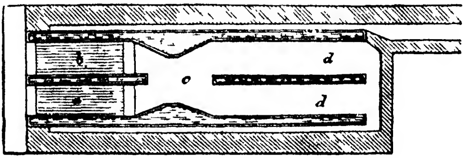


Fig. 106.

is supplied with fresh coal. The smoke arising from this mingles in the space *c* with the hot gases from *a*, and is ignited. It is usual to allow 10 square feet of evaporating surface for every horse-power of the engine.

240. A large amount of boiler-surface is obtained in small space by constructing the boiler as represented in section in

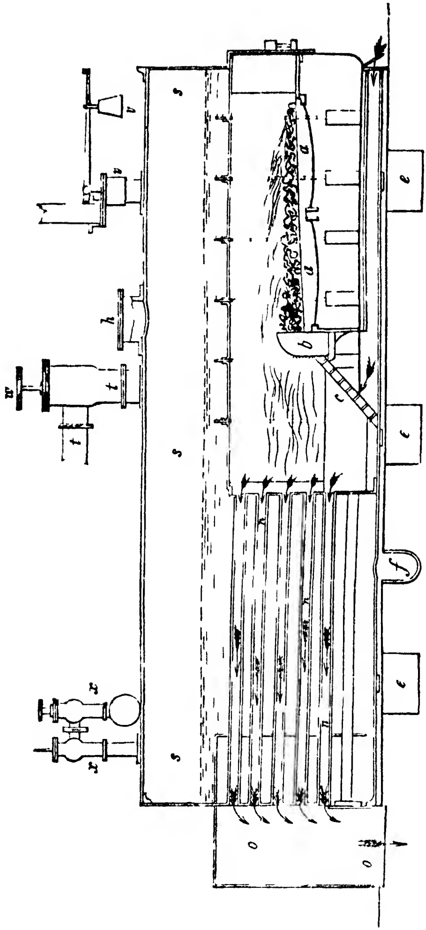


Fig. 107.

fig. 107. The furnace or fire-box with its grate *aa*, is within the water-space of the boiler. When the smoke and gases arising

from the fuel are not sufficiently consumed by the air entering through the furnace-door, a regulated supply may be admitted by the valve *d* along a tube and through openings in the partition *c*, behind the bridge *b*; the combustion being thus complete, the heated products rush through a number of tubes of from 2 to 3 inches diameter, which pass through the boiler to the chamber *oo*, communicating with the chimney. The

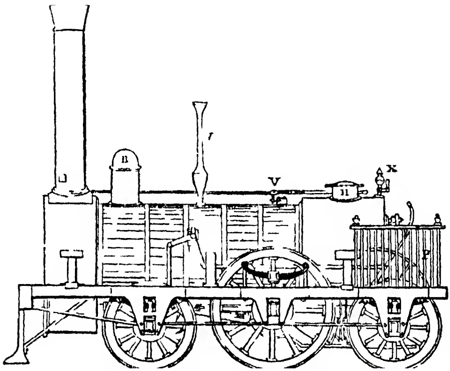


Fig. 108

boilers of locomotives are constructed on this *multi-tubular* principle.

241. A number of tubes, valves, and other appliances are required for indicating the strength of the steam in the boiler and other parts, the height of the water in the boiler, and for regulating the supply of the latter, &c. Of these we can only afford to notice the important matter of the *safety-valve*, and the ingenious contrivance of the *governor*. Fig. 109 represents a simple form of safety-valve: *aa* is a tube entering the boiler, and closed by a valve *cc*, which is pressed down by means of a lever. The weight of *f* and its distance from the fulcrum *e*, are so adjusted as to produce a certain pressure—say 75 lbs., on the valve. The area of the valve being supposed a square inch, this will balance a pressure of 75 lbs. per inch, or 5 atmospheres, from the steam

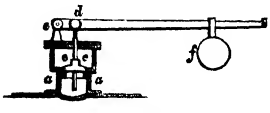


Fig. 109

within the boiler; and if the elasticity of the steam rise above that degree, the valve will be forced up, and allow the steam to escape.

242. The nature of the governor, one of Watt's many inventions, will be understood from fig. 110. An upright spindle has two balls jointed to it, and revolving along with it, the spindle being set in motion by a pulley in connection with some shaft of the machinery. At the ordinary velocity of the engine, the balls have a certain divergence; but when, as often happens, the resistance to the machinery becomes for a time less, and the speed consequently increases, the

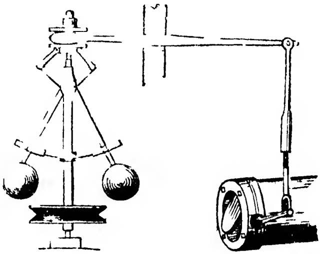
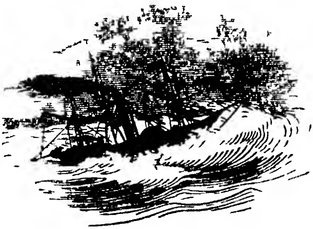


Fig 110

balls, having their centrifugal force increased, begin to diverge further. In doing so, the upper ends of the rods by which they hang pull down a boss that slides upon the spindle, and is clasped by the forked end of a lever. This lever is connected with a *throttle-valve*, or circular disc, in the tube that conveys the steam to the cylinder, and the depression of the end of the lever has the effect of closing more or less the passage for the steam, and thus diminishing the driving-force.



EXERCISES.

Article 17.

1. In fig. 2, the force Q or Q′ is 100 lbs., what is its moment in each of the positions BQ and BQ′, CB being 15 feet and CB′ 12 feet? *Ans.* 1500, and 1200

2. The length of AC, in the same fig., is 6 feet, what must the force P be, in order that its moment may equal that of Q in the former case? *Ans.* 250 lbs.

Arts. 24 to 30.

3. In the lever ACB, fig. 2, AC is 10 feet, CB, 30 feet; P is 85 lbs., and Q is 28 lbs.: shew by art. 26 whether or not the two forces are in equilibrium.

4. If the above forces are not in equilibrium, how must one of the arms, say CB, be altered in length to produce equilibrium, everything else remaining the same?

Ans. CB must be $30\frac{5}{14}$ feet.

5. Or, how must the force P be altered?

Ans. P must be 84 lbs.

Arts. 35, 36.

6. In fig. 12, AB is 15 feet, BF, 4 feet, and W is 1000 lbs., what weight has the hand to support? *Ans.* $210\frac{10}{19}$ lbs.

7. What weight is supported by the hand, in fig. 13, AB being 15 feet, BF, 4 feet, and W, 1 cwt.? *Ans.* 532 lbs.

Art. 37.

8. In fig. 14, the block of stone is a ton weight, the part of the crow-bar from B to F is 9 inches, and from F to A, 6 feet; what force at A will be sufficient to bear up the edge of the block, so as to make it ready to begin to turn on its other edge?

NOTE.—It is to be borne in mind here, that the crow-bar has not to sustain the whole weight of the block. The base of the block is, as it were, a lever of the second kind; the fulcrum of which is at the opposite end from that resting on the crow-bar; the crow-bar is the power lifting up the long arm of this

lever, and the weight of the stone, which is concentrated in its middle point or centre of gravity, acts midway between the fulcrum and the power. The crow-bar then has to sustain only one-half the weight of the stone. *Ans.* 140 lbs.

Art. 49.

9. The bale, in fig. 18, weighs 1000 lbs., the length from P to F is 5 feet, and from F to where the corner of the bale rests on the bar is $1\frac{1}{2}$ feet, what pressure at P is required to begin the tilting up? (The weight on the bar becomes less and less as the bale is turned up on its edge.) *Ans.* 150 lbs.

Art. 51.

10. The cask, fig. 20, weighs 3 cwt., the distance from the shoulder of the foremost man to the point of suspension is 5 feet, and from that of the last man 4 feet: what weight does each support?

Ans. { Foremost man, $149\frac{1}{3}$ lbs.
Last man, $186\frac{2}{3}$ lbs.

11. The strengths of two horses are as 9 to 7; at what point ought the 6 feet long cross-bar, at the ends of which they are pulling, to be attached to the load, in order that each may have a share of the draught proportioned to its strength?

Ans. The two arms of the lever are $40\frac{1}{2}$, and $31\frac{1}{2}$ inches.

Art. 61.

12. In a combination of three levers, the arms next the power are 12, 20, and 8, and the others 5, 4, and 3; and P is 100 lbs.; what is W? *Ans.* 3200 lbs.

Art. 67.

13. A force, Q, of 900 lbs., and a force, P, of 700 lbs., act on a point, A, in directions at right angles to each other; what is the magnitude of their resultant? *Ans.* 1140 lbs.

Art. 70.

14. Two balls, A and B, weighing respectively 15 and 48 lbs., are joined by a rod 12 feet long; required the distance of C, their common centre of gravity, from A? *Ans.* $9\frac{1}{7}$ feet.

15. The centre of the moon is 240,000 miles distant from that of the earth, and its mass is $\frac{1}{90}$ of the earth's mass; where, in the line joining their centres, is the common centre of gravity of the two bodies?

Ans. $2637\frac{1}{3}$ miles from the earth's centre.

Art. 75.

16. The spokes of a capstan are each 6 feet long, measured from the axis of the capstan; and the diameter of the part on which the chain is wound is 3 feet; four men turn it, each exerting a pressure of 200 lbs.: what is the strain upon the chain? *Ans.* 3200 lbs.

Art. 77.

17. The handle of a crane for lifting blocks of stone describes a circle whose radius is 2 feet; the pinion has a radius of 3 inches, and drives a wheel $2\frac{1}{2}$ feet radius; and the axle of this wheel, on which the chain is wound, is 4 inches thick: what pressure on the handle will sustain a weight of a ton? *Ans.* $18\frac{2}{3}$ lbs.

Art. 89.

18. In fig. 41, W = 1000 lbs., what is the tension of the cord in the hand? *Ans.* $142\frac{6}{7}$

Art. 90.

19. With 5 movable pulleys, arranged according to the Second System, what weight will a power of 100 lbs. sustain? *Ans.* 3200 lbs.

Art. 92.

20. In fig. 42, W = 1000 lbs., and each of the movable pulleys weighs 2 lbs.; required P? *Ans.* $126\frac{3}{4}$ lbs.

Art. 98.

21. A train weighs 30 tons; with what force does gravity urge it down an incline of 1 in 25? *Ans.* $1\frac{1}{5}$ tons.

Art. 100.

22. Two weights of 10 and 15 lbs. balance each other on two planes, as in fig. 47; the length of the longer plane is 6 feet, 7 inches: required the length of the other? *Ans.* 4 feet, $4\frac{2}{3}$ in.

Art. 111.

23. The lever, in fig. 54, is 4 feet long, and the breadth of a thread of the screw is $\frac{1}{2}$ inch, what pressure on the books results from a force of 100 lbs. applied to the lever, neglecting friction, and taking the circumference at three times the diameter? *Ans.* 57,600 lbs.

Art. 113.

24. In fig. 55, the circumference described by the handle is 12 feet, the distance between the threads of the screw $1\frac{1}{4}$

inch, the radius of the toothed wheel $2\frac{1}{2}$ feet, and of its axle 5 inches: what weight is supported at W for every pound of pressure on the handle? *Ans.* $691\frac{1}{5}$ lbs.

Art. 117.

25. A hempen cable has a diameter of 4 inches; what weight will it bear? *Ans.* 75,398·4 lbs.

Art. 126.

26. What weight, uniformly distributed over a bar of cast-iron $6\frac{3}{4}$ feet long, 1 inch thick, and $2\frac{1}{2}$ deep, will just be sufficient to break it; the bar being fixed at one end and loaded at the other? *Ans.* 1265·6 lbs.

Art. 127.

27. A beam of ash, 15 inches deep and 4 inches broad, projects 8 feet from a wall; what weight of bricks laid upon it uniformly will it sustain, in addition to its own weight, taking a cubic foot of ash at 48 lbs.? *Ans.* $37,982\frac{1}{2}$ lbs.

Art. 128.

28. What weight will the beam in last example sustain when it rests on both ends and is loaded in the middle, neglecting its own weight? *Ans.* 152,250 lbs.

29. A bar of cast-iron, 2 inches square and 15 feet long, is supported at both ends; what weight applied at its middle will break it? *Ans.* 1440 lbs.

30. The length of a plank of American birch (the constant for which is 1500) is 10 feet, its breadth 5 inches, and the weight necessary to break it, when supported at both ends, is 1500 lbs.; what is its thickness or depth?

NOTE. As d is required, and not W, the formula requires to be modified, so as to make d stand alone on one side of the equation, thus: $d^2 = \dfrac{l \times W}{\text{Const.} \times b} \times \frac{1}{4}$. *Ans.* 2·45 inches.

Art. 162.

31. A pulley, 4 inches in radius, is connected by a band with a drum, $2\frac{1}{2}$ feet in radius, which revolves 40 times a minute; required the rate of the pulley?

Ans. 300 revolutions per minute.

32. The centres of two toothed wheels, that are to work into one another, are to be 4 feet apart, and their rates of speed are to be as 20 to 3; required their radii?

Ans. $41\frac{17}{23}$ inches and $6\frac{6}{23}$ inches.

Art. 164.

33. In fig. 81, let the diameter of F be 2 inches, of G, $1\frac{1}{2}$ inch, of B, 8 inches, and of C, 10 inches; and let F (supposed to be the driver) have a speed of 3 revolutions per minute: required the speed of C? *Ans.* $\frac{9}{80}$ of a revolution per minute; or, very nearly one revolution in 9 minutes.

Art. 194.

34. What is the traction, on a macadamised road, of a wagon weighing 3 tons, up an incline of 1 in 20?

Ans. 560 lbs.

35. What is the traction in descending the same incline?

Ans. 112 lbs. less than nothing; that is, the wagon requires to be held back, or *dragged*, with that force.

Arts. 197-200.

36. A cistern, 20 feet long, 15 feet broad, and 10 feet high, has to be filled from a source 40 feet below; what is the amount of work, the weight of a cubic foot of water being $62\frac{1}{2}$ lbs.? *Ans.* 7,500,000

37. In digging a cylindrical shaft or well, 10 feet in diameter and 92 feet deep, what is the work of raising the excavated material to the surface, supposing its mean specific gravity to be twice that of water?

NOTE. The depth from which the material has to be lifted is to be reckoned from the centre of gravity of the mass; it is as if the whole were lifted from half the depth of the shaft.

Ans. 41,547,660

Art. 202.

38. 440 cubic feet of water per minute have to be pumped to the height of 50 feet; what must be the horse-power of the engine, supposing the whole of it effective? *Ans.* $41\frac{2}{3}$ H. P.

39. What must it be, if the pump yield only ·55 of the work applied to it? *Ans.* $75{\cdot}\dot{7}\dot{5}$ H. P.

40. How many tons of coal ought an engine of 24 H. P. to lift from a pit 260 feet deep in 12 hours? *Ans.* $979\frac{11}{91}$

Art. 207.

41. What is the work accumulated in a ram weighing 4 cwt., that has fallen from a height of 30 feet? *Ans.* 13,440

Art. 208.

42. A hammer, 20 lbs. weight, strikes with a velocity of 90 feet, what is its work? *Ans.* $2518\frac{26}{193}$

43. A train, weighing 300 tons, has a velocity of 30 miles per hour, how far will its own impetus carry it on, on level rails? *Ans.* 8426 feet.

Art. 210.

44. How many cubic feet of water will an engine of 9 H. P. raise in 10 hours to the height of 20 feet, working a pump the modulus of which is ·45? *Ans.* 64,152 cubic feet.

Art. 229.

45. The area of the piston of a steam-engine is 3000 square inches, the length of the stroke is 4 feet, the number of strokes per minute 15, and the pressure of the steam 20 lbs. per square inch (over that of the atmosphere); required the theoretical H. P. of the engine? *Ans.* $109\frac{1}{11}$ H. P.

THE END.

Edinburgh:
Printed by W. & R. Chambers.

NATURAL PHILOSOPHY.

THIRD TREATISE.

HYDROSTATICS—HYDRAULICS—PNEUMATICS.

NOTICE.

The pupil having been made acquainted with the general Laws of Matter and Motion, and with the Elements of Mechanics, theoretical and practical, is now introduced to a knowledge of the more simple Laws of Fluids, usually comprehended under the three heads of *Hydrostatics*, *Hydrodynamics*, and *Pneumatics*. In drawing up the present treatise, the same principles have been followed as in the two preceding. See the Prefatory Notice to the treatise on *Mechanics*.

CONTENTS.

HYDROSTATICS—HYDRODYNAMICS—PNEUMATICS.

1. The properties of matter treated of under Mechanics are such as arise chiefly from the *solid* state of bodies. But consequences, scarcely less important, arise from the other two states in which matter presents itself—from the *liquid* condition, as in the case of water, and the *gaseous* or aëriform condition, as in the case of air; and these form the subjects of the present treatise.

2. All matter that we are acquainted with, presents itself in one or other of the three forms mentioned; and the same substance may generally be made to assume them all in succession. Thus, liquid water may become solid ice, or aëriform steam. These different *states of aggregation*, as they are called, depend upon the way in which the particles, or molecules, are united. From the fact that a force is required to separate the parts of a solid body, it is inferred that an attractive force is at work drawing or holding them together—the attraction of *cohesion*. From the fact, again, that force is necessary to restrain the molecules of a gas from separating further than they are, it is inferred that a repulsive force is at work among them. Attractions and repulsions, thus acting between the ultimate molecules of bodies, are called *molecular forces*. Since the same body can generally exist both as a solid and as a gas, it is assumed that both forces are inherent in the molecules of all substances. When the attractive force preponderates, the solid state is the result; when repulsion preponderates, the gaseous state results; an equilibrium, or nearly so, of the two forces produces a liquid.

3. Liquids and gases agree in this, that their particles seem at liberty to glide about among one another without friction; they *flow*, and hence both classes of bodies are called *fluids*.

There are very different degrees of fluidity. Some fluids are thick and viscid; such as tar, honey, and some metals in a state of fusion. Viscid fluids are, in general, not homogeneous; they consist of solid granules floating in a real fluid. Alcohol and ether are more fluid than even water. The most perfect fluidity belongs to the gases.

4. But what chiefly distinguishes gases from liquids, is elasticity. A cubic foot of any gas may readily be compressed into half a foot; double the pressure will reduce it to a quarter of a foot; and when the pressure is removed, the gas returns to its original bulk. But no ordinary pressure produces any sensible compression on water or any other liquid. Hence gases have been denominated *elastic fluids*, and liquids, *non-elastic*. It is on account of this difference that the mechanical properties of the two classes of fluids are treated of apart.

5. We have said that liquids cannot be sensibly compressed by any ordinary force; and this is so far true, that both in the theory of hydrostatics and in practice they are assumed to be perfectly incompressible. But the assumption is not absolutely correct. They are slightly compressible under great pressure. By sinking a vessel in the ocean to the depth of 6000 feet, where every square inch supports a weight of 2648 pounds, it is found that twenty cubic inches of water contained in the vessel are reduced to nineteen cubic inches, or the volume of water is diminished by one-twentieth. A pressure equal to that of the atmosphere, or fifteen pounds on the square inch, reduces a million cubic inches of water to forty-five or fifty inches less.

6. The phenomena of liquids are considered under two heads, according as the pressures to which the liquids are subjected produce rest or motion. The laws of liquids at rest or in equilibrium form the subject of *Hydrostatics* (from two Greek words signifying *water* and *to stand*); those of liquids in motion form the subject of *Hydrodynamics* (from the Greek words for *water* and *power*). *Hydraulics* is sometimes used in the same sense as Hydrodynamics, but has more especial reference to the flow of water in *pipes* (Gr. *aulos*).

7. In treating of the mechanical properties of liquids, water, as the most common and important, is taken to represent the whole class. We have nothing to do here with the fact, that water is composed of two gases—that belongs to chemistry. The atoms of oxygen and hydrogen having united in pairs to form compound atoms or molecules of water, the science of Hydrostatics depends on the way in which these compound molecules, or particles, join together to form a mass. It is obviously different from the way in which a heap of sand is composed. For one thing, the sand, however fine, may still be seen to be composed of separate solid grains, which, under the microscope, become real blocks of stone; while the molecules of water are quite invisible, and must be inconceivably minute. The particles of sand, again, move over one another, it is true, but with considerable friction or resistance, as is seen by their remaining standing up in a heap; the particles of water slide upon one another without the least apparent friction, and when not confined, spread out into a thin film.

8. Another point of difference is, that the separate particles of sand have no cohesion among themselves; the cohesion among the molecules of water, though weak, is still sensible, as is seen in a drop hanging suspended at the end of a rod. A liquid differs from a solid, not so much in the force with which the particles of either resist separation, as in the force with which the particles of the solid resist changing their relative position. We may conceive two molecules of water, in the form of two minute balls, in contact; and when one of them is pushed, even with the slightest conceivable force, it slides round on the other, adhering to it as it moves. Two molecules of a solid resist this sliding motion, and if forced to change their points of contact, can no longer cohere; as if they only attracted each other by certain sides. It is this property that enables solids to retain a fixed shape; *fluids have no shape, but what they receive from the containing vessel.*

9. Not only have the particles of a liquid an attraction for one another—the attraction of *cohesion*—but there is an attraction between most liquids and solids; this is the cause of a liquid *wetting* a solid. The particles of one liquid, also, often adhere to those of another; and gases similarly adhere to liquids. This species of attraction is known as *adhesion*. The combined action of *adhesion* and *cohesion* gives rise to many interesting

phenomena; among others, to *capillary* attraction and *osmose*. See *Matter and Motion*.

10. It is the perfect *mobility* of the particles of liquids that gives them the mechanical properties considered in Hydrostatics. The fundamental property may be thus stated: WHEN A PRESSURE IS EXERTED ON ANY PART OF THE SURFACE OF A LIQUID, THAT PRESSURE IS TRANSMITTED UNDIMINISHED TO ALL PARTS OF THE MASS, AND IN ALL DIRECTIONS. Most of the other propositions of Hydrostatics are only different forms or direct consequences of this truth.

11. The proposition may be experimentally proved in a variety of ways. If, for instance, a bladder is filled with water, and tied, and then pressed down with one hand; the other hand, if applied to the bladder, will be pushed out with corresponding force, and that whether resting on the top, the sides, or under. Suppose, again, a close box B filled with water, and having a tube *a* inserted into the upper cover, of an inch in area, and with a plug or piston fitting into it. If the piston *a* is now pressed down upon the water with a force equal to a poundweight, the water, being unable to escape, will react upon the piston with the same force; but it obviously will not press more against *a* than against any other part of the box, therefore every square inch of the interior surface of the box is pressed outward with the force of a pound. If, then, there is another tube inserted in any part of the box with a plug of the same area, as at *b*, it will require a force of a pound to keep this plug in its place. (We leave out of account at present the pressure upon *b* arising from the *weight* of the water in the box above it, and consider only the pressure propagated by the forcing down of the plug *a*.) However many plugs of the same size there were, each would be pressed out with the same force of a pound; and if there were a large plug of four times the area, as at *c*, it would be pressed out with a force of four pounds.

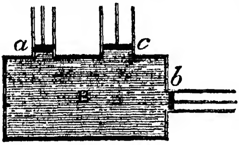


Fig. 1.

12. We have only, then, to enlarge the area of the piston *c* to obtain any multiplication of the force exerted at *a*. If the area of *c* is 1000 inches, that of *a* being one inch, a pressure of one pound on *a* becomes a pressure of 1000 pounds on *c*; and if we make the pressure on *a* one ton, that on *c* will be 1000 tons. This seemingly wonderful multiplication of power has received the name of the *hydrostatic paradox*. It is, however, nothing more than what takes place in the lever, when one pound on the long arm is made to balance 100

pounds on the short arm. The law of virtual velocities holds in the one case as well as in the other: when we think of the machine in motion, we see that what is gained in power, is lost in time. If the piston in *a* descend one inch, it will raise the piston *c* only the one-thousandth part of an inch; for a descent of one inch in *a* dislodges a cubic inch of water, forcing it into the box; this causes a cubic inch of water to rise into *c*, where it is spread out over a thousand times as much surface, and therefore has only the thousandth part of the depth.

13. If the pressure we have supposed exerted on the piston *a* arose from a pound of water poured into the tube above it, it would continue the same though the piston were removed. The pound of water in the tube is then pressing with its whole weight on every square inch of the inner surface of the box—downwards, sidewise, and upwards—and, if the box is twenty inches each way, so as to have upwards of 2000 inches of surface, this one pound of water is tending to burst it with a force of about a ton. That this is no mere theory, may be proved without much difficulty, by fitting a long small tube, *b*, into the top of a cask, *a*, as represented in fig. 2. If the tube is only the twentieth of an inch in area, and is made long enough to contain a pound of water, it gives a bursting force

Fig. 2.

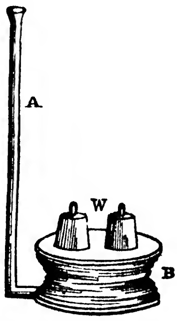


Fig. 3.

of twenty pounds on every square inch of the inner surface—a strain which no ordinary cask could resist.

14. The apparatus called the *hydrostatic bellows* acts on the same principle (see fig. 3). It consists of two stout circular boards connected together by leather in the manner of a

bellows, B. The tube A is connected with the interior; and a person standing on the upper board, and pouring water into the tube, may lift himself up. If the area of the upper board is 1000 times that of the tube, an ounce of water in the tube will support 1000 ounces at W.

15. If we conceive a mountain having a cavity in its interior, as at B in fig. 4, without any outlet except a crevice, A,

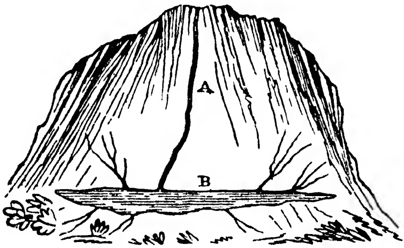


Fig. 4.

extending upwards to the surface; the rain descending and filling the crevice to a great height, might be sufficient to burst the mountain asunder.

16. It is on the principle explained with regard to the two pistons *a* and *c*, in fig. 1, that the very useful machine called the *Hydraulic Press*, invented by Bramah, is constructed. Fig. 5 represents the essential parts of the machine, the details of construction being omitted. F is the cavity of a strong metal cylinder E, into which the piston D passes water-tight through the top. A tube G leads from the cylinder to a force-pump H; and by means of this, water is driven from the tank T into the cavity F, so as to force the piston D upwards. The piston supports a table on which are placed the bales, books, or other articles to be pressed; and the rising of the table presses them against the entablature AA, which is fastened to the pillars B, B.

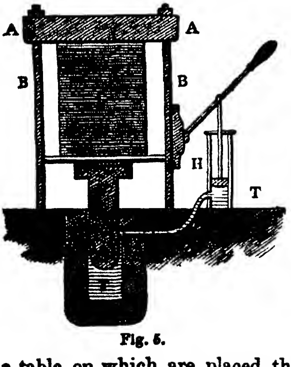


Fig. 5.

17. The power of the hydraulic press is readily calculated. Suppose that the pump has only one-thousandth of the area of F, and that, by means of its lever handle, the piston of the pump is pressed down with a force of 500 pounds, the piston of the barrel will rise with a force of one thousand times 500 pounds, or more than 200 tons. The rise, however, will be slow in proportion to the gain of power.

PRESSURE OF WATER ON SURFACES IN VARIOUS POSITIONS.

18. Water has weight like solids, and this weight produces pressure; but the quality by which it transmits pressure in all directions, makes its weight be felt in many respects differently from that of solids. Thus, suppose a cylindrical vessel filled with a piece of ice exactly fitting it; the whole weight of the ice will rest on the bottom, the sides being unaffected. But if the ice is now melted, the bottom will sustain the same weight as before, and the sides will at the same time be pressed outwards with a certain force. Any object immersed in water is also pressed on all parts of its surface in contact with the water. We have now to determine the amount of this pressure caused by the weight of water in various circumstances.

19. *The pressure of water increases in intensity with the depth, without regard to the shape or size of the cavity or vessel containing it.*

20. Suppose the water in the two vessels represented in fig. 6 to be divided into layers of an inch deep. If the area of the tube C is one square inch, and that of the larger cylinder, A, ten inches, the top layer in the tube will contain a cubic inch of water, and that in the cylinder ten cubic inches. Now, these layers rest on the surfaces of the layers below them; the second layer in the tube sustains the weight of a cubic inch of water; that in the cylinder, ten cubic inches. But in the last case the weight is equally distributed over ten square inches of surface; in both cases, then, the pressure on a square inch of the upper surface of the second layer is the weight of a cubic inch of water, or about 252 grains. And this pressure is not merely downwards. The film of particles sustaining this weight react on the upper layer, pushing it upwards, and also push sidewise against each other and against the sides of the vessels, all with equal force (art. 10). In like

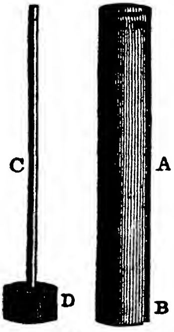


Fig. 6.

manner, each square inch of the surface of the second layer, in either vessel, has to bear the weight of two cubic inches of water, and the particles at that depth push upwards and laterally with corresponding intensity. The pressure thus increases with the depth equally in the narrow vessel and in the wide.

21. If we suppose the vessels to be each a foot deep, it is evident that on every square inch of the bottom of the cylinder there will rest a pile of twelve cubic inches (weighing about half a pound troy), and therefore the pressure on the whole bottom will be the weight of ten such columns. But from the shape of the other vessel, the lower part of which is enlarged, it is not so evident what the pressure on the bottom is. One thing is clear, that, on the square inch in the middle of the bottom immediately under the tube, there rests a column of twelve cubic inches. Now, there must be the same pressure among the particles of the water lying around the bottom of this column, as among its own particles, otherwise there would be a flow towards the part where the pressure was less. Throughout the whole of the enlarged space D, there is the same pressure as there is at the same level immediately under the tube. Therefore, if the enlarged part of the vessel is of the same width as the cylinder, having an area of ten square inches, the pressure on the whole bottom is equal to the weight of ten times twelve cubic inches of water, exactly as in the cylinder at B.

22. It thus appears that *the pressure on the horizontal bottom of a vessel is as the area of the bottom and the perpendicular height of the liquid*, and that without regard to the shape of the vessel.

23. Take the case of two vessels of equal capacity, and shaped as in figs. 7 and 8. In fig. 7, the bottom is pressed with the

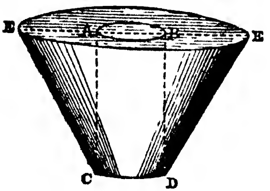


Fig. 7.

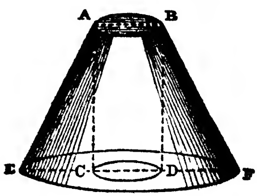


Fig. 8.

weight of the column of water ACDB. In fig. 8, the portion CD, equal to the whole base of the other, sustains an equal

column; but from the law of liquids the same pressure is extended over the whole bottom. Now, if the diameter EF is three times the diameter CD, the larger bottom is nine times the area of the smaller. Therefore, the pressure on the bottom of fig. 8 is nine times the pressure on that of fig. 7, although both vessels contain the same quantity of water. If the bottoms of the vessels were separate from the sides, and kept in their places by springs or weights, it could be shewn experimentally that such is the case.

24. A strange consequence would seem to follow from this. If the two vessels in fig. 6 were placed on the opposite scales of a beam, the bottoms of both being equally pressed, it would seem as if they must balance each other. Similarly, we might expect fig. 8 to weigh nine times as much as fig. 7. But that such cannot be the case appears from the following considerations: In the upright cylinder in fig. 6, the lateral pressure being directly horizontal, neither presses the sides down upon the bottom nor lifts them up; but with the irregular vessel it is different. Where the small tube C enters the wide part D—say two inches from the bottom—the pressure of the column is equal to ten cubic inches of water on the square inch; and as this pressure acts upwards as well as downwards, the horizontal cover of the wide part is pressed *upwards* with a force of ten cubic inches on the square inch. Its area, the tube being one inch, is nine inches, so that the whole upward pressure is 9×10 cubic inches. The cover being attached to the sides, and the sides to the bottom, a part of the pressure on the bottom is thus counteracted, and only the difference—the difference between 10×12 and 9×10—or thirty cubic inches of water—remains to weigh down the scale. The confined water is in this case like a person in a covered box who should place his shoulders against the top and press against the bottom with his feet; he would thus add nothing to the weight of the box beyond his own dead weight. With regard to figs. 7 and 8, the pressure on the sides EC and ED of the former being at right angles to those sides, is partly downwards (see fig. 11), and thus adds to the direct pressure on the bottom; while in fig. 8 the pressure on the sides is partly upwards, so as to relieve a part of the bottom pressure. The result is, that the pressure or weight on the scale is the same in both.

25 *Pressure of Liquids on the Sides of Vessels.*—We begin with the case of an upright side as the simplest. Let AB represent a vessel with upright rectangular sides filled with water, and suppose the perpendicular depth, AC, of the water to be six inches. A square inch of the bottom at Cd is evidently pressed with the weight of a column of six cubic

inches of water. Now, as water presses equally in all directions, the square inch from C to 5 of the side adjacent, will also be pressed; but evidently not to the same extent, owing to the gradual diminution of the weight of the water as we advance upwards. The particle of water in the corner at C presses equally downwards and outwards, and as its pressure downwards arises from the weight of the column of particles above it from C to A, we may represent its outward pressure by the line Ce, made equal to CA. At 5, the downward pressure, and therefore also the outward pressure, is one-sixth less, and will therefore be represented by 5l, made equal to 5A. If el is joined, the figure Cel5 represents the sum of the outward pressures of all the particles from C to 5, and also the mode of their gradual diminution. Had the pressure continued constant, the sum would have been expressed by the parallelogram Cf, which is equal to Ad, the weight on the bottom. From this it appears that the pressure on a square inch of the side next to the bottom is less than that on a square inch of the bottom itself, by the weight of half a cubic inch of water. For it is clear that fle, which represents the difference of the pressures on the lines 5C and Cd, is half a square inch, and completing the parallelopiped on the square 5C, we should find the portion of it corresponding to $lfe = \frac{1}{2}$ a cubic inch.

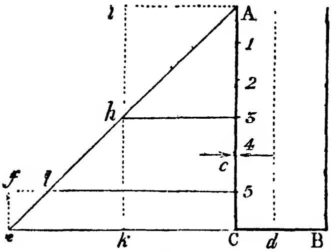


Fig. 9.

26. By similar reasoning, the pressure at any point on the side is represented by a horizontal line equal to the depth of the point from the surface, or the *head of water* of the point, as it is called. Thus, at 3, the pressure is 3h, equal to 3A. Now, by the property of similar triangles, the extremities of all such lines, e, l, h, are in the same straight line with A; and by joining them, we have a triangle, CeA, representing the amount and mode of distribution of the lateral pressure on the side CA.

27. It will make the nature and amount of this pressure yet clearer if we conceive A′C′ (fig. 10) to be a vertical slip or stave of the side, an inch broad, laid horizontally, and solid columns of water (supposed rigid or turned to ice) standing on each square inch; the columns being shaped at top like C′e′l′5,

and gradually diminishing in height to A′. The vertical pressure on A′C′ that would arise, in this supposed case, from the weight of the columns of water, corresponds exactly to the lateral pressure on AC in the original position.

28. What is true of one vertical row of square inches in the side, is true of all the vertical rows of which it is composed; and therefore the pressure on the whole rectangular side is equal to the weight of a wedge-shaped mass of liquid, A′C′e′, whose base is equal in area to that side. But the content of A′C′e′ is evidently equal to that of a uniform rectangular column of mean depth, 3h′. If, then, we suppose the length of the side to be 10 inches, its depth being 6 inches, the area will be 60 square inches; and this multiplied by 3, the mean depth, gives 180 cubic inches of water as the mass supposed to rest on the side in its horizontal position, and the weight of which is equivalent to the lateral pressure in the upright position. The truth thus arrived at may be expressed in the following general proposition:

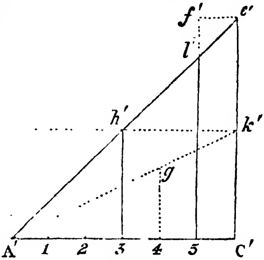


Fig. 10.

29. *The lateral pressure of a liquid, perpendicular to the side of a vessel, is equal to the weight of a column of the liquid whose base is the side and height equal to the depth of the middle point, or centre of gravity of the side.*

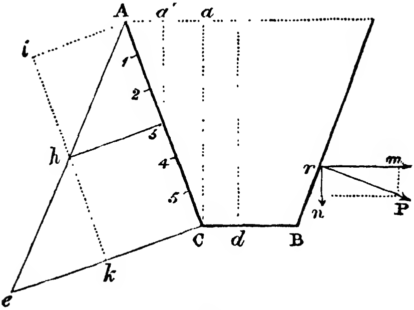


Fig. 11.

30. In the case of a vessel with a sloping side, AC (fig. 11),

the pressure perpendicular to AC is still represented by a wedge-shaped mass, ACe; but here Ce is not equal to CA, but to Ca, the perpendicular depth of the water; and 3h to 3a', the mean perpendicular depth. In all cases, when speaking of depth or head of water, it is perpendicular depth that is meant; and this being understood, the proposition of art. 27 is equally true of a sloping side (whether in the direction of fig. 7 or of fig. 8) as of a vertical.

31. We have as yet considered only rectangular sides, in which the middle point, or point of mean pressure, lies midway between the surface and the bottom, or at half the depth. But in the case of a vessel with a triangular end or side, like ABC (fig. 12 or 13), it is evident that the middle point of the figure is not at m, the middle of the vertical line Bd. For in fig. 12 the number of points or units of surface that lie below m, and are therefore pressed with greater force, is far less than of those that lie above m, and sustain less pressure. The real middle point of any surface is its centre of gravity; and the centre of gravity of a triangular surface (see *Matter and Motion*, par. 147) is found by drawing a line from any angle, B, to the middle of the opposite side, d, and taking the point g at one-third of the distance from d to B. The mean depth, then, or mean head of water, in fig. 12, is dg; and in fig. 13, Bg. Taking, as represented in the figures, the simple case of isosceles triangles, in which the lines Bd, bisecting the base AC, are also perpendicular to it. If AC is, in both figures, 20 inches, and Bd 18 inches, the area in both cases is $20 \times 18 \div 2 = 180$ square inches. The head of water in fig. 12 (supposing the side to be perpendicular) is $18 \div 3$, or 6 inches; and thus the pressure is $180 \times 6 = 1080$ cubic inches of water. In fig. 13, the head of water is 12 inches, so that the pressure is $180 \times 12 = 2160$ cubic inches.

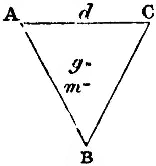


Fig. 12.

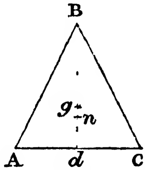


Fig 13.

32. The pressure of a liquid on a surface is the same whether the surface forms part of the bottom or sides of the vessel containing the liquid, or belongs to a body immersed in the liquid. If an empty box, having for one of its sides a triangle like ABC, fig. 12, is sunk in water to any depth, the water will exert a pressure tending to force in the side, whether it be uppermost or undermost, vertical or oblique; and the amount of the pressure could be shewn, as in the cases already examined,

to depend on the area of the surface and the depth of its centre of gravity from the surface of the water. Thus, were the supposed box sunk, no matter in what position, so that the point g on its side ABC were 5 feet, or 60 inches, deep; it would sustain a pressure equal to the weight of $180 \times 60 = 10{,}800$ cubic inches of water. We may therefore generalise the proposition respecting the pressure of liquids, so as to apply to a surface in any position and of any form. Thus:

33. *The pressure of a liquid on any surface immersed in it, is equal to the weight of a column of the liquid whose base is the surface pressed, and whose height is the perpendicular depth of the centre of gravity of the surface below the surface of the liquid.*

34. A cubic foot of pure water weighs very nearly 1000 oz. avoirdupois, or 62·5 lbs. More exactly, a cubic inch of distilled water, at the temperature of 62° Fahrenheit and barometric pressure of 30 inches, weighs 252·46 grains; and a cubic foot weighs 62·32 lbs., or 997 oz. The cubic foot of sea-water may be taken on an average at 1026 oz.

35. The above rule gives the pressure in volume of water; but this is easily reduced to weight by multiplying the number of cubic feet of water by 1000 (see art. 34), which gives the weight in ounces, or by 62·5, which gives the weight in pounds.

Ex. 1. Required the pressure on a lock-gate 12 feet wide, the depth of the water being 10 feet? The area of the surface pressed is $12 \times 10 = 120$ square feet; 120 multiplied by 5 (the depth of the middle point of the surface) gives 600 cubic feet of water as the pressing column; and $600 \times 62{\cdot}5 = 37{,}500$ lbs., is the pressure in weight.

Ex. 2. What is the amount of pressure on the sides of a cubical vessel full of water, the side of the base being 3 feet? The pressure on one side is that of $9 \times 1\frac{1}{2} = 13\frac{1}{2}$ cubic feet of water, and on all the four sides, $13\frac{1}{2} \times 4 = 54$ cubic feet, or 3375 lbs. The pressure on the bottom, in this case, is $9 \times 3 = 27$ cubic feet $= 1687{\cdot}5$ lbs.; and in like manner it will be found, that *in any cubical vessel, the pressure on the sides is twice that on the bottom.*

36. The circumstance of pressure increasing in proportion to depth, makes it necessary to increase the breadth of embankments for dams and canals from the top downwards; also to increase the strength of the lower hoops of large vats, to prevent their bursting. It likewise teaches the propriety of making dams, ponds, canals, and vessels for liquids generally, as shallow as is consistent with convenience or their required purpose. In every case, it is important to recollect that the

degree of pressure on the sides is irrespective of shape or size of the contents, and depends exclusively on the perpendicular depth of the liquid.

37. When the poet speaks of 'the *broad* ocean leaning against the land,' we are apt to think of the broadness as adding to the weight that the land has to sustain. But the pressure against a square foot of the shore of the broad Atlantic is no greater than against a foot of the shore opposite Dover, or a foot of the side of a canal, at the same depth. The pressure against the gates of a canal is the same whether the next lock is a mile or whether it is fifty yards off. It might be brought within a foot, or even a fraction of an inch, and the weight on the gate would remain unchanged so long as the depth was the same. This being the case, it is really no more difficult to embank the calm ocean than a small lake of the same depth; except for the violence of the waves, the same strength of dike would resist the weight in both cases.

38. At the depth of a foot, the pressure on a square inch of surface is a column of 12 cubic inches, weighing, in the case of fresh water, very nearly 7 oz., and, in the case of salt water, slightly more than 7 oz. This gives, at the depth of 7 feet, a pressure of 49 oz., or about 3 lbs. As water is so very slightly compressible (see art. 5), we may assume, without any great error, that its density is uniform at all depths, and, therefore, that the pressure will go on increasing regularly at the rate of 3 lbs. on the square inch for every 7 feet in depth. It is thus easy to calculate the pressure at any given depth. At a thousand feet, for instance, it will be $\frac{1000}{7} \times 3 = 428{\cdot}6$ lbs., or about 29 atmospheres. At the depth of a mile, or 5280 feet, it will be $\frac{5280}{7} \times 3 = 2263$ lbs., or 150 atmospheres.

39. *Centre of Pressure.*—We have seen that a vertical line or section AC (fig. 9) of a vessel containing a liquid is pressed outwards by a set of forces increasing in intensity at every point from A to C. Now, there is evidently a point somewhere in AC at which, if a single pressure of sufficient strength is applied on the outside in the opposite direction, it will counteract the effect of all the outward pressures at the several points on the inside, and hold the line in its place. This point is called the *Centre of Pressure;* it is the point of application of the resultant of all the elementary pressures exerted at the different points.

40. As the pressures on the opposite sides of the centre of pressure must balance as on a fulcrum, it is clear that it cannot be on the middle of AC, but nearer the bottom, where the pressures are the greatest. Since the lateral pressure on CA (fig. 9)

is exactly the same as the vertical pressure on A′C′ (fig. 10), arising from the weight of a triangular plate of rigid water A′e′C′ (see art. 27), we may determine the exact position of the point in question by supposing the two fulcrums at A′ and C′ removed, and A′C′ (fig. 10) balanced on a single fulcrum. That fulcrum must evidently be under the centre of gravity of the triangular plate; now, the centre of gravity, g, of a triangle A′e′C′ is found by joining A′ with k', the middle of one of the sides, and taking $k'g$ equal to $\frac{1}{3}$ of A′k'; the fulcrum will therefore be at the point 4, which is $\frac{1}{3}$ from C′ towards A′. Thus, the centre of pressure on the vertical section AC (fig. 9) is at c, $\frac{1}{3}$ from the bottom; and for the whole of a rectangular side, the centre is $\frac{1}{3}$ from the bottom of the vertical line at the middle of the length.

41. For surfaces of other forms than rectangular, the investigation of the centre of pressure is too difficult for introduction here. On a triangular side in the position of ABC (fig. 12), it is determined to be at m, $\frac{1}{2}$ the depth; in the position of fig. 13, it is at n, $\frac{1}{4}$ from the bottom.

42. Each stave of a vat or cask may be considered as a rectangular side; the position, therefore, where a single hoop has the greatest effect in resisting the pressure, is at $\frac{1}{3}$ from the bottom. At that point, too, the staves should be made the strongest, if they are wished to combine the greatest strength with lightness.

THE SURFACE OF LIQUIDS.

43. *The surface of a mass of liquid at rest is a perfect level.* Two or more points are on the same level when they are equally distant from the earth's centre. Let EAPBS represent a portion of the earth's circumference, C its centre, and L, T, two points above the surface equally distant from C; then L and T are on the same level; and if an arc of a circle is described from C through these two points, any other points, as Q, R, in that arc are on the same level with L and T.

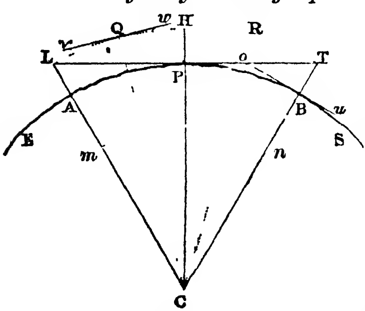


Fig. 14.

In like manner, the points A, P, B, in the earth's circumference, are on the same level; and so are *m* and *n*, below it. A level line, then, is, strictly speaking, not a straight line, but a portion of a circle; and similarly, a level surface is not a plane, but a portion of a spherical surface.

44. The direction of gravity—that is, the direction of a plumb-line—is always towards the earth's centre. Thus, at T or at B, the direction of gravity is the line TBC; at Q, it is the line QC; and so of the other points in the figure. These lines marking the direction of gravity are perpendicular to the earth's surface, or to any other level surface, as LHT. A straight line, as TB, is perpendicular to a curve when it is perpendicular to the line B*u* drawn touching the curve at the point B. The lines TB, HP, LA, that mark the direction of gravity, are called *vertical* lines; and lines at right angles to them, such as *o*B*u*, LPT, *v*Q*w*, are *horizontal* lines.

45. When we confine our attention to a small arc of a very large circle, it does not differ sensibly from a straight line. Owing to this, 'horizontal' and 'level' are used, in common language, to signify the same thing. It is only when we take in a considerable space that the difference becomes appreciable. When considering a mass of water in a vessel or pond, then, the surface may be assumed to be straight or plane, and all vertical lines, or lines shewing the direction of gravity, as parallel to one another.

46. The familiar fact that the surface of a liquid at rest is always level or horizontal—in the popular sense—admits of explanation; it can be shewn to be a consequence of the primary property of 'pressing equally in all directions.' For let *da* and *cb* be vertical lines, or lines in the direction of gravity; and *ab* a plane at right angles to that direction, or horizontal. A particle of the liquid at *a* is pressed by the column of particles above it from *a* to *d*; and the like is the case at *b*. Now, since the liquid is at rest, these pressures must be equal; for if the pressure at *b*, for instance, were greater than at *a*, there would be a flow of the water from *a* towards *b*. It follows that the line *ad* is equal to *bc*, and hence that *dc* is parallel to *ab*, and therefore horizontal. The same might be proved of any two points in the surface; therefore the whole is in the same horizontal plane. Or we might prove the same thing by the following consideration: if we could conceive a portion of the surface

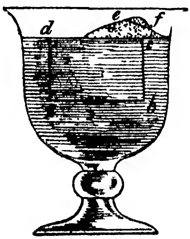


Fig. 15

heaped up, as at ef; besides causing unequal pressure, and therefore disturbance in the mass below, the particles on the summit would have a tendency to flow down as on an inclined plane. Particles of sand so heaped up have this tendency; and are only prevented from spreading out into a perfectly level surface by their imperfect mobility.

47. When a vessel, AB (fig. 16), containing water, is made to rotate on an axis, xx, the surface assumes a hollow form. Any molecule of the water, as m, is acted on by two forces; its own gravity or weight, represented in the figure by mg, and the centrifugal force arising from rotation, represented by mc. The resultant of these two, mr, shews the real direction in which the molecule, m, presses. Now, the molecules of the surface can be at rest only when the direction of the pressure of each is at right angles to the surface at that point; and it is shewn by mathematics that this is the case when the surface is parabolic. Such, accordingly, is the form that water assumes in the case supposed.

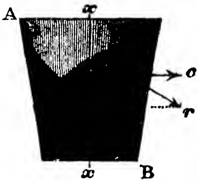


Fig. 16.

48. It seems so natural that the surface of an unbroken mass of liquid should be all of one height, that we seldom think of it as requiring explanation. But when the surface of a liquid is not continuous, being contained in separate vessels, with communication between them, it requires consideration, and experience to convince us that the several parts of the surface must in all cases stand at the same level. A common tea-pot affords the most familiar illustration of this truth. Since pressure in a liquid depends exclusively on the depth, without regard to the shape or size of the mass, the pressure on a square inch at the level of communication is the same under A as under B; thus there is equilibrium. If an inch of additional water is put on the top at B, the pressure at the bottom is increased by that much, and a flow takes place along the line of communication till the whole again stand at the same height, the surfaces coinciding with a horizontal line AB.

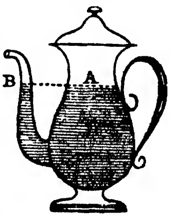


Fig. 17.

49. Fig. 18 represents six vessels communicating with one another, or rather one vessel consisting of six compartments of different shapes and sizes; and it can be shewn in the same way as with a continuous mass of water (art. 46), that the

surface in all the compartments must form part of the same horizontal line AB.

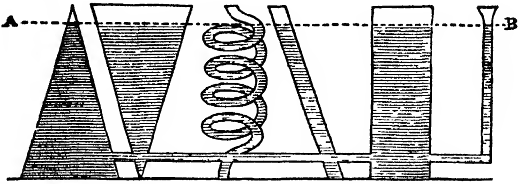


Fig. 18.

50. We have considered the surface of liquids as perfectly flat or straight; and a small surface is practically so. But in the case of extensive surfaces, such as that of the ocean, or of a large lake, it becomes evident that the shape is round or spherical, partaking of the general form of the earth. Even within a small space, as the vessel, fig. 15, the lines of direction of gravity, *ad*, *cb*, are not absolutely parallel; they really converge towards the centre of the earth; although, owing to the enormous distance of that centre compared with the small distance between the lines, the amount of their convergence is not discernible. As all the columns of pressure between *da* and *cb* converge in like manner, it can be shewn that they cannot be all equal and also at right angles to *ab* at every point, unless *ab* and *dc* be both portions of circles concentric with the earth.

51. When a considerable extent of surface is in question, such as that embraced by a portion of sea or ocean, the convergence of the lines of gravity, and the convexity of the surface, become manifest; as in fig. 14. The difference of the directions of gravity at P and at A is visible to the eye; and that the surface may keep constantly at right angles to the pressure, it must curve round in proportion. It is impossible that it could extend in a straight, horizontal surface from P to L. For let us conceive PL to be a solid plane with a thin stratum of liquid laid upon it; at P the force of gravity presses the particles right against the plane, and they have no tendency to flow to either side. But at L the circumstances are different. Gravity there does not press right against the plane, but slanting, towards C, and thus gives the particles a motion along the plane to P. The water is, in fact, on an inclined plane all the way till it get to P, and will flow in that direction until the surface is everywhere at right angles to gravity.

52. It is only when a sheet of water is stretched out to an

extent of several miles that the convexity becomes conspicuous. It is very perceptible on the ocean when a ship is approaching on the horizon; first, the masts and sails of the ship are seen, and lastly, the hull. The convexity of the land is not so conspicuous, in consequence of the many risings and fallings in the surface. It is only in extensive plains that the roundness can be perceived in the same manner as at sea.

The amount of the earth's convexity can be calculated exactly. It is very nearly eight inches in each mile; that is, if PL (fig. 14) is a mile long, the point L will be eight inches above the surface.

In constructing canals and railways, allowance must be made for the convexity of the earth; for the channel of a canal, or a line of level railway, does not form a straight line, but a curve; it is not a part of the tangent or PL, but of the curve PA.

53. *Levelling.*—The most common instrument for determining levels is the *spirit-level.* It consists of a glass cylinder, *ac*, slightly convex on the upper side, and filled with spirits, all except a small space.

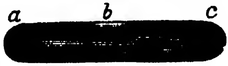


Fig. 19.

It is placed for protection in a wooden or brass case, the bottom of which is exactly parallel with the tube. If the instrument is laid on the upper surface of a stone slab or a beam, and if one end of the slab or beam is higher than the other, the liquid in the tube will seek the lowest position, and the air-bubble will move towards the higher end; when the bubble rests in the middle, as at *b*, the surface is horizontal or level. To ascertain levels at a distance, the spirit-level is fixed below a small telescope, the tube of the telescope and the glass tube being made exactly parallel. The telescope being fixed on a stand, is adjusted by screws until the air-bubble of the spirit-level stands exactly in the middle, and is thus made perfectly horizontal. A pole being now set up at the spot where the level is to be found, the telescope is directed to it, and the point of the pole which appears in the centre of view is in a horizontal line with the eye. If the distance is short, this point may be considered as the level; but if the pole is as far as a mile off, the natural level at which water would stand, if there were a sheet of it all the way, will be eight inches below. The difference between the *apparent level* and the *true level*, as they are sometimes called, varies for different distances, and may either be calculated or taken from a table.

BUOYANCY AND FLOTATION.

54. *When a solid is immersed in a liquid, it displaces exactly its own bulk of the liquid.* This requires no demonstration; as soon as we conceive clearly what is meant, the truth of the proposition is self-evident. Of course, it is only of solids that are neither melted nor penetrated by the liquid that the proposition holds good.

55. If a cubic inch of wood or of a metal is wholly immersed in a vessel previously full of water, the liquid that occupied the space where the solid now is, must overflow; or, if the vessel was not full, the level of the liquid must rise. This affords a ready means of measuring solids, even the most irregular in shape. To ascertain what bulk of solid metal there is in a gold chain: have a cylindrical vessel with a base of known area—say a square inch—filled to a certain height with water; drop in the chain, and note the rise of the liquid. If it rise an inch, the chain contains a solid inch of gold, and so for any fraction of an inch. Archimedes is said to have been the first to think of measuring solid bodies in this way, and to have applied it in detecting the adulteration of a gold crown.

56. Every one must have experienced that a heavy body, held in the hand, becomes lighter when it is immersed in a liquid. We have now to consider how this loss of weight arises, and what is its amount.

57. Conceive, first, that a body of water, AB, forming part of the contents of a vessel, is separated from the rest by an imaginary film; this water will clearly remain suspended like any other portion of the mass. Now, as its weight is pressing it downwards, there must be an equal force acting upon it upwards. That force arises from the pressure of the surrounding liquid, which is acting upon it in all directions, upwards, downwards, and laterally. The horizontal pressures have each an opposite and equal resultant, and thus mutually destroy one another; but the under surface being at a greater depth than the upper, there remains a balance of upward pressure. This balance or result must be exactly equal to the weight of the conceived isolated body, otherwise, it could not keep it at rest. The upward pressure, then, on any separate portion AB, say a cubic foot, within a mass of liquid, is equal to the weight of that portion. This is true, whatever be the shape of the isolated body. If we now suppose the mass of water, AB, to become

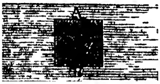


Fig. 20.

solid ice without change of bulk, the same pressure will act upon it as before; and if we further conceive the cubic foot of ice to become a cubic foot of gold, the upward pressure upon it will remain the same—namely, the weight of a cubic foot of water, or the weight of the water it displaces. To this extent the water supports it, and renders it lighter.

58. It appears, then, that *a solid body in a liquid loses as much weight as an equal bulk of the liquid weighs.*

If a cubic foot of the liquid and of the solid have equal weights, the solid will lose all its weight, or will remain in the liquid wherever it is put; if a cubic foot of the liquid weigh more than one of the solid, the solid will not only lose all its weight, but will rise up, and that with a force equal to the difference; if a cubic foot of the liquid weigh less than one of the solid, the solid will lose weight, but will still sink.

59. It follows that, when two liquids, or, generally, when two fluids (for the principle is true of gases, as well as of liquids) of different specific gravity, are put together into the same vessel, the lighter will float on the top of the heavier, provided they are of a kind that do not intermix. For if we suppose AB to be a mass of oil surrounded by water, being lighter than the same bulk of water, it must rise up; and as it cannot remain in a heap at the top, it diffuses itself in a horizontal stratum.

60. A body of alcohol or spirits in similar circumstances would take the same position for an instant; but the attraction existing between alcohol and water, soon causes a mutual and equal diffusion of the one through the other.

61. When a solid swims, or rises and *floats* on the surface of a liquid, the next problem of hydrostatics is to determine how much of it will be below the surface. We have already seen that any solid in a liquid is pressed upward with a force equal to the weight of the water whose room it occupies. Now, a floating body must be pressed up with a force equal to its own weight, otherwise it would sink lower; hence,

62. *A floating body displaces its own weight of the liquid.* A solid, as AB in fig. 21, sinks until the space occupied by the part B immersed would contain an amount of water equal in weight to the whole solid AB.

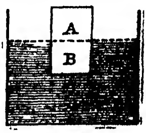


Fig. 21.

63. By measuring how many cubic feet of water a floating body, such as a ship, displaces, we can thus know its weight, by allowing 1000 ounces, or 62·5 lbs., for every cubic foot. If we knew previously the weight of the ship and rigging, the difference would give the weight of the cargo.

64. As the buoyancy of a body thus depends on the relation between its weight and the weight of an equal bulk of the liquid, the same body will be more or less buoyant, according to the density of the liquid in which it is immersed. A piece of wood that sinks a foot in water, will sink barely an inch in mercury. Mercury buoys up even iron. Sea-water is denser than fresh water, in the proportion of 1026 to 1000. A ship, then, that carries 1026 tons on the sea, would carry only 1000 tons on a fresh-water lake.

65. The human body has almost the same weight as an equal bulk of water. When the lungs are full of air, it is slightly lighter, and will float with a bulk of about half the head above water. A person, then, that cannot swim, but has presence of mind to lie flat on the back, with the back part of the head submerged, may, by keeping the lungs full, continue to float with the face above water; but if any other part of the body, as a hand, is raised above the surface, the whole head goes immediately under. Swimmers, by the action of hands and feet against the water, keep the whole head, and often more, above the surface.

66. A body which would sink of itself, is buoyed up by attaching to it a lighter body; the bulk is thus increased without proportionally increasing the weight. This is the principle of life-preservers of all kinds. The most common are those which consist of pieces of cork, or other very light material, attached to the upper part of the body. But air-tight bags are preferable, as they may be said scarcely to encumber the body when empty, and, as danger approaches, they can be inflated with ease by being blown into. Life-boats have large quantities of cork in their structure, and also air-tight vessels made of thin metallic plates; so that, even when the boat is filled with water, a considerable portion of it still floats above the general surface. The bodies of some animals, as sea-fowl, and many other species of birds, are considerably lighter than water. The feathers with which they are covered add very much to their buoyancy. Fishes are enabled to alter their buoyancy by means of an air-bag, which they can inflate at pleasure by an apparatus for generating gases.*

* The bodies of most fishes are nearly of the specific gravity of water, and therefore, if living in it without making exertion, they neither sink nor swim. When this subject was less understood, many persons believed that fishes had no weight in water; and it is related as a joke at the expense of the philosophers, that a king having once proposed as a task to his men of science to explain this extraordinary fact, many profound disquisitions came forth, but not one of the competitors thought of trying what really was the fact. At last a simple

67. The heaviest substances may be made to float by shaping them so as to make them displace more than their own weight of water. A flat plate of iron sinks; the same plate, made concave like a cup or boat, floats. By making the sides of an iron vessel double, with a considerable air-tight space between the plates, the vessel would be prevented from altogether sinking, even when filled with water.

68. The buoyant property of liquids is independent of their depth or expanse, if there be only enough to surround the object. A few pounds of water might be made to bear up a body of a ton-weight; a ship floats as high in a small dock as in the ocean.

69. *Stability of Floating Bodies.*—Conceive *abd* (fig. 22) to be a portion of a liquid turned solid, but unchanged in bulk; it will evidently remain at rest, as if it were still liquid. Its weight may be represented by the force *cg*, acting on its centre of gravity *c*; but that force is balanced by the upward pressure of

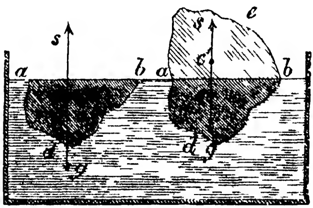


Fig. 22.

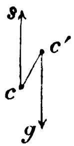


Fig. 23.

the water on the different parts of the under surface; therefore, the resultant of all these elementary pressures must be a force, *cs*, exactly equal and opposite to *cg*, and acting on the same point *c*, for if it acted on any other point, the body would not be at rest. Now, whatever other body of the same size and shape we suppose substituted for the mass of solid water *abd*, the supporting pressure or buoyancy of the water around it must be the same; hence we conclude, that *when a body is immersed in a liquid, the buoyant pressure is a force equal to the weight of the liquid displaced, and having its point of application in the centre of gravity of the space from which the liquid is displaced.* This point may be called the *centre of buoyancy.*

man [who doubted the fact] balanced a vessel of water in scales, and on putting a fish into it, shewed a scale preponderating, just as much as if the fish had been weighed alone.—*Arnott's Elements of Physics.*

70. We may suppose that the space *abd* is occupied by the immersed part of a floating body *aebd* (fig. 22). The supporting force, *cb*, is still the same as in the former case, and acts at *c*, the centre of gravity of the displaced water; the weight of the body must also be the same; but its point of application is now *c′*, the centre of gravity of the whole body. When the body is floating at rest or in a state of equilibrium, this point must evidently be in the same vertical line with *c*; for if the two forces were in the position of *cs*, *c′g* (fig. 23), they would tend to make the body roll over. The line passing through the centre of gravity of a floating body and the centre of gravity of the displaced water is called the *axis of flotation.*

71. The equilibrium of a floating body is said to be *stable*, when, on suffering a slight displacement, it tends to regain its original position. The conditions of stability will be understood from the accompanying figures. Fig. 24 represents a body

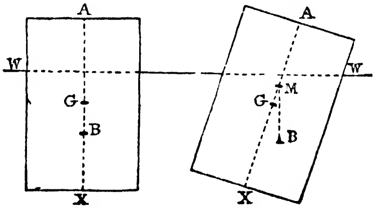


Fig. 24. Fig. 25.

floating in equilibrium, G being its centre of gravity, B its centre of buoyancy, and AGB the axis of flotation, which is of course vertical. In fig. 25 the same body is represented as pushed or drawn slightly from the perpendicular. The shape of the immersed portion being now altered, the centre of buoyancy is no longer in the axis of figure, but to one side, as at B. Now, it is evident, that if the line of direction of the upward pressure—that is, a vertical line through B—meets the axis above the centre of gravity, as at M, the tendency of the two forces is to bring the axis into its original position, and in that case, the equilibrium of the body is stable. But if BM meet the axis below G, the tendency is to bring the axis further and further from the vertical, until the body get into some new position of equilibrium. There is still another case; the line of support or buoyancy may meet the axis in G, and then the two forces counteract one another, and the body remains in

any position in which it is put; this is called *indifferent* equilibrium. In a floating cylinder of wood, for instance, B is always right under G, in whatever way the cylinder is turned.

72. When the angles through which a floating body is made to roll are small, the point M is nearly constant. It is called the *metacentre;* and its position may be calculated for a body of given weight and dimensions. In the construction and lading of ships, it is an object to have the centre of gravity as low as possible, in order that it may be always below the metacentre. With this view, heavy materials, in the shape of ballast, are placed in the bottom, and the heaviest portions of the cargo are stowed low in the hold.

SPECIFIC GRAVITIES OF BODIES.

73. If a lump of sulphur is weighed in air, and found to be two ounces, when weighed in water it will be found to weigh only one ounce. Now, as we know that the weight which a body loses in water is exactly the weight of an equal bulk of water (art. 54), we infer that a body of water equal in size to the lump of sulphur weighs an ounce. Bulk for bulk, then, sulphur is twice as heavy as water, or their weights are as 2 to 1. In the same way, if a piece of cast iron, of, say thirty-five pounds' weight, is weighed in water, it will lose about five pounds; cast iron, then, is about seven times as heavy as water, or their weights are as 7 to 1. By thus referring substances to water as a standard, we get a convenient view of their comparative densities, or specific gravities, as the term is.

74. *The specific gravity of a body, then, is its weight compared with that of water.* In tables of specific gravities, that of water is generally expressed by 1, but sometimes, also, by 1000. Thus, with water as 1, the specific gravity of sulphur, by the last art., is 2; of iron, 7; with water as 1000, these substances would be 2000, and 7000 respectively. Similarly, gold is found to weigh about 19⅓ times as much as an equal bulk of water; its specific gravity therefore is, according to the one scale, 19⅓, according to the other, 19,333.

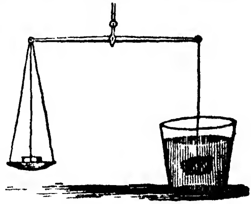


Fig. 26.

75. The apparatus used in such experiments is called the hydrostatic balance, a simple form of which is represented in fig. 26. With such simple means, one might readily detect whether a precious metal, such as gold, is

alloyed or not. We know that the weight of a piece of pure gold is to its loss of weight in water as $19\frac{1}{3}$ to 1; if, then, the object said to be gold is found, on trial, to lose more than this proportion, it is a proof that it is adulterated with some more bulky metal. If it is known what the alloying metal is, it is possible to calculate how much of it enters into the composition.

76. Instruments for readily indicating the specific gravities of liquids are called *hydrometers*, from two Greek words signifying *water* and *measure*. The name *areometer* (Gr., measurer of *rarity*) is also applied. Hydrometers are of various kinds; but the general principle on which they act will be understood from the accompanying figure. *b* is a glass ball, with a smaller ball, *c*, below it, containing shot, in order to make the instrument float upright; the fine stem, *ed*, being uniform and graduated. Suppose the hydrometer first immersed in water, it sinks until the water displaced equals the weight of the whole instrument; if placed next in a liquid rarer than water, it must sink further, so as to displace more of the liquid, before it is in equilibrium. By knowing what part of the volume of the whole instrument each division of the stem is, we get the comparative volumes of water and of the liquid that sustain the same weight; and then the specific gravities are inversely as the volumes. It is evident that the thinner the stem, the more delicate will the instrument be. Hydrometers are made to indicate a difference in specific gravity of 1 part in 40,000.

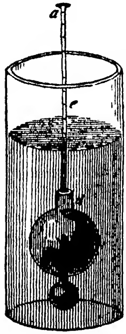


Fig. 27.

77. For determining the specific gravities of bodies lighter than water, and of such as melt in it, various expedients must be had recourse to; but the principle in all is the same. In the case of a piece of wood, for instance, a heavy body is taken sufficient to sink the light body when united with it. Having ascertained the weight of the heavy body in water, the two together are then weighed in water. The weight of the two will be less than the weight of the heavy body alone; and this difference, added to the weight of the light body in air, will evidently be the weight of the water displaced by it. This, compared with its own weight in air, gives its specific gravity.

78. To determine the specific gravity of a liquid, take a known weight of any solid that does not melt in the liquid or in water, and that is heavier than either, and observe its loss when weighed in water, and also when weighed in the liquid. These losses are the weights of equal bulks of water

and of the liquid, and therefore express their comparative densities; and if the loss in the liquid is divided by that in water, the quotient will be the specific gravity of the liquid, that of water being 1. Thus, a cubic inch of lead loses 253 grains when weighed in water, and only 209 grains when weighed in rectified spirit; therefore a cubic inch of rectified spirit weighs 209 grains, an equal bulk of water weighing 252; and so the specific gravity of water is about a fourth greater than that of the spirit.

79. The heaviest substance known is platinum, whose specific gravity is 22. There are metals, again, lighter than water, as potassium and sodium. One liquid, mercury, has a specific gravity of 13½; ether is only 0·715. The lightest substance known is hydrogen gas. The standard of comparison for gases is atmospheric air, which, at 60°, is 800 times lighter than water. Its specific gravity is taken at 1000. Hydrogen is fourteen times lighter than air, while carbonic acid and chlorine are more than twice as heavy as air. The vapour of iodine is eight times the weight of atmospheric air.

80. Spirits, such as brandy, whisky, gin, &c., consist of alcohol and water in varying proportions; and as the excise-duty is charged according to the amount of the alcohol, it is important to be able to tell at once what proportion or percentage of alcohol is present in any spirit. Alcohol, or spirit of wine, without any water—absolute alcohol, as it is called—has a specific gravity of ·796, water being 1. The stronger any spirit is, then—that is, the less the proportion of water it contains—the lighter it is; and in this way the hydrometer becomes the means of measuring the strength of spirits. The hydrometer used by the officers of excise is of the kind known as Sikes's Hydrometer. What is called 'proof-spirit,' consists of equal weights of alcohol and water, and has a specific gravity of ·92. When placed in this liquor, the exciseman's hydrometer sinks to a point on the scale marked *proof*. In a stronger, and, therefore, lighter spirit, it will sink further, and the surface of the spirit will be so many degrees 'above proof.' The contrary takes place with a weaker spirit.

81. As heat expands liquids, rendering them specifically lighter, great attention must be paid to the temperature of those experimented upon, as otherwise wrong results will be obtained. Spirituous liquors are especially affected by temperature, and would appear considerably stronger when tested at a high temperature than at a low, if this difference were not taken into account. The standard temperature is 60°, and observations taken at any other temperature have to be reduced to the standard by certain rules.

(In *Matter and Motion*, a table is given of the specific gravities of a few of the more usual substances.)

The following rules, founded on the principles above explained, will enable the student readily to solve the more important problems in which the specific gravities of bodies are concerned.

82. *Problem I.*—To find the specific gravity of a body heavier than water.

Rule.—Find the weight of the body in air and also in water, and divide the weight in air by the difference between the two weights. Let w be the weight in air, and w' that in water, and s the specific gravity; then the rule expressed in a formula is, $s = \frac{w}{w - w'}$.

Ex. A piece of silver weighed 33·6 ounces in air, and 29·56 ounces in water; what is its specific gravity?

$$Ans.\ s = \frac{33{\cdot}6}{33{\cdot}6 - 29{\cdot}56} = \frac{33{\cdot}6}{3{\cdot}04} = 11{\cdot}05.$$

83. *Problem II.*—To find the specific gravity of a body lighter than water.

Rule.—Find the weight of the body in air, and the weight in water of another body, which, when attached to the former, will make it sink; find also the weight in water of the compound body; then, calling these three weights w, w, and W, respectively, $s = \frac{w}{w + \mathrm{w} - \mathrm{W}}$.

Ex. A piece of ash weighs 60 lbs. in air, and to it is attached a piece of copper which weighs in water 40 lbs., and the compound weighs in water 25 lbs.; what is the specific gravity of the ash?

$$Ans.\ s = \frac{60}{60 + 40 - 25} = \frac{60}{75} = {\cdot}8.$$

84. *Problem III.*—To find the weight of a body when its cubic content is given and its specific gravity.

Rule.—Multiply the content or volume in cubic feet by the specific gravity of the body, and this product by 1000; the result is the weight in ounces. Or, calling s the specific gravity, v the volume or content in cubic feet, and w the weight in ounces, $w = v \times s \times 1000$. By substituting 62·5 for 1000, the weight is found in lbs.

Ex. What is the weight of a bar of cast iron, its breadth and thickness being 4 and $2\frac{1}{2}$ inches, and length 8 feet, and its specific gravity 7·25?

Here, $v = \frac{1}{3} \times \frac{5}{24} \times 8 = \frac{5}{9}$ cubic foot.

$$w = \tfrac{5}{9} \times 7{\cdot}25 \times 62{\cdot}5 = 251{\cdot}7 \text{ lbs.}$$

85. *Problem IV.*—To find the cubic content of a body when its weight is given.

Rule.—Divide the weight of the body in ounces by 1000 times its specific gravity, and the quotient is the content in feet. Or, $v = \frac{w}{1000s}$.

Ex. Find the content of an irregular block of sandstone weighing 1 cwt., its specific gravity being 2·52.

$$v = \frac{112 \times 16}{2520} = \cdot 71 \text{ feet.}$$

86. *Problem V.*—To find the quantity of either of the ingredients in a compound of two ingredients, when the specific gravities of the compound and of the ingredients are given.

Let W, w, w', denote the weights of the compound and of the two ingredients; and S, s, s', their specific gravity respectively, s being that of the denser ingredient; then

$$w = \frac{(S - s') \times s}{(s - s')\,S} \times W, \text{ and } w' = \frac{(s - S)\,s'}{(s - s')\,S} \times W.$$

Ex. A composition weighing 56 lbs., having a sp. gr. of 8·784, consists of tin and copper of the sp. gr. of 7·32 and 9 respectively; what are the quantities of the ingredients?

$$w = \frac{(8\cdot 784 - 7\cdot 32) \times 9}{(9 - 7\cdot 32) \times 8\cdot 784} \times 56 = \frac{1\cdot 464 \times 9}{1\cdot 68 \times 8\cdot 784} \times 56$$

$$= \frac{13\cdot 176}{14\cdot 75712} \times 56 = 50\text{; and hence } w' = 56 - 50 = 6\text{;}$$

Or, there are 50 lbs. of copper and 6 of tin.

HYDRODYNAMICS.

87. Hydrodynamics treats of the laws of the motion of liquids; the flow of water from orifices and in pipes, canals, and rivers; its oscillations or waves; and its resistance to bodies moving through it. The term Hydraulics is sometimes applied to this part of the subject, from the Greek word *aulos*, a pipe. The application of water as a moving power, forms the practical part of the subject.

88. *Efflux.*—If three apertures, A, B, C, are made at different heights in the side of a vessel (fig. 28) filled with water, the liquid will pour out with greater impetuosity from B than from A, and from C than from B. The velocity does not increase in the simple ratio of the depth. The exact law of dependence is known as the theorem of Torricelli; the demonstration is too abstruse for introduction here, but the law itself is as follows:

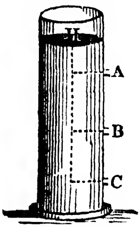


Fig. 28.

89. '*Particles of fluid, on issuing from an aperture, possess the same degree of velocity as if they had fallen freely, in vacuo, from a height equal to the distance of the surface of the fluid above the centre of the aperture.*' The jet from B, for instance, has the same velocity as if the particles composing it had fallen in vacuo from H to B. Now, the velocity acquired by a body in falling is as the time of the fall; but the space fallen through being as the *square* of the time (see *Matter and Motion*, art. 136), it follows that the velocity acquired is as the square root of the space fallen through. In the first second, a body falls 16 feet, and acquires a velocity of 32 feet. If C, then, is 16 feet below H, a jet from C flows at the rate of 32 feet; and if A is at a depth of 4 feet, or one-fourth that of C, the velocity of the jet at A will be half the velocity of that at C, or 16 feet. In general, to find the velocity for any given height, multiply the height by 2×32, and extract the square root of the product. This rule may be expressed by the formula $v = \sqrt{2gh}$, in which v signifies the velocity of the issue, g the velocity given by gravity in a second, or 32 feet, and h the height of the water in the reservoir above the orifice. This last quantity is technically called the *head* or *charge*.

90. That this theory of the efflux of liquids is correct may be proved by experiment. Let a vessel, MB (fig. 29), have an orifice

situated as at *o*; the water ought to issue with the velocity that a body would acquire in falling from M to the level of *o*. Now, it is established in the doctrine of Projectiles (see *Matter and Motion*, art. 137), that when a body is projected vertically upwards with a certain velocity, it ascends to the same height from which it would require to fall in order to acquire that velocity. If the theory, then, is correct, the jet ought to rise to the level of the water in the vessel at M. It is found in reality to fall short of this; but not more than can be accounted for by friction, the resistance of the air, and the water that rests on the top in endeavouring to descend. When the jet receives a very slight inclination, so that the returning water falls down by the side of the ascending, ten inches of head of water may be made to give a jet of nine inches.

91. A stream of water spouting out horizontally, or in any oblique direction, obeys the laws of projectiles, and moves in a parabola (see *Matter and Motion*, art. 139); and the range of the jet for any given velocity and angle of direction may be calculated precisely as in projectiles. Take, for instance, a jet issuing horizontally at D (fig. 29). Its velocity in the horizontal

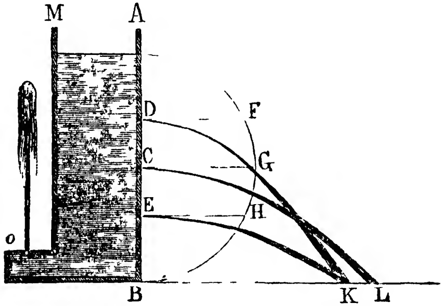


Fig. 29.

direction DF can be found from knowing the head of water AD. But its horizontal motion does not interfere with its downward motion by gravity; so that it must descend to the level of B somewhere on the line BL, in the same time that it would fall perpendicularly from D to B. Now, the time of this fall can be calculated by knowing the height DB; and thus knowing the time that the water must take to reach K, and the rate of its horizontal motion, we can find the length of BK, or the range of the jet.

92. The range of horizontal jets is readily determined by practical geometry. On AB (fig. 28) describe a semicircle; from D, the orifice of the jet, draw DF perpendicular to AB, and make BK equal to twice DF; then it can be proved by the laws of falling bodies and the properties of the circle, that the jet must meet BL in the point K. If BE is equal to AD, the perpendicular EH is equal to DF; and therefore a jet from E will have the same range as that from D. Of all the perpendiculars, CG, drawn from the middle point C, is the greatest; therefore, the jet from C has the longest possible range.

93. The area of the orifice and the velocity of the flow being known, it is easy to calculate the quantity of water discharged in a given time. Thus, suppose the area to be 1 square inch, and the velocity 20 feet a second, it is evident that there issues in a second a cylinder or a prism of water 1 square inch in section and 20 feet long, the content of which is $1 \times 240 = 240$ cubic inches. In any given time, then, as three minutes (= 180 seconds), the discharge is $240 \times 180 = 43{,}200$ cubic inches.

94. It has as yet been assumed that the water in the vessel or reservoir is kept constantly at the same height, and that thus the velocity is constant. We have now to consider the case of a vessel allowed to empty itself through an orifice at the bottom. As the surface of the water sinks, the velocity of the discharge diminishes or is retarded; and when the vessel is of the same area from top to bottom, it can be proved that the velocity is *uniformly* retarded. Its motion follows the same law as that of a body projected vertically upwards. Now, when a motion uniformly retarded comes to an end, the space described is just half what the body would have passed over, had it gone on uniformly with the velocity it had at the outset. Therefore, when the vessel has emptied itself in the way supposed, the quantity discharged is half what would have been discharged had the velocity been uniform from the beginning.

95. This enables us to find the time in which a vessel of given height and area will empty itself by an orifice of given area. Suppose the height of the vessel 9 feet, its area 20 square feet, and the area of the orifice 4 square inches. By art. 88, the velocity of efflux at the beginning is $v = \sqrt{2gh}$ $= \sqrt{2 \times 32 \times 9} = 24$ feet per second. This gives a discharge of $4 \times 288 = 1152$ cubic inches per second; or $\frac{1152 \times 60}{1728} = 40$ cubic feet per minute. Now, the content of the vessel is $20 \times 9 = 180$ cubic feet; and at a uniform rate of 40 feet a minute, this would be discharged in $\frac{180}{40} = 4\frac{1}{2}$ minutes. But

the uniform velocity would discharge twice the content of the vessel, or 360 cubic feet in the time that the retarded velocity discharges once the content. The time of emptying the vessel with the retarded velocity is therefore twice as great, or 9 minutes. The rule expressed in a formula is, $t = \frac{A}{a} \sqrt{\frac{2h}{g}}$; where t = the time in seconds, A = area of vessel, a = area of orifice, h = height of vessel, and $g = 32$.

96. *The 'Contraction of the Vein.'*—When, by means of the area of the opening and the velocity thus determined, we calculate the number of cubic feet or of gallons that *ought* to flow out in a given time, and then measure the quantity that actually does flow, we find that the actual flow falls short of the theoretical by at least a third. In fact, it is only the central part of the jet, which approaches the opening directly, that has the velocity above stated. The outer particles approach from all sides, with less velocity; they jostle one another, as it were, and thus the flow is retarded. In consequence of this want of uniformity in velocity and direction among the component layers of the jet, as they enter the orifice, there takes place what is called a 'contraction of the vein' (*vena contracta*); that is, the jet, after leaving the orifice, tapers, and becomes narrower. The greatest contraction is at a distance from the orifice equal to half its diameter; and there the section of the stream is about two-thirds the area of the opening. It is, in fact, the section of the contracted vein that is to be taken as the real area of the orifice, in calculating by the theory the quantity of water discharged. If the wall of the vessel has considerable thickness, and the orifice is made to widen gradually inwards, in the proportions of the contracted vein, the stream does not suffer contraction, and the area of the orifice where it is narrowest may be taken as the actual area of discharge.

97. *Adjutages.*—It has as yet been supposed that the issue is by means of a simple opening or hole in the side or bottom of the vessel; but if the flow takes place through a short tube, the rate of discharge is remarkably affected. Through a simple opening, in a thin plate, the actual discharge is only about 64 per cent. of the theoretical; through a cylindrical conducting-tube, or *adjutage*, as it is called, of like diameter, and whose length is four times its diameter, the discharge is 84 per cent. The effect is still greater if the discharge-tube is made conical both ways, first contracting like the contracted vein, and then widening. The effect of a conducting-tube in increasing the discharge is accounted for by the adhesion of the water to its sides, which widens out the column to a greater area than it would naturally have. It has thus a tendency to form a vacuum

in the tube, which acts like suction on the water in the reservoir, and increases the quantity discharged.

98. The flow is more free if the orifice is in the bottom of the vessel, than in the side on a level with the bottom. If the discharge-tube is made to project inwards beyond the thickness of the walls of the vessel, the velocity is much impeded, owing to the opposing currents produced by the water approaching the opening.

99. *Pipes.*—When a conduit pipe is of any considerable length, the water issues from it at a velocity less than that due to the head of water in the reservoir, owing to the resistance of friction. With a pipe, for instance, of $1\frac{1}{3}$ inch in diameter, and 30 feet long, the discharge is only one half what it would be from a simple orifice of the same diameter. The rate of reduction depends upon the diameter of the tube, its length, the bendings it undergoes, &c. The resistance to the flow of water in pipes does not arise properly from friction, as understood of solids, but from the adhesion of the water to the sides of the pipe, and from the cohesion of the watery particles among themselves; it makes little difference, therefore, whether an earthenware pipe, for instance, be glazed or not. Large projections form an obstacle; but mere roughness of surface is filled up by an adhering film of water, which is as good as a glaze. The resistance increases greatly with the narrowness of the pipes. Engineers have formulas, deduced in great part from experiment, for calculating the discharge through pipes of given length and diameter, and with a given head; but the subject is too complicated for introduction here.

100. If water flowed in a conduit pipe without friction or other obstruction, so that its velocity were always equal to that

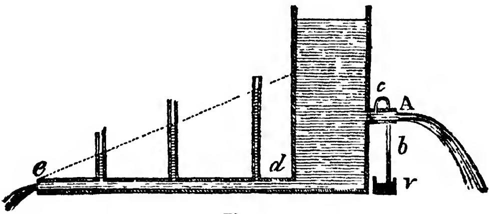


Fig. 30.

due to the head of water, there would be no lateral or bursting pressure on the walls of the pipe; and if the pipe were pierced, the water would not squirt out. Accordingly, with a short

tube or adjutage (see art. 97), which, instead of obstructing, increases the flow, there is not only no lateral outward pressure on the walls of the tube, but there is actually a pressure inwards. If a hole is made in the wall of a cylindrical adjutage, A (fig. 30), and the one end of a small bent tube, *bc*, is inserted in the hole, while its other end is dipped in a vessel of water, *V*, the water will be sucked up the tube, shewing the tendency that the adjutage has to form a vacuum (see art. 97).

101. But when the velocity of discharge is diminished by the friction of a long pipe, or by any narrowing, bending, or other obstruction in the pipe, then that portion of the pressure of the head of water that is not carried off in the discharge, becomes a bursting pressure on the walls of the pipe. This pressure is unequal at different parts of the pipe. At the end *e*, where the water issues free and unobstructed, it is next to nothing, and gradually increases towards the reservoir at *d*, where it is equal to the difference between the head of water in the cistern, and the head due to the velocity with which the water is actually flowing in the pipe. The principle now explained accounts for the fact, that pipes often burst or begin to leak on the motion of the water in them being checked or stopped.

102. *Rivers.*—The effects of friction between liquids and solids are nowhere so conspicuous as in the flowing of rivers. The natural tendency in the water to descend at a certain speed, is checked by friction, by bends in the course of the stream, and by projections on the banks and bottom. From these causes, the water in a river flows with different velocities at different parts in any vertical section across the current. It flows at a slower rate of speed at and near the bottom than at the surface, and also slower at the sides than in the middle. The place at which the velocity is greatest is called the *line of current;* it is generally at the deepest part of the channel. The surface of a river is not quite level; it is slightly convex, the highest part of the convexity being at the line of current. This arises from the fact, that flowing water does not press so much laterally as if it were at rest; the more slowly flowing side particles thus press in upon the central portions that are flowing fast, and heap them up until there is equilibrium of lateral pressure.

103. To get the mean velocity of a river or other stream, find first the surface velocity in the line of current by observing the rate in feet per minute at which a floating body is carried down. The float should be such as barely to reach the surface, so as not to be affected by the wind. As an approximation to the truth, the mean velocity may be taken at $\frac{4}{5}$ths of the

greatest surface velocity; and if this is multiplied by the area in feet of the cross-section of the stream, the product is the discharge in cubic feet per minute.

104. The abrasion of the banks and bottom of a canal or river depends upon the velocity. A velocity of 30 feet per minute will not disturb clay with sand and stones; one of 40 feet will sweep along coarse sand; of 60, fine gravel; of 120, rounded pebbles; and of 180, angular stones. The action on the banks is much lessened by making them sloping or shelving. Oblique currents in rivers or canals are caused by the presence of obstructions in the water-way, as rocks or stones. The water striking against these is deflected, and thrown against the bank opposite; hence those parts of the sides are much abraded. It is this unequal wearing away of the banks of rivers that gives them their serpentine character; which is most observable in flat countries, as where the velocity of the current is considerable, it overcomes comparatively small impediments, and dashes forward in a straight course.

105. In constructing an artificial channel for a stream, the form that offers least obstruction to the flow of the water is a semicircle, there being less surface of friction for the same area than with any other figure. A semi-hexagon is, for a similar reason, better than a rectangle.

106. *Resistance of Water to Bodies moving through it.*— This is greatly affected by the shape of the body, which ought to have all its surfaces oblique to the direction of the motion. When a cylinder terminates in front in a hemisphere, the resistance is only one-half what it is when the cylinder terminates in a plane surface at right angles to the axis; and if instead of a hemisphere, the termination is an equilateral cone, the resistance is only one-fourth. If a globe is cut in halves, and a cylinder, whose length and the diameter of whose base are each equal to the diameter of the globe, is fixed between them; this cylinder with hemispherical ends experiences less resistance than the globe alone, the diminution being about one-fifth of the resistance of the globe. Ship-builders have become, in recent times, more alive than formerly to the importance of length and of sharp angles both at bows and stern, in order that vessels may sail well. The best form in all respects for ships is still a disputed point, though it seems to be agreed that swift-swimming fish present the nearest approach to the true model.

107. The resistance offered to a body moving in water increases in a higher ratio than the simple one of the velocity. One part of the resistance arises from the momentum that the body has to give to the water it displaces. Moving at a

certain rate, it displaces a certain quantity; moving at twice that rate, it displaces twice the quantity in the same time. But not only does it displace twice the number of particles of water; it also has to displace them with twice the velocity; the pressure of the resistance is thus not merely doubled, but quadrupled or squared. Similarly, when the velocity is tripled, the resistance arising from the simple displacement of water becomes nine times as great.

108. Another part of the resistance of liquids to bodies moving in them is owing to the cohesion of the particles, which have not to be thrown aside merely as separate grains, but to be torn asunder. In addition to this, when the velocity is considerable, the water becomes heaped up in front, and depressed at the other end from not having time to close in behind, thus causing an excess of hydrostatic pressure against the direction of the motion. Owing to the combination of these causes, the real law of the increase of resistance is difficult to investigate, and the results of experiments are not a little discordant.

109. The law, however, established (107) with regard to that part of the resistance arising from the displacement of the water, has an important practical application. We have seen that when the speed of a vessel is doubled, the pressure against her bows must be at least four times as great as before. Now, this fourfold pressure has to be overcome over twice the space in the same time; and this amounts to doing eight times the amount of work in one hour or in one minute that was done before, implying, of course, an eightfold working or impelling power. We thus arrive at the important conclusion, that *the impelling power required for a vessel increases as the cube of the velocity*. If an engine of 20-horse power is sufficient to impel a steamer 6 miles an hour, then to impel the same vessel 12 miles, or at twice the velocity, would require $2^3 \times 20$, or $8 \times 20 = 160$-horse power. It thus appears that to do 12 miles in one hour, requires eight times as much expenditure of coal as to do 6 miles in one hour, or it may be put thus: to do 6 miles in half an hour costs four times as much coal as to do the same distance in a whole hour.

WATER-POWER.

110. When any surface, as a piece of board, is moved through water, it experiences a resisting pressure, which we have seen (art. 107) must increase as the square of the velocity. Now, it is evident that the same pressure will be exerted on the board if it is held motionless in a stream of water running

with a velocity equal to that of the board's motion in the former case. As it is this pressure exerted by flowing water that is the source of power when water acts by its impact, it is necessary to be able to estimate exactly its amount on a given surface, and with a given velocity.

111. Water acquires velocity by descending from a height, or by flowing from an orifice or pipe under pressure; and we have seen (art. 89) that a pressure or head of a certain height gives it the same velocity that it would have acquired by falling through the same height. In either case, the flowing water has accumulated in it the 'work' (see MECHANICS, art. 204), or mechanical effect done upon it by gravity, and is capable of exerting the like effect on any body opposed to it. Therefore, a stream of water issuing from an orifice exerts on a square foot of surface a pressure equal to the weight of a column of water of a square foot in base, and whose height is that of the water in the reservoir. If the head of water, for example, is 20 feet, the force of impact on a square foot is $20 \times 62\frac{1}{2}$ (lbs.) = 1250 lbs.

112. When the velocity of the stream is given, the head due to that velocity is found by the formula $h = \frac{v^2}{2g}$. Thus for a velocity of 40 feet a second, the head is $\frac{40^2}{64} = 25$ feet, giving a pressure on the square foot of $25 \times 62\frac{1}{2} = 1562\frac{1}{2}$ lbs. Since 64 does not differ greatly from $62\frac{1}{2}$, it gives a result sufficiently accurate for most purposes to avoid the multiplication by the one and the division by the other, and to take the square of the velocity as the pressure in pounds on a square foot. By this rule, the pressure due to a velocity of 40 feet per second is $40^2 = 1600$ lbs.

113. *The Undershot Wheel.*—By an undershot wheel is meant a wheel with float-boards dipping into a flowing stream or current which sweeps under the wheel. The moving water exerts a pressure on the float-boards arising from its velocity; but in computing the amount of the pressure, it is necessary to consider that the float-boards are not at rest; and that the water acts on them, not by its absolute but by its relative velocity—that is, by the excess of its velocity above the velocity of the float-boards. If, for instance, the stream has a velocity of 8 feet in a second, and the float-board a velocity of 3 feet, the velocity of impact is only 5 feet. A velocity of 5 feet gives, by art. 112, a pressure on the square foot of 25 lbs.; and if the float-board is 4 feet wide and $1\frac{1}{2}$ feet deep, making an area of 6 square feet, the whole pressure is 150 lbs. A pressure of 150 lbs. exerted over 3 feet a second (the velocity of the float-board), gives $150 \times 3 = 450$ units of

work done on the float-board in a second; and $450 \times 60 =$ 27,000 are the units of work in a minute, being $\frac{27,000}{33,000}$, or $\frac{9}{11}$ of a horse-power, which is estimated at 33,000 units of work in a minute. Although more than one float-board are immersed, it is only one that receives the impact of the water at any particular time.

114. Were the float-boards to move at the same velocity as the water, there would be no pressure, and therefore no mechanical effect or work; also, were the float-boards at rest, although there would be pressure, there would be no work, because there is no motion given to the point of resistance (see MECHANICS, art. 197). Now, between those two extremes there must evidently be some relation of the two velocities that will give the greatest possible effect to be got from the stream. According to the theory of the undershot wheel given in art. 113, the maximum effect is produced when the float-boards are made to move with $\frac{1}{3}$ the velocity of the stream. Thus in a stream with a velocity of 8 feet, let the float-boards (of the size supposed in art. 113) have a velocity of $\frac{8}{3}$ or $2\frac{2}{3}$ feet, the pressure per square foot is $(8-\frac{8}{3})^2 = (5\frac{1}{3})^2 = \left(\frac{16}{3}\right)^2 = \frac{256}{9}$ lbs., and on the whole float-board, $\frac{256}{9} \times 6 = \frac{512}{3}$ lbs. Then $\frac{512}{3} \times 2\frac{2}{3} = \frac{512}{3} \times \frac{8}{3} = \frac{4096}{9} = 455$ (nearly) = work done in a second; and $455 \times 60 = 27,300$, the work done in a minute. The result is thus greater than when the velocity of the wheel was supposed to be 3 feet (art. 113). If, again, the velocity were taken less than $2\frac{2}{3}$ feet, the effect would be, in like manner, less. Other authorities, however, consider that a velocity $\frac{1}{2}$ that of the current gives the greatest effect.

115. It is thus evident that an undershoot wheel of the kind supposed, can never receive but a fraction of the effect due to the head of water; for the water leaves the float-boards, retaining at least half its velocity. Owing to this and other causes of loss, it is found by experiment that the work done on the wheel does not exceed from $\frac{1}{4}$ to $\frac{1}{3}$ of the effect due to the fall.

116. *Poncelet's Wheel* is a contrivance for extracting more of the power of a running stream than is done by the ordinary undershot wheel, in which the float-boards are at right angles to the circumference. The float-boards or palettes are curved, and are so disposed that the stream enters upon them without shock, and loses its whole velocity before leaving them. The effect on the wheel is stated to be ·75, or $\frac{3}{4}$ of the effect due to the head.

117. *Bucket-wheels.*—In bucket-wheels, the water acts by

its weight, and not by impact. This mode of applying water-power is to be preferred, wherever a sufficient fall can be had. When the channel or spout is carried over the top, the wheel is called an *overshot* wheel. This arrangement is now seldom followed. It is found preferable to make the diameter of the wheel a foot or two greater than the fall, and to deliver the water a little below the summit, as at the upper channel in fig. 31. The water may enter the bucket with greater impulse by the former arrangement, but the dash makes the bucket overflow before it is full, which more than counterbalances the advantage. When sufficient fall is not available, the water is sometimes laid on as low as the level of the axis as at *c*. The term *breast*-wheel is generally applied where the water is delivered anywhere above the level of the axis without being carried over.

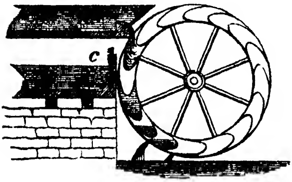


Fig. 31.

118. The absolute working-power possessed by water descending from a height, depends upon the quantity and the height of the fall. If the descent is 12 feet, one pound of water is urged over 12 feet by a pressure of a pound, which makes 12 units of work, and is equivalent to the work of lifting one pound to the height of 12 feet. Therefore, the absolute work inherent in a ton of water having a descent of 20 feet, is $2240 \times 20 = 44{,}800$ units. If the stream deliver 10 tons per minute, $44{,}800 \times 10 = 448{,}000$ is its work per minute; and as a rate of 33,000 units of work in a minute is what is meant by a horse-power, the power of such a stream is equal to $\frac{448{,}000}{33{,}000} = 13\frac{1}{2}$ horse-power.

119. But water acts fully on a bucket-wheel only from the point where the bucket is filled to the point where it begins to flow out, or for about $\frac{2}{3}$ of the fall. Owing to this and other causes of loss, the proportion of the absolute power of the fall rendered available seldom exceeds from 55 to 65 per cent. It is advantageous, with a view to economising the power, to give a small velocity to the circumference of the wheel, not exceeding 3 or 4 feet per second.

120. *Fourneyron's Turbine.*—This is a horizontal water-wheel, invented by a French mechanician named Fourneyron. A column of water, confined by a cylinder, rests above the middle part of the wheel, and rushing out horizontally from

under the cylinder all round, strikes against the oblique palettes of the wheel, and makes it revolve. This kind of wheel, when well constructed, yields from 70 to 75 per cent. of the power of the water.

121. *Barker's* or *Segner's Mill.*—When a vessel filled with water has no outlet, the outward pressure on the sides has no tendency to push the vessel out of its place, because the pressure on any spot is counteracted by an exactly equal pressure on some other spot. But when an opening is made, as at A, B, or C (fig. 28), the pressure is less on that side of the vessel than on the other, and if the vessel were standing on a piece of wood floating on water, it would be seen to move in a direction opposite to the jet. A jet of air or steam has a similar effect to one of water. A tea-kettle mounted on wheels, with a spirit-lamp below it to make it boil, retreats from the current of steam that issues from the spout. This phenomenon is similar to the recoil of a musket, and may be considered as an instance of the law of reaction.

122. It is on this principle that water is made to act as a moving power in what are called reaction-wheels—as, for example, in the machines known as Barker's or Segner's mills. A drawing of a Barker's mill, in its simplest or typical form, is annexed (fig. 32). A is a wide metal pipe resting at its lower end by the spindle T, on a metal block B, and kept in a vertical position by the spindle S, at its upper end, which passes through the frame of the machine, so that it can easily revolve round its axis. Near its lower end, two smaller pipes or arms, C, C, are inserted, which project horizontally from it; and these have near their outer extremities, holes cut in them opening towards opposite sides. The water is supplied by the pipe P, which opens over a funnel-like expansion at the upper part of A; and the quantity is so regulated, that while the pipe A is kept nearly full, no more is admitted than issues from the lower orifices The reaction caused by the

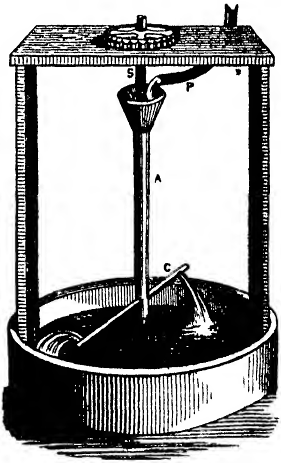


Fig. 32.

water gushing from the arms, forces them backwards, and gives to the whole machine a rotatory motion. As soon as this motion ensues, centrifugal force comes into play, which, throwing the water out towards the ends of the arms, increases the rapidity of its discharge, and also its reacting power. When the wheel is in action, the water thus acts under the influence of two forces—the one being the pressure of the column in A, and the other the centrifugal force generated by the rotation of the wheel itself. The motion of the wheel is transmitted by the spur-wheel fixed to the spindle S, to the machinery which is to be driven by it; or, in the case of a corn-mill, the spindle passes directly through the lower millstone, and is firmly fixed into the upper one.

123. The power in a Barker's mill is manifestly increased by heightening the water-column or by lengthening the arms —the former increasing the pressure of the water, the latter the leverage at which this pressure acts. In the mill shewn in fig. 32, the column in A cannot be advantageously heightened, for the higher it rises, the greater must be the weight which the conical spindle T has to sustain, and the greater, consequently, the friction. From this circumstance, such mills are found, in practice, to yield but a small mechanical effect—the friction consuming too large a proportion of the work of the wheel. Hence, in the reaction-wheels now in use, the original Barker's mill has been so modified as to allow of the water being conducted from the reservoir below the arms instead of above. This is effected by making the vertical pipe revolve below in a stuffing-box at its junction with the conduit, and above by a pivot moving in a fixed frame. By this arrangement, the friction attending the rotation is reduced to a minimum, for not only is the weight of the water-column placed out of account, but also a large proportion of the weight of the wheel itself, which is borne up by the upward pressure of the water. The mechanical performance of wheels thus arranged is said to be highly satisfactory.

124. The power of these mills may be also increased by using curved (fig. 33) instead of straight arms. With straight arms, a considerable loss of force is incurred by the sudden change of the direction of the current when it leaves the arm, which does not take place to the same extent with curved arms, where this change is effected gradually. In Whitelaw's mill (called the Scottish turbine), the form of Barker's mill generally met with in Scotland, there are three instead of

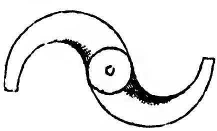


Fig. 33.

two curved arms of this description. Opinions still greatly differ as to the merits of Barker's mills, even in their best forms; some considering them as the most perfect means of applying water-power, while others think they have no advantage over undershot wheels, working with the same water-supply.

125. *Water-column Machines.*—We shall understand the nature of those machines, if we conceive that in a steam-engine, instead of steam, water, under the pressure of a high column, were admitted alternately above and below the piston; the motion of the piston is then communicated to a pump, or other apparatus. They are usually applied for raising water to a considerable height.

126. *The Hydraulic-ram* is a simple and conveniently applied mechanism, by which the momentum or weight of falling water can be made available for raising a portion of itself to a considerable height. In the annexed figure (fig. 34), which represents a section of Montgolfier's hydraulic ram, R is the reservoir from which the water falls; RS, the height of the fall; and ST, the horizontal tube which conducts the water to the engine ABHTC. E and D are two valves, the former of which closes its cavity by ascending, the latter by descending; and FG is a pipe reaching within a very little of the bottom CB. The valves are such that the water at its normal pressure cannot support their weight; the valve E is prevented from falling below a certain point by a knob above *mn*. When the water is allowed to descend from the reservoir, after filling the tube BHS, it rushes out at the aperture *mn*, till its velocity in descending RST becomes so great as to force up the valve E, and close the means of escape. The water being thus suddenly checked, and unable to find a passage at *mn*, will produce a great action on every part of the containing vessels, and by its impact raise the valve D. A portion of water being admitted into the vessel ABC, the impulse of the column of fluid is expended, the valves D and E fall; the opening at D being thus closed, and that at *mn* opened. The water now rushes out at *mn* as before, till

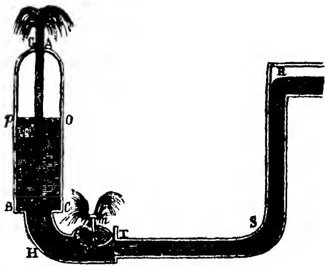


Fig. 34.

its motion is again stopped by its carrying up the valve E, when the operation is repeated, the fluid impulse opening the valve at D, through which a portion of the water passes into ABC. The valves at E and D thus alternately closing and opening, and water at every opening of D making its way into ABC, the air therein is condensed, for it has no communication with the atmosphere after the water is higher than the bottom of the pipe FG. This condensed air, then, exercises great force on the surface, *op*, of the water, and raises it in the tube, FG, to a height proportioned to the elasticity of the imprisoned air.

127. The principles of the hydraulic-ram are susceptible of a very extensive application. In well-constructed rams, the mechanical effect obtained should be from 65 to 75 per cent. of the force supplied. For raising comparatively small quantities of water, such as for single houses, farmyards, &c., the ram is the best mechanism yet introduced. But the concussion, and consequent deterioration of the valves, places a limit to the use of the mechanism when applied to raise large quantities.

PNEUMATICS.

128. Pneumatics, from the Greek word *pneuma*, 'breath,' or 'air,' is a general term for the science of aëriform fluids, embracing both Aërostatics and Aërodynamics (see art. 4). The laws of the motions of fluids, whether aëriform or liquid, are very imperfectly understood; what we do know about them consists mostly of observed facts only partially, if at all, explained—that is, not yet reduced to the form of science, properly so called. The most interesting of these motions are those that take place on a grand scale in the atmosphere; for which we refer to the treatise on Meteorology. What follows relates chiefly to the weight, pressure, and elasticity of gases; in other words, to the statical part of the subject.

129. Modern science has made us acquainted with a number of distinct *airs*, besides common air; these are now usually called *gases*, and the name *air* is confined to the fluid composing our atmosphere. This itself is not a simple substance; it is a mixture of two distinct gases, oxygen and nitrogen. It belongs to Chemistry to explain the different nature of these two; it is only with the mechanical effects that we have at present to do, and in this respect the mixture acts as if it were a simple gas. Aëriform fluids differ from one another in weight as well as liquids do; the other properties treated of in pneumatics are common to them all. Whatever, therefore, is established regarding air, is to be understood as applying to gases in general.

130. An *atmosphere* of air surrounds the globe like a continuous ocean, many miles in height. Air, in small quantity, appears *colourless*, owing to its extreme rarity; but when we look through a considerable depth of it, it is seen to be really blue. Hence the peculiar tint of the sky, and the blue colour of distant mountains. The blue is purer and brighter the less vapour is mixed with it. Were it not for the atmosphere, the heavens would appear as a perfectly black vault; and on very high mountains, where the greater part of the air is below, they are said to approach to black.

WEIGHT OF AIR.

131. The idea of air as a material fluid possessing weight and exerting pressure on everything immersed in it, is modern. The ancients thought of it as essentially *light*, or without weight, and as wanting generally the characteristics of matter; and they used the same words for 'breath,' 'air,' as for 'spirit,' 'life.' Yet it really possesses all the properties

of matter, as surely as water does. A portion of it may be much compressed, indeed, but cannot be squeezed to nothing; it still occupies space, and is therefore *impenetrable* (see Matter and Motion). It also resists sensibly a flat body, such as a fan moving through it; and when itself in motion, as in a strong wind, it is felt to have a powerful momentum or moving force.

132. With regard to the *weight* of air, nothing in the history of science is more remarkable than that men should have lived so long subject to the great pressure which this weight occasions, without discovering it. The fact is, that air acts so little on the senses, that it does not make us aware directly of its existence. The effects produced by its weight were therefore attributed to other causes. The facts for instance, that go by the name of *suction*, are all owing to the pressure of the atmosphere. But when water was seen to rush up a pipe from which the air was withdrawn, it was explained by saying that 'nature abhors a vacuum'—that is, an empty space. This explanation continued to satisfy philosophers till the middle of the seventeenth century. Some mechanicians near Florence, having to construct a pump of unusual length, found, to their surprise, that the water refused to rise higher than thirty-four feet. This led to the conjecture that the weight of the atmosphere was the cause of water rising in the pump; and Torricelli, the pupil of Galileo, confirmed the conjecture by a happy experiment.

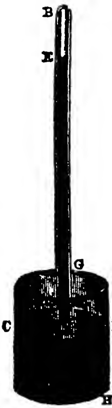


Fig. 35.

133. Mercury weighs nearly 14 times as much as water. If, now, he argued, the atmospheric air can support a column of 34 feet of water, it must also be able to sustain a column of mercury of about 1-14th that height. The experiment is easily made. A glass tube of upwards of 30 inches in length, and closed at one end, is filled with mercury; and the finger being firmly pressed on the open upper end, the tube is inverted, and the end closed by the finger is plunged into a vessel containing mercury, CD. The finger being now withdrawn, the liquid in the tube descends, however long the tube may be, till it stands at the height of about 30 inches above the level of that in the vessel.

134. All doubt on the subject was shortly after put an end to by the celebrated Pascal, who caused the Torricellian tube, as it was called, to be carried up the Puy-de-Dôme, a mountain

in France. By ascending a mountain, a part of the atmosphere is left below; if it is the pressure of the superincumbent atmosphere, then, that sustains the column, the mercury must sink further and further as the elevation is greater. Such was found to be the case, and the question was set at rest for ever.

135. That air is ponderable, or has weight, can be put to direct proof. By means of an air-pump, to be afterwards described, vessels of a certain construction can be emptied of air which, in ordinary circumstances, fills every space not occupied by other matter. When a hollow globe of glass or of copper, holding a cubic foot, is thus emptied of air, and weighed, on admitting the air again, it is found to be about an ounce and a quarter heavier than before. As a cubic foot of water weighs 1000 ounces, water, at the ordinary height of the barometer, is thus about 800 times heavier than air. It is sufficiently accurate for most purposes to consider a cubic foot of air as weighing one ounce.

136. *Weight of the whole Atmosphere.*—The tube of Torricelli gives us an exact measure of the weight and pressure of the whole atmosphere. If the tube (fig. 35) is an inch in area, the column of mercury, PE, 30 inches high, weighs about fifteen pounds. This weight rests on the mercury within the tube at P, the level of the general surface, and, by the law of liquid pressure already established, would press it down, and raise the level of the surface without, were not that surface equally pressed by the atmosphere. The atmosphere, then, rests with a weight of fifteen pounds on every square inch of surface. By multiplying the number of inches on the surface of the globe by fifteen, the product is the weight of the whole atmosphere in pounds. It is the same as the weight of an ocean of mercury spread over the whole globe to the uniform height of 30 inches, or of one of water 34 feet deep, or of one of oil 37 feet deep. It has been calculated to amount to 77,670,000,000,000,000 tons.

THE BAROMETER.

137. In the Torricellian tube above described, we have the well-known instrument called a *barometer* (from two Greek words signifying 'weight,' and 'measure'). The weight of the atmosphere is liable to fluctuations, and these are marked by the rising and sinking of the top of the column at E. The height of the column varies, at the level of the sea, from 28 to 31 inches, the average height being 29·7. A scale is attached to the tube to mark the rise and fall. It is important to observe, that the height of the column is counted from the level of the mercury in the open vessel, and that this level is

not steady. If the column sink, for instance, part of the mercury in the tube descends into the basin, and raises the level of the surface. When the basin is comparatively wide, the rise is insignificant, and is seldom attended to; but where great accuracy is required, either the scale is so constructed as to allow for the alteration of level, or the bottom of the basin is made flexible, and can be raised up and down by a screw, so as to bring the surface of the mercury always to the same point on the outside of the tube.

138. Barometers are of various constructions, according to the uses they are intended for. The common *weather-glass* consists of a glass tube, upwards of 30 inches in length, closed at one end, and bent upwards at the open end. The mercury requires to be introduced with great care into the tube, and boiled in it to expel all air; so that when it sinks down, there may be a perfect vacuum. A wheel-barometer is in very general use as a weather-glass: *c*, fig. 36, represents Hooke's wheel-barometer, which differs from the common weather-glass principally in having a cistern at the top of the closed end, to increase the amount of variation in the lower branch. A small piece of iron or glass, *f*, floats on the open surface, with a thread attached to it, which passes over a small wheel, *a*, fixed to a horizontal axis to which it is kept tight by a small weight *c* hanging at the other end. A pointer, *p*, is fixed at the other extremity of the horizontal axis, which moves to the right or left of the dial, *dd*, according as the mercury falls or rises in the lower branch. The chief merit of this barometer is the great sweep of the index, compared with the minute variations of the mercurial column. The height of the column sustained is here to be counted from its level in the bulk down to a point on the tube on a level with *f*, the surface of the fluid in the open end. Such instruments are not capable of any great accuracy in ascertaining the actual height of the column; they indicate, however, generally whether it is high or low, and whether it is rising or falling; and these are the chief points in prognosticating the weather. (See METEOROLOGY.)

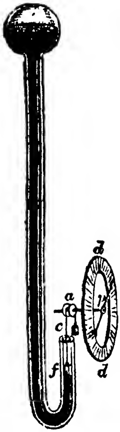


Fig. 36.

ELASTICITY OF GASES.

139. The laws which regulate the pressure of gases are essentially the same as those already established with regard

to liquids. The fundamental fact of *transmitting pressure equally in all directions*, is as true of the one class of substances as of the other. But the effects in the case of gases are much modified by their peculiar property of *elasticity*, which requires careful consideration. The elasticity of gases involves two things—compressibility and expansibility, and in the case of common air, both properties are without any known limit.

140. *Air unlimitedly Compressible.*—The compressibility of air follows a remarkable law. Let *ab* be a cylinder 12 inches long, closed at *b*, and open at *a*, having an air-tight piston, *d*, moved up and down by the rod *c*. Before the piston is inserted, the air in the cylinder sustains the usual pressure of the atmosphere, which is resting upon it at the open end *a*; and if the area is an inch, the amount of the pressure is 15 pounds. When the piston is placed on the open end, and about to enter, the direct action of the outer air is cut off, but it continues to act on the upper side of the piston, so that the column inside still sustains 15 pounds. Let the rod be then loaded with weights till the piston descend through half the cylinder, so as to squeeze the confined air into half its original bulk; it will be found that 15 pounds have been laid on. The piston is now forced down with a weight of 15 pounds, in addition to the weight of the outer atmosphere; the pressure on the confined air is thus exactly *doubled*, and the effect is to compress it into *half* the space.

Fig. 37.

141. If the weight is *tripled* by adding 15 pounds more, the piston descends two inches further to *e*, and the air is reduced to *one-third* of its original bulk. An additional 15 pounds, making in all *four* atmospheres, sinks it to *f*, *one-fourth* from the bottom; and *twelve* atmospheres leave the air confined between *b* and *g*, *one-twelfth* of the whole space. This regular progression holds good, without variation, for atmospheric air up to at least twenty-seven atmospheres.

142. It follows from this that *the elastic force or tension of compressed air—that is, the force with which it seeks to expand—is exactly equal to the compressing force, and inversely, as the space it occupies.* At *f*, the air in the cylinder reacts upon the piston with a force of four atmospheres; at *d*, where it occupies twice the space, it presses up the piston with a force of only two atmospheres; at *a*, the under side of the piston is pressed with a force of one atmosphere, so as exactly to balance the weight of the external air. The law now explained is known as *Mariotte's Law*, from the name of its discoverer.

143. *Air expansible without limit.*—The spring of air is not like that of a compressed feather or of a piece of India-rubber, which loses its tension as soon as it regains its original condition; air cannot be said to have any original volume, for it is always striving to occupy a larger space. Suppose an opening at the bottom of the cylinder (fig. 37), and let the piston be at *f*; its upper and under sides sustain each 15 pounds, for the external atmosphere is admitted to both; and air, like water, presses equally in all directions, up as well as down. If the opening is now closed by a plug or a stop-cock, cutting off the external air from acting on the under side, the piston does not move, because the confined air presses on the under side, not by its weight, but by its expansive force, as much as the external air on the upper.

144. Let the piston be next drawn upwards, and the air not only follows it, but continues to press upon the under side of it, though with a diminishing force. When the piston has risen to *d*, the air occupies twice its original bulk, and its tension or pressure against the under side of the piston is reduced to one-half, or $7\frac{1}{2}$ pounds; and if its volume is again doubled by drawing the piston to *a*, its elastic force falls to one-fourth, or $3\frac{3}{4}$ pounds. Thus the law of Mariotte holds for rarefied air as well as for condensed air. If the piston is drawn up with the hand, the gradually diminishing elastic force of the air within the cylinder is sensibly felt. What the hand has to sustain beyond the resistance of friction, is the difference between the two pressures on the opposite sides of the piston. That on the upper side continues a constant weight of 15 pounds; at starting from *f*, the two are equal, but the difference gradually increases until at *a*, it is $11\frac{1}{4}$ pounds.

HEIGHT OF THE ATMOSPHERE.

145. If we assume air at the level of the sea to be, in round numbers, 800 times lighter than water, then, since water is nearly 14 times lighter than mercury, it follows that air is 14×800, or above 11,000 times lighter than mercury. In order, therefore to balance the column of mercury in the barometer, which is $2\frac{1}{2}$ feet high, a column of air of the above density would require to be 11,000 times $2\frac{1}{2}$ feet, or 27,500 feet high, which is nearly five miles. And this would actually be the height of our atmosphere if air were like water, of uniform density throughout its whole depth.

146. It is evident, however, that the atmosphere is many times higher than five miles. Light clouds are seen floating

above the highest mountains, such as Kinchinjunga, which exceeds 28,000 feet. And not only this, but we know, from the elastic property of air above explained, that the atmosphere cannot be uniform to the top, but must become rarer and rarer as we ascend. An ascent of about 500 feet from the level of the sea, makes the barometer fall half an inch, shewing that a sixtieth part of the weight of the atmosphere is left below. The air at this height, then, is relieved of $\frac{1}{60}$ of the pressure that confines it at the sea-level, and by the law of elasticity, becomes proportionally enlarged in volume; so that to get above another sixtieth part of the mass of the atmosphere, we must ascend more than 500 feet; and so on in increasing ratio. At the height of about three miles, one half the mass of the air is below; the density at that point is reduced to one-half, and an ascent of 500 feet would produce only half the fall of the barometer that is produced at the earth's surface. The upper half of the atmosphere must thus occupy a depth many times that of the lower half.

147. The regular progression in the increasing rarity of the successive strata of the atmosphere, which would arise from Mariotte's law, is true only on the supposition that the temperature is the same throughout. But the capacity of air for heat increases with its rarity, so that the temperature of the atmosphere diminishes as we ascend. (See METEOROLOGY, art. 13). This loss of heat diminishes the elastic force of the air in the upper regions, and makes it expand less under the diminished pressure than it would otherwise do. Cold, then, may be said to check the indefinite upward extension of the atmosphere. As this cold goes on increasing, and also the distance between the atoms of the air, the repulsive force that tends to separate the atoms further and further, must at last become so weak, that the gravity of the upper layer of atoms is able to resist it, and to keep them from rising higher. It would thus appear from theory, that there is a definite boundary to the atmosphere; and from observations made as to the effects on rays of light in passing through the upper regions of the air, it is concluded that the limit of its height is forty or fifty miles.

148. *Measurement of Heights by the Barometer.*—Though the same difference of elevation does not at all stages in the ascent give the same rise or fall of the barometer, yet, by means of the known law of expansion, it is possible, from knowing the heights of the barometer at two stations, and a number of other circumstances, to calculate the elevation of the one above the other. Besides the height of the barometer at the two stations, it is necessary to observe the temperature,

for this affects not only the length of the column of mercury in the barometer, but also the density of the atmosphere, both of which, especially the latter, must be allowed for. When great accuracy is required, the latitude of the place has also to be taken into account; for the force of gravity varies from the equator to the poles (see MATTER AND MOTION), and this affects the density of the air.

149. For the investigation of the actual method of calculating heights by the barometer, we must refer the reader to Part II. of the treatise on Practical Mathematics. The following simple formula, which leaves temperature and latitude out of account, gives an approximation to the height: $h = 10{,}000 \times (\log. p - \log. p')$, p being the barometric pressure at the lower station, p' that at the upper, and h the height in fathoms of the upper station above the lower. If the height thus found is increased by $\frac{9}{4000}$ of itself for every degree that the mean temperature (that is, the mean of the temperatures at the two stations) is above 32°, the result will be not very far from the truth.

Ex. The barometric pressures at the bottom and top of a mountain were observed to be $p = 31{\cdot}725$ and $p' = 27{\cdot}866$; and the temperatures of the air 65°·75 and 54°·25; required the difference of height.

Here logarithm of 31·725 = 1·50140
" " 27·866 = 1·44508
difference = ·05632

and $h = 10{,}000 \times {\cdot}05632 = 563{\cdot}2$ fathoms $= 3379{\cdot}2$ feet, the height approximately.

The mean temperature is $\frac{65{\cdot}75 + 54{\cdot}25}{2} = 28°$;

and $28 \times \frac{9}{4000} \times 3379{\cdot}2 = 212{\cdot}8$

$\therefore\ 3379{\cdot}2 + 212{\cdot}8 = 3592{\cdot}0$, = the height more nearly.

THE AIR-PUMP.

150. The air-pump, by which air is removed from vessels, acts by means of the expansibility of the fluid. There are various forms of the instrument; fig. 38 represents one of the simplest. R is the glass-receiver, or vessel to be exhausted, standing on a smooth plate P, and fitting so exactly, that no air can penetrate between them. From an opening in the middle of the plate, a tube passes down to the sole of the frame, and then along to the end, where it opens into the bottoms of the two pumps or syringes, as is seen in fig. 39, which represents the syringes A A′ in section. The piston-rods, R R′, are alternately raised and depressed by turning the pinion *o*

backwards and forwards by the handle H (which for facility in working is often made double, as represented in fig. 38). As the piston, P, descends, the valve, V, is closed, and that in the piston opens and allows the enclosed air to escape. When the

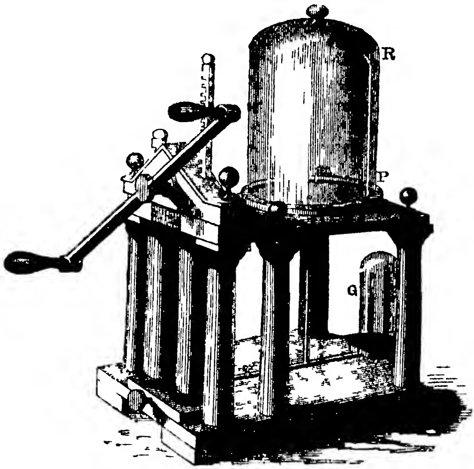


Fig. 38.

piston has reached the bottom, and begins to ascend, the valve in P closes, and a vacuum would be formed, were it not that the air in the receiver, R, and the connecting tube being relieved of the pressure of the outward atmosphere, expands, and opening the valve at V, fills the barrel below the piston. Thus, when the piston reaches the top of the barrel, the air that was at the outset confined in the receiver and tube, now fills a larger space, and is therefore so much rarer. The action of the other syringe is exactly the same; and the two do not interfere with each other, as the valve at V′ is always shut when that at V is open. Some air-pumps have only one syringe; and the chief use of two is for rapidity of exhaustion. Although every stroke

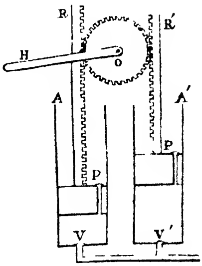


Fig. 39.

of each piston removes a barrelful of the enclosed air, it must be remembered that that air is continually becoming rarer, so that the absolute quantity removed becomes less and less for every stroke. The whole of the air could never thus be removed, even in theory; and in practice, a limit is put to the degree of rarefaction by the elasticity of the remaining air becoming too feeble to raise the valves. The syphon-gauge at G shews to the eye how far the exhaustion has proceeded. It is constructed on the principle of the bent-tube barometer, fig. 36. When the process of rarefaction has reached a certain point, the mercury begins to descend in the closed leg of the tube; and if the exhaustion could be made complete, the two surfaces would be on the same level.

151. The *condensing pump*, or syringe, represented at *ik*, fig. 40, is the counterpart of the exhausting air-pump. By means of it, a receiver can be charged with an amount of air that would fill it many times at the ordinary density, and experiments can thus be tried on bodies enclosed in the receiver, under the pressure of many atmospheres.

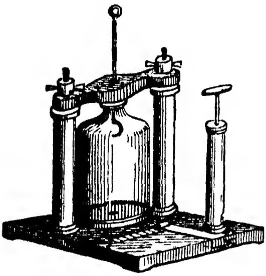


Fig. 40.

152. A condensing syringe is used for charging the chamber of air-guns, which consists of a strong hollow copper ball. The valve of this chamber being then suddenly opened by the trigger, a portion of the highly compressed air is allowed to escape into the barrel behind the ball, which it propels with great velocity.

ILLUSTRATIONS OF THE PRESSURE OF THE ATMOSPHERE.

153. By means of the exhausting air-pump, many interesting experiments are performed, illustrative of the pressure of the atmosphere. If a bladder, half full of air, and tightly tied at the neck, be placed under the receiver, as the exhaustion proceeds, the bladder will expand by the removal of the external pressure, and seem as if ready to burst. Dried raisins, or shrivelled apples, will in like manner swell out, and have all the plumpness of new fruit; and an egg, by the expansion of its confined air, will explode. A mouse placed below the receiver, and deprived of air, immediately dies from want of breath.

154. It is the atmosphere that retards the falling of bodies of

a light and porous nature; and therefore, in the exhausted receiver of an air-pump, all such bodies descend with the same velocity as those of a heavy compact nature. A coin and a feather let fall at the same instant of time from a hook within the top of an exhausted receiver will strike the bottom at the same moment.

155. *Boiling-point under different Pressures.*—If water is boiled in a glass flask, and then allowed to cool several degrees below the boiling-point, on being placed under the exhausted receiver, it will again begin to boil briskly, and will continue to do so for some time. This shews the connection between the pressure of the air and vaporisation. Under the usual pressure, water boils at 212°, alcohol at 172°, ether at 98°, oil at 600°. But as the pressure is removed, the boiling-point in each case becomes lower. On the top of Mont Blanc, water is found to boil at 187°; and in a complete vacuum, liquids in general are stated to boil at a temperature 140° lower than in the open air.

156. Increased pressure, again, raises the temperature of the boiling-point. This is taken advantage of in extracting gelatinous and oily matter from bones, &c. The bones are boiled in a strong vessel where the steam may be confined, and made to press on the liquid. A pressure of two atmospheres raises the temperature to 250°. In distilling liquors, and in making vegetable extracts, on the other hand, it is often desirable to avoid such a high heat as 212°, which is apt to injure the substances, and in such cases the processes are carried on under a partial vacuum. In boiling down the juice of the sugar-cane to the crystallising point, this expedient is employed with great advantage.

157. It thus appears that boiling water is not equally hot at all places on the earth. At Quito, for instance, which is at an elevation of 9000 feet, water boils at 194°; and this temperature is too low for cooking many substances. Even at the same place, boiling water is hotter on a day when the barometer stands above 30 inches, than when it stands at 28 inches.

158. *Heights measured by Boiling-point of Water.*—Vapour of water or steam has a certain tension, or elastic force, according to its temperature. Thus, at 32°, its force is able to sustain 0·2 inch of mercury; at 80°, it supports 1 inch; at 150°, 7·42 inches; at 180°, 15·5 inches; at 212°, 30 inches, or the whole pressure of the air. Now, whatever is the temperature of vapour that can support a given pressure, the same is the temperature at which water boils under that pressure. By observing, therefore, the temperature at which water boils, we can find from a table of the elastic force of vapour at different

temperatures, the pressure in inches of mercury, to which it is subject at the time, and thus make the thermometer a substitute for the barometer.

159. Accordingly, heights are often determined by the boiling temperature of water. Beginning at the level of the sea, a fall of 1° in the boiling-point corresponds to a difference of elevation of 510 feet, or 51 feet for one-tenth of a degree. At an elevation of 2500 feet, the difference for a degree is 520 feet; at 5000 of elevation, it is 530; and at 17,000, it is 590. An approximation may be made for medium elevations, by multiplying 530 feet by the number of degrees of difference in the boiling-point between the two stations. Thus, if the boiling-point at the foot of a mountain were 210°, and at the top 202°, the height above the station at the bottom would be about 530 × 8, or 4240 feet. In order to allow for the temperature of the air, &c., certain tables are necessary.

160. The experiment of boiling water under diminished pressure may easily be performed without an air-pump. Take a common olive-oil flask, and fit it with a good sound cork. Fill it one-third with water; and when it has been brought to boil briskly over a spirit-lamp, or a gas flame, insert the cork, withdrawing it from the flame at the same instant, and then place it in a support, bottom upwards. The ebullition will cease after a short time; but by pouring cold water over the upper part of the flask, it can be renewed again and again. The explanation is, that the space above the water in the flask is filled with nothing but steam, and when this is suddenly condensed by cooling the glass, the surface of the water is relieved from pressure, and the boiling is renewed, until the steam generated again cause as much pressure as to check it.

161. *Gases dissolved in Liquids.*—Liquids contain a certain amount of air and other gases dissolved in them. The molecules of the dissolved gas adhere apparently to those of the liquid with a certain force, sufficient to condense the gas into much less than its natural bulk; and a liquid can hold more of a gas thus imprisoned the greater the pressure it is under. This is most strikingly seen in the case of effervescing liquids, such as beer. Carbonic acid gas is generated in the beer by fermentation; under the usual pressure of the atmosphere, the liquid can hold a certain amount of this dissolved, and the rest escapes. But when a bottle of it is closely corked, the pressure on the surface continues to increase till it is equal perhaps to two atmospheres, and enables the liquid to hold an additional amount of gas in solution. If the cork is now removed, this additional portion separates from the liquid to which it was adhering in a dense, perhaps liquid form, and starts into its

own aëriform shape, appearing as minute globules all through the mass, and rising towards the surface.

162. After standing in an open glass for some time, the beer becomes apparently flat and dead, as if all the gas had escaped. But that this is not the case, may be shewn by placing it under the receiver of the air-pump; on the first stroke, specks of gas make their appearance in all parts of it, and it soon froths up, as if newly drawn.

163. When a glass of common spring or river water is in like manner placed under the receiver, it is seen to contain a considerable amount of common air dissolved in it. As the exhaustion proceeds, minute globules of air make their appearance all through the water, and rise in a stream to the top. A dense crop of bubbles is observed to form on the sides of the glass; this arises from a film of air having been condensed on the glass by adhesion (see MATTER AND MOTION, art. 72) before the water was poured in, and from part of it now escaping under the diminished pressure.

PUMPS.

164. The effect of atmospheric pressure on water affords a convenient method of raising it above its ordinary level; this is effected by pumps, which may be termed both hydraulic and pneumatic machines. Fig. 41 represents the outline of a common suction-pump. It consists of a cylinder, furnished with a piston, A, made to fit air-tight. In this piston there is a valve opening upwards, but here indicated as closed. At the bottom of the cylinder or barrel there is another valve, B, opening upwards; and from the bottom proceeds a feeding-pipe, and dips more or less into the water. When the piston is raised, the air under it is rarefied more and more at each stroke, and the pressure of the air upon the surface of the water on the outside of the pipe causes the water to rise inside where the pressure is lessened. The valve B is at the same time opened upwards, and the water, after several strokes, rushes in above it. When the upward stroke of the

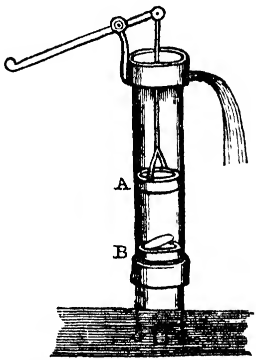


Fig. 41.

piston is complete, it is again depressed—the water passes through the valve in the piston, and on the next stroke it is discharged at the spout.

165. In this form of pump, the greatest height to which the water can be raised, counting from the level of the water in the well to the bottom valve, B, is, in theory, 34 feet. In practice, however, owing to the imperfect vacuum, the limit is usually from 20 to 28 feet. But with the *forcing-pump*, fig. 42, there is no such limit. The mode of its action will be understood from the accompanying outline. A feeding-pipe connects the water and the barrel, as in the suction-pump; but the piston, P, is solid, or without a valve, and a pipe, called the ascension-pipe, provided with a valve opening from the barrel, enters near the bottom. When the piston is lifted, the valve, S, closes, and the water rises through the feeding-pipe and suction-valve, on the same principle as in the suction-pump. The piston being raised to the top of the barrel, and the space below it being filled with water, it is now pressed down; the valve V closes, that at S opens, and the water is forced up the pipe to the required height.

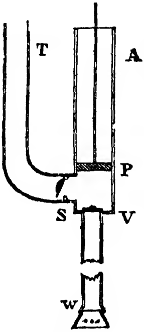


Fig. 42.

166. *The fire-engine.*—The fire-engine is an application of the force-pump. The principle of its action, and its connection with the pumps, &c., will be easily understood by the aid of the annexed diagram (fig. 43), where *a* represents in section a piston ascending, *d* the other piston descending, *f* the pipe or hose communicating with the water-supply, *g* the hose that conveys the issuing stream to the fire, *bc* the level of the water in the air-chamber, *e* the space above filled with compressed air. The rising piston raises the water from *f* to fill its cylinder; the descending piston forces the water contained in its cylinder into the bottom of the air-chamber, and thereby compresses the air in *e*. The pistons rise and descend alternately. The compressed air reacts by its elasticity, and pressing upon the surface *bc*, forces the water through the

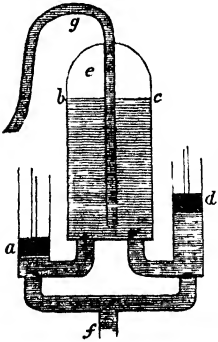


Fig. 43.

hose g. In the space e, above bc, the whole of the air that formerly filled the chamber is supposed to be compressed. Assuming this to be one-third of its original bulk, its pressure will be about 45 lbs. to the square inch, and this pressure will be continuous and nearly steady, if the pumps act with sufficient force and rapidity to keep the water at that level. As air may be compressed to any extent—and its elasticity is increased in exactly the same proportion—the force that may be stored in the compressed air is only limited by the force put upon the pumps, and the strength of the apparatus.

167. It is clear that, both in the suction-pump and in the force-pump, the motion of the water is intermittent; which circumstance causes a great waste of power. It is calculated that in the majority of lifting and forcing pumps, from 55 to 80 per cent. of the power is expended in giving a velocity to the water in the first instance, which velocity has to be suddenly checked, so that it is not available as working-power. The evil of the intermittent nature of the action is aggravated in pumps of ordinary construction by a variety of causes. First, the small size and faulty construction of the valves. By increasing the size of these, the sudden change in the section and velocity of the stream would be avoided. As the variation of the velocity is dependent upon the ratios of the suction-pipe and force-pipe to the barrel, attention should be paid to attain to an approximation of these. This want of due proportion is a second cause of loss of power.

168. A third cause is the neglect to make the extremity of the suction-pipe where the water enters it, such as to render the contraction of the vein as small as possible. In HYDRAULICS, we have seen that the discharge of water from a pipe depends upon the shape of the orifice. It has been found that an equal effect is obtained by attending to the shape of the orifice at which it enters. By expanding, then, the lower extremity of the suction-pipe where it enters the water into the shape of a cone, the diameter of the wider end being 1·2 times that of the narrower, the contraction of the vein may be nearly destroyed. By similarly expanding the discharging orifice, a discharge will be obtained greater than that due to the section of the pump.

169. A great shock, and also loss of power, occurs at the beginning and end of each stroke, especially when a force-pump is worked with great velocity. A remedy for this evil has been sought in the application of a second air-vessel communicating with the suction-pipe immediately below the barrel, or with the top of the suction-pipe and the bottom of the barrel. The commencement of each stroke is eased by

a supply of water from this air-chamber to the space beneath it. The influx of the water into that space is aided by the pressure of the condensed air in the air-chamber; and when the stroke is completed, the state of condensation of this air is, by the momentum of the water in the suction-pipe, restored, causing it to rush through the passage by which that pipe communicates with the air-chamber.'

170. *The Syphon.*—The action of the syphon will be readily understood from the principles already illustrated. It is simply a bent tube, with one end inserted in a liquid, as represented in fig. 44. To begin the action, the air is withdrawn from the tube by means of the mouth or a syringe, when the liquid enters and fills it, as in other cases of suction. When the suction is stopped, there are two forces acting on the liquid in the tube—the pressure of the atmosphere on the liquid in the vessel, forcing it in the direction from C towards B and A, and the pressure at A, forcing it in the opposite direction. These two pressures are of themselves equal, and when the lengths of the columns of liquid in the two legs are equal, there is no flow; but when one column, as AB, is longer than the other, its weight destroys the balance,

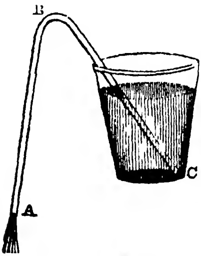


Fig 44.

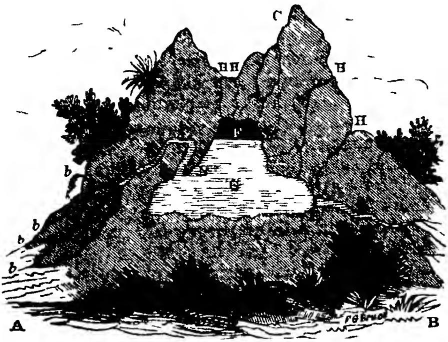


Fig. 45.

and produces a flow towards itself. The length of the column BC, is counted from the level of the liquid, not from the mouth

of the tube. It is clear that the height of B above the surface of the water can never exceed thirty-four feet.

171. *Intermitting Springs.*—The principle of the syphon has been applied to explain the phenomenon of intermitting springs (fig. 45). A mountain, ABC, is supposed to have an internal cavity DFE, into which water trickles through small channels H, H, H. An outlet situated as EKL would form an ordinary spring, which would flow so long as there was any supply from the channels. But if we suppose the only outlet to have a syphon form as NPI, there could be no flow until the water rose as high as the top of the syphon at F. When once the flow began, however, it would continue until the level of the water fell to the mouth at *a*; air would then enter the channel, and the springs at *b*, *b*, would intermit until the water rose again to the level of F.

172. *Pressure of the Atmosphere on the Human Body.*—The body of a man has, on an average, a surface of about 2000 square inches. At the rate, then, of fifteen pounds to the square inch, the whole pressure which a person of an ordinary size sustains on the surface of his body is 30,000 pounds, or nearly fourteen tons. It gives a wrong notion of this pressure to speak of it as a load. It has, on the contrary, a buoyant effect, and makes a man lighter, or press less on the earth, than he would without it (see below). Air presses on all sides of a body immersed in it. It is, therefore, a compressing or crushing force, and does not at all act as a weight.

173. The reason why this compression is not felt, and does not in effect squeeze the body into less bulk, is that there is an equal pressure *outwards* at all parts. In fig. 1, a force of a pound pressing in the plug *a*, is felt as an equal force pressing out the plug *b*. Now, the blood and other liquids in the body, though contained in tubes, transmit pressure through all their ramifications, as the water in the box B does through its mass; and the direct pressure of the atmosphere on any spot of the outer surface, tending to squeeze in the sides of the tubes containing the fluids, is met by an equal pressure on the part of the fluids, tending to force them out; which outward pressure itself is caused by the atmosphere pressing on other parts of the system.

174. We are as little sensible, in ordinary circumstances, of this outward pressure of the fluids on the vessels of the body, as we are of the compressing force of the atmosphere. But it makes itself felt whenever the external pressure is removed from any part, as in the familiar act of sucking the finger, or more strikingly in the operation of *cupping*. A small bell-shaped glass, in which the air has been rarefied by burning a

slip of paper, or by other means, is suddenly applied to a flat surface of the body. The small portion of air under the glass, as it cools, presses with diminished force on the surface of the skin; and the fluids within, released from the usual check, swell out the vessels and skin, and if a wound has been made with a lancet, the blood is forced out.

175. When the usual pressure of the atmosphere is much diminished over the whole body by ascending great heights, uneasy sensations are often felt, occasioned most likely by air contained in the liquids and in cavities of the solid parts of the body, seeking to expand.

176. *Boy's Sucker.*—A boy's sucker affords a good illustration of the operation of atmospheric pressure. A piece of moist leather is pressed against a smooth stone, so as to exclude all air from between them. They are now held together with a force of fifteen pounds on the square inch. By beginning at the edge, they could be separated easily, because the air fills up the interstice as it is formed. But if a string is attached to the middle of the leather, as represented in fig. 46, and pulled upwards, separation is resisted. If, by a strong pull, a portion—say a square inch —of the leather is raised up, there is a vacuum under it, and the weight of the atmosphere on the upper side of that inch has to be borne up by the hand. With regard to the stone, again, we have to consider that before the vacuum was formed, the atmospheric pressure on its under side was exactly equal to that on its upper side; the two counteracted one another, and produced no effect, either in increasing or lessening the weight of the stone. But the effect of the vacuum is to remove the pressure from an inch of the upper surface, so that on the square inch of the under surface opposite the vacuum there is an upward pressure of fifteen pounds which is not counteracted; and by this amount of force the stone is pushed up. If not too heavy, it may thus be lifted by means of the sucker. Correctly speaking, the sucker does not lift the stone; it only lifts the weight of a column of the atmosphere from off the stone, and thus allows the remaining atmospheric pressure to push it up.

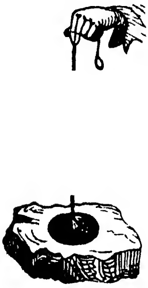


Fig. 46.

177. The feet of flies, and some other insects, are formed on the principle of the sucker; and by this means they can walk on the ceiling of a room, or on an upright smooth pane of glass. When deprived of the extremities of its legs, on which the apparatus for adhesion is situated, a fly can no longer

climb an upright surface, or walk on a roof with its back downwards, though it can walk on the surface of a table without apparent difficulty. Limpets and other testaceous animals adhere to rocks by causing a partial vacuum within their shells, which they accomplish by contracting themselves into less bulk.

178. *Bird-cage Fountain—Vent-peg*, &c.—It is surprising at first sight, that in ink-bottles of the kind represented in fig. 47, and in bird-cage fountains, which are of much the same make, the liquid should stand as high as the top at A without flowing out at C. But the very same cause sustains it that sustains the column of mercury in fig. 35, and keeps it from flowing out at C—the atmospheric pressure, namely, on the surface at C, while the top at A is close, like the top of the tube in fig. 35. The greater width of the bottle makes no difference, the pressure of liquids being not as width, but as depth (see art. 13). The atmospheric pressure on C would be able to keep up the liquid in the bottle till it reached the height of thirty-three feet or thereby.

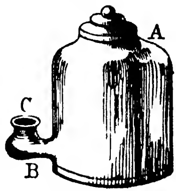


Fig. 47.

179. The ink is introduced into a bottle of this kind by inclining it over, and pouring in gradually at C. When the ink in the tube CB sinks down by use to the level of the horizontal communication, a bubble of air gets in, and rising to the top at A, fills part of the space, and forces down a portion of ink into C.

180. On the principle now explained, if the lid of a tea-pot were quite close, so as to exclude the pressure of the atmosphere from the top of the liquid within, the liquid could not be poured out. It is for this reason that in drawing liquor from an opening in the lower part of a cask, there must be a vent-peg at the top to admit air.

181. Every one knows that when a full bottle is turned mouth downward, the liquid cannot flow out but in proportion as air can pass in. It is instructive to mark the difference between the case of a common bottle inverted, and the case of the ink-bottle, fig. 46. The atmosphere pressing upon the downward surface of the liquid in the neck of the bottle, prevents it from *descending* as strongly, as it prevents the ink from *ascending* in the tube at C. But air being lighter than the liquid, bubbles of it enter the downward surface, and ascend to the top, where their elastic pressure overcomes that of the outward air, and forces down the liquid; while, before a

particle of air could get to the top of the ink-bottle, it would have to *descend* through the liquid in the tube C, which the comparative specific gravity of the two fluids prevents.

182. A liquid does not flow at all even from a downward opening in a vessel close at the top, provided the air is prevented from entering the surface in bubbles, and rising up through it. If we conceive a thin disk of caoutchouc inserted in the lower end of the tube, fig. 35, so as to slide up and down without resistance and yet be air-tight, the tube might be lifted from the cistern without the mercury falling out. A simpler way of illustrating the same principle is the following:—Take a common wine or ale glass, with as even edges as possible, fill it brimful with water, and on the top lay a round disk of smooth stiff writing-paper, rather larger than the mouth of the glass. Above this, place a book, and pressing it down, invert the whole, so that the glass shall stand on the book with the mouth downwards and perfectly horizontal. In this position, the glass may be lifted up from the book with the paper adhering to its mouth, and the water hanging in it apparently without support.

BUOYANCY IN AËRIFORM FLUIDS.

183. The principles explained in arts. 57—60, apply as well to gases as to liquids. If a jar of carbonic acid gas, and another of hydrogen gas are emptied out in the atmosphere of a room, the carbonic acid descends in a stream towards the floor, and the hydrogen ascends to the ceiling. They do not remain, however, in these positions, as many suppose. There are no gases related to one another as water and oil, or as water and mercury are, so as to remain in strata, the one lying upon the other. Every gas tends to diffuse itself uniformly through the whole of the space accessible to it; the presence of another gas only makes this diffusion take place more slowly than it would do if the space were a vacuum. Thus in a short time both the carbonic acid and the hydrogen are found equally mixed with the air in every part of the room.

184. When a body of air is rendered warmer than the surrounding air, it is expanded, and thereby becomes lighter, and must ascend on the same principle as hydrogen ascends, or as cork rises in water. The unequal diffusion of heat over the earth's surface is thus constantly disturbing the equilibrium of the atmosphere, and causing the movements in it that constitute winds (see METEOROLOGY).

185. *Ventilation.*—It is on the same law that we usually depend for the ventilation of houses. Ventilation consists of

two operations—the removal of foul air and the introduction of fresh, neither of which operations can go on without the other going on at the same time. Foul air is removed in either of two ways—viz., by the draughts of ascending currents produced by difference of temperature, or by mechanical force, such as pumping; the former being the more common, and the only method applicable to private houses.

186. The column of air in the chimney of a lighted fireplace, being expanded and comparatively light, exerts less than the prevailing pressure on the air immediately under and about its base. Hence the air below and around it pushes it up and flows in to take its place, the velocity of the movement being proportioned to the height of the chimney and the degree of heat. Thus, although it is often convenient to speak of the air being *drawn* or *sucked* into the chimney, the force does not lie in the chimney, but in the greater pressure of the air behind. A heated chimney is thus a ventilator, the only objection to which is its producing draughts, and its inability to prevent the heated breath—which ascends at once to the ceiling—vitiating to some extent the air in the room before it is carried off.

187. The ascent of foul air to the top of the room, suggests that its exit should be in that direction rather than low down at the chimney-mouth. Some think that the carbonic acid of the breath being heavier than air, must be at the bottom of the room; but, in fact, the breath being lighter than the air around it, goes upwards at once; and the carbonic acid in it does not tend to separate from it and fall by its greater weight, but rising with it, it tends, by the law of diffusion of gases, to spread itself equally all over the room, which it would in time do even were it at first lying on the floor. It is on the principle of the foul air ascending at first to the top of the room, that Dr Arnott's ventilating valve is contrived.

188. Dr Arnott's contrivance above referred to is as follows: An aperture is cut in the wall over the chimney as near to the ceiling as convenient. In this is suspended a valve opening inward to the chimney, but not in the other direction, by which means a return of smoke is prevented. The valve is so balanced on its centre of motion, that it settles in the closed position, but is easily opened. This apparatus operates by virtue of the draught in the chimney. When that is active, a stream of air from the top of the apartment passes through into the chimney, and is carried off. A wire descends to a screw or peg fixed in the wall, by which the opening of the valve may be limited or altogether prevented.

189. There is generally more or less draught in a chimney

even without a fire, from the air within being slightly warmer than that without; and this may be increased by burning jets of gas in the vent. When fire in this or any other way is used to produce a current of air, the ventilation is said to be by fire-draught. The plan has been exemplified with success in mines, where a fire being lighted at the bottom of a shaft, air is drawn off in all directions around and sent up the shaft, to replace which fresh air is constantly pouring down other shafts. Many of our large public buildings, such as St George's Hall in Liverpool, and the Millbank Penitentiary, London, are ventilated by fire-draught.

190. *Ventilation by Fans and Pumps.*—The fan-wheel has long been used in factories as a means of ventilation. It is essentially the same as the barn-fanners; the air is drawn in at the centre of the wheel, and flies off at the circumference by centrifugal force. The fan is placed at the top of a flue into which branches from all parts of the establishment proceed; and when it is set in motion, it draws off the air from every apartment communicating with it. In the use of the fan-wheel, however, as of the air-pump or bellows invented by Dr Hales, a great deal of power is wasted by 'wire-drawing' the air—that is, making it squirt through small valves or other openings.

191. Dr Arnott has shewn how this may be obviated by the following form of a ventilating pump; *efhg* (fig. 48) is the outer cylinder; the piston consists of another cylinder, *abdc*, suspended within it like a gas-holder, and dipping into water, WW. The outer cylinder is closed at top and bottom, but has at *e*, *f*, *g*, *h*, curtain-valves of oiled silk resting against wire-gauze, those at *e* and *g* being suspended outside, at *f* and *h*, inside. As the cylinder-piston, A, descends, the curtains at *g* open, and allow the air to issue, while air is drawn in at *f*; when it begins to ascend, the former valves close, and air is now drawn in at *h*, and expelled at *e*. Thus air is continually entering at the side *fh*, and being expelled at the side *eg*. By connecting, then, the side *fh* with the outer air, and the side *eg* with a system of pipes leading to the apartments of a building, fresh air may be forced in, and the foul

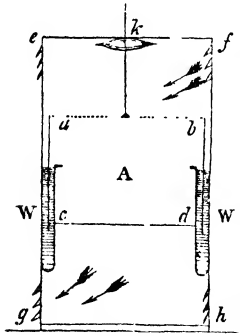


Fig. 48.

air blown out; or by reversing the connection, the foul air may be pumped out of the apartments, which of itself will draw in fresh air, as before explained. Very little power is required to move the piston up and down.

192. The introduction of fresh air, as stated in art. 183, necessarily goes on with the removal of foul, whether the latter be by fire-draught or fan-wheel. As foul air is removed fresh enters, whether by chinks in the doors or windows, or by forcing its way through channels in the walls. The objection to this natural method of supply from without, is the coolness of the fresh air introduced. To obviate this, there are various devices for warming the fresh air before allowing it to circulate through sitting or other rooms.

193. For a house with fireplaces of the usual construction, the simplest and most effective expedient is to admit the fresh air into the entrance-hall, and there warm it by means of a low-temperature stove, or by warm pipes, heated by water from the kitchen-range; its passage into the various rooms can then be provided for by regular channels behind the skirting or otherwise. It is unaccountable that in this country the plan of warming the lobby and staircase is so seldom resorted to: its healthfulness and economy are indisputable.

194. *Buoyancy in Aeriform Fluids.*—A body in air displaces its own bulk of the air, and is borne up with a force equal to the weight of the air so displaced. If the body's own weight is less than this, it must rise up, as a piece of cork does in water; if greater, the body only loses a portion of its weight, as a stone in water.

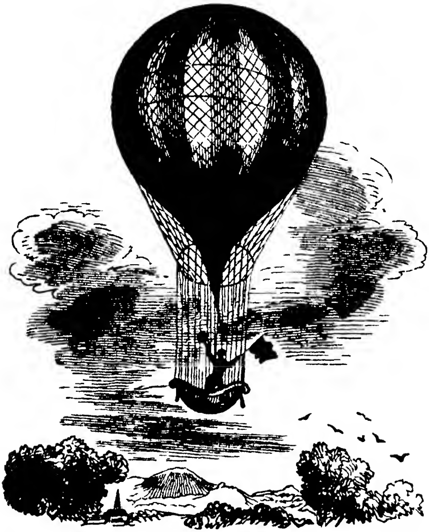
195. In weighing bodies in air, the fact that the air deprives them of part of their weight, does not interfere with the accuracy of the process so long as the substance is nearly of the same density as the weight against which it is balanced, because then both are equally affected. But where there is great difference of density, it leads to sensible error. A pound of feathers—as it is sometimes paradoxically stated—is heavier than a pound of lead; that is, it contains more matter. If a quantity of feathers occupying a cubic foot of space, were balanced in a vacuum against a piece of metal, the balance would be destroyed on taking them into the air.

196. *Air-power.*—The theory of air-resistance is the same, to some extent, as that of fluid-resistance in general. When, however, its elasticity and compressibility are taken into account, air becomes a power capable of a variety of peculiar applications, as in the air-gun, fire-engine, and hydraulic-ram.

197. The action of air in motion (wind) in propelling ships,

loses its buoyancy by cooling; if blown with hydrogen, it continues ascending till it bursts. A body floating in air rises till it arrives at a stratum of the atmosphere whose density is the same as its own.

203. A balloon is a bag made of varnished silk, and surrounded by a net-work, from which a car is suspended. Balloons were originally filled with heated air, and had a fire suspended in the opening below to keep up the temperature.



So far as lightness is concerned, hydrogen would be the best gas for the purpose; but, from its cheapness, common coal-gas is generally employed, whose density is rather more than half that of air.

204. The air contained in a room fifty feet long, thirty wide, and twenty high, weighs upwards of a ton. A balloon of this capacity (30,000 cubic feet) would thus displace a ton of air, while the gas contained in it weighed only half a ton; it would therefore, at first, ascend with a force equal to the difference, or half a ton after deducting, of course, the weight of the silk, and other parts of the apparatus. The greatest elevation ever reached

in a balloon was by Gay Lussac; the barometer sunk to 12 inches, and the height was computed at 23,000 feet, or 4⅓ miles. The ascent and descent of a balloon can be regulated by means of valves for allowing gas to escape, and of ballast which can be thrown out at pleasure; but the impossibility of guiding the horizontal motion, renders the art of *aërostation*, as it is called, only a matter of amusement.

EXERCISES.

Art. 16.

1. In the construction of the Britannia Bridge, the area of a horizontal section of the piston, DF (see fig. 5), of one of the hydrostatic presses employed, was equal to 355 times the horizontal section of the force-pump, H, and the lever was worked by a force of five-horse power, acting at a leverage which was to the leverage of the piston-rod of the force-pump as 8 : 1; find the force with which the table is raised.

Ans. 14200 horse-power.

2. If the radius of the cylinder in Bramah's press is 10 feet, that of the force-pump 3 inches, the whole lever 12 feet, and the piston-rod fastened one foot from the fulcrum; and if the power be 30 lbs., find the force with which the table is pressed upwards. . . . . Ans. = 576000 lbs.

Art. 33.

1. A sluice-gate 4 feet wide is completely shut down. On the one side, the water is 6 feet deep; on the other side, 2. Compare the pressures on the two sides. . . Ans. 9 : 1.

2. A rectangular cistern 3 feet deep, 3 broad, and 6 long, is full of water. What is the total pressure on its sides?

Ans. 5062½ lbs.

3. An equiangular triangle is immersed in water, so that one side is vertical; compare the pressures on the three sides.

Ans. 1 : 2 : 3; 2 corresponding to the vertical side.

4. A right cylinder 3 feet high, and whose base has a radius of 1 foot, is full of water. What is the pressure on the bottom? . . . . . . Ans. 589·05 lbs. nearly.

5. A straight line a yard long is immersed vertically in a fluid; divide it into three portions that shall be equally pressed.

Ans. 20·7846 inches; 8·6094 inches; 6·606 inches.

6. Two squares, whose sides are respectively 10 and 7 feet, are immersed vertically in a fluid, a side of each being horizontal. The first square has its upper side at a depth of 7 feet below the surface, and the second at 14 feet; compare the pressure on each. . . . . . . Ans. 10 : 7½ nearly.

7. A cubical vessel, 3 feet in the side and filled with water, is sunk to the bottom of a river 12 feet deep; compare the external and internal pressures. . . . Ans. 7 : 1.

8. A cubical vessel is filled with fluid, and held with one of its diagonals vertical; compare the pressures on the sides.

Ans. Pressure on upper : Pressure on lower : : 1 : 2.

9. Two spheres, whose radii are as 12 : 5, are just immersed in a fluid; compare the pressures on them. Ans. 1728 : 125.

10. A right cone, with its axis vertical, is just immersed in a fluid, first with its base, second with its vertex upwards; compare the pressure on the convex surface in the two cases.

Ans. 1 : 2.

11. If a cubical vessel be filled half with water and half with mercury, what is the ratio of the pressure on the sides to the pressure on the base, the specific gravity of water being 1, and that of mercury 13, the base being horizontal? Ans. 8 : 7.

Art. 39.

If a rectangle, 10 feet by 18, be just immersed vertically in a fluid, with its longer side vertical; find the position of the centre of pressure. . . . Ans. 12 feet from surface.

Arts. 54 to 62.

A ship in fresh water can carry 200 tons; what can she carry in sea-water, the latter being denser than the former in the proportion of 1026 to 1000? . . . Ans. 205·2 tons.

Art. 82.

A piece of copper weighs 31 grains in air, and 27½ grains in water, as ascertained by the hydrostatic balance; what is the specific gravity of copper? . . . . Ans. 8·857.

Art. 83.

A piece of elm, which weighs 920 grains in air, being attached to a piece of metal which weighs 911 grains in water, is found to weigh 331 grains in water; what is its specific gravity? . . . . Ans. 0·613.

Art. 84.

What is the weight of a bar of copper whose breadth and thickness are 4 and 2 inches respectively, and length 1 foot, and specific gravity 8·85? . . . . Ans. 30·729 lbs.

Art. 85.

1. What is the content of a mass whose weight is half a ton, and specific gravity 4? . . . Ans. 4·48 cubic feet.

2. A hollow copper sphere, whose internal diameter is 2 feet, just floats in water; find its thickness, the specific gravity of the copper being 8·788. . . . . Ans. ·493 inches.

Art. 86.

1. A composition weighing 50 lbs. has a specific gravity of 10·2, that of its ingredients being 10 and 13 respectively. What are the quantities of the ingredients?

Ans. 45·7516 of the one, and 4·2484 of the other nearly.

2. Required the internal radius of a spherical shell of iron, $\frac{1}{10}$ inch thick, which, when filled with alcohol, shall just float in water, assuming the specific gravities of the iron, water, and alcohol to be 7·2, 1, and 8 respectively.

Ans. Radius 9·4 inches.

3. The weight of a globe in air $= W$ and in water $= w$; find its diameter and specific gravity, that of water being S, and that of air s.

Ans. Diam. $= \left\{ \frac{6}{\pi} \cdot \frac{W - w}{S - s} \right\}^{\frac{1}{3}}$; and specific gravity $= \frac{WS - ws}{W - w}$.

4. Hiero's crown, weighing, let us say, 1680 grains, was weighed by Archimedes in water along with an equal weight of gold and silver; suppose the crown lost $\frac{1}{14}$, the gold $\frac{4}{77}$, and the silver $\frac{2}{21}$ of their respective weights; find the grains of gold and silver in the crown. Ans. 924 grains of gold, 756 of silver.

Art. 93.

A cylinder, whose diameter is 5 inches, and altitude 1 foot, is filled with fluid issuing into it through an aperture ·125 inch diameter, in $1\frac{1}{2}$ minutes; find the velocity of the fluid at the aperture. . . . . . . Ans. $17\frac{7}{9}$ feet.

Art. 95.

1. Three persons divide a cylindrical pipe of wine between them, which is emptied through a small orifice in the bottom in three-quarters of an hour, each taking as his share what is discharged in one quarter of an hour. In what proportion must they pay for it? Ans. In the proportion of 5 : 3 : 1.

2. In what time will a vessel 12 feet high, and whose area is 10 square feet, empty itself by an orifice at the bottom, whose area is 6 square inches. . . . . Ans. 3·464 minutes.

Art. 198.

What is the absolute work inherent in a cwt. of water having a descent of 100 feet? . . . . . Ans. 11200 units.

Art. 199.

If an engine of 100 horse-power can impel a steamer 8 miles an hour, how many horse-power would be needed to propel it 16 miles an hour? . . . Ans. 800 horse-power.

Art. 199.

If a diving-bell, whose cubic content = 240 feet, be sunk to the bottom of a harbour 25 feet deep, and the water has risen 3 feet in the bell; find the spaces occupied by the air and the water. . . . . . . Ans. 144 and 96 cubic feet.

NATURAL PHILOSOPHY.

FOURTH TREATISE.

ACOUSTICS.

NOTICE.

The following treatise is intended to present a systematic exposition of the nature of Sound, the mode of its production, and the laws which determine its propagation. Great care has been taken to render the treatment, both in point of expression and sequence, as simple and intelligible as the nature of the subject will permit; while under most of the sections the laws are further illustrated by means of examples, diagrams, and descriptions of actual phenomena.

CONTENTS.

ACOUSTICS.

GENERAL EXPLANATIONS.

1. That branch of science which treats of the nature of sound, the mode of its production, and the laws which determine its propagation, is termed *acoustics*, from a Greek word signifying *to hear*. The phenomena of sound are of great interest, whether we consider those that are natural—as the voices of animals, the purling of the stream, and the rushing of the waterfall; or those that are artificial—as the varied sounds of musical instruments. It must therefore be an attractive inquiry to ascertain the cause of these phenomena; to discover by what organic apparatus, and through what medium, the human voice, for example, is capable of conveying even the thoughts of one individual to the mind of another—a process which we would consider to be perfectly astonishing, were it not constantly familiar to us; and to determine what are the remarkable relations that subsist among those artificial sounds, arranged and regulated by the rules of musical composition, by which we are enabled to express, and that often powerfully, a variety of our strongest feelings and deepest sentiments.

2. The atmosphere which envelops the earth, and in which all terrestrial beings are immersed, as it were, at the bottom of an ocean, is the great vehicle for conveying sound. When anything contained within it is violently struck, so as to be put into a tremulous motion—that is, to vibrate—it communicates vibrations to the adjoining air, and these are but the commencement of a line of waves all around, extending to great distances. In this manner a body can act upon things far removed from itself. Many other substances, however, besides

air, are capable of conveying sound. Every body possessing elasticity exhibits this quality; it being by virtue of its elasticity that atmospheric air is a sounding medium. Hence solids and liquids may have the same property.

3. If an elastic spring be fixed at one extremity, and the other extremity bent from its natural position of rest, and then set free, it will move backward and forward with a degree of rapidity dependent on its length and thickness, its density and elastic force; and its vibrations will be *isochronous*—that is, of equal duration, whatever be their extent. As the simplest case of propagation of waves of air, suppose the spring is fixed at one end of a long tube, these waves will be propagated along the air contained in it with the same velocity as in unconfined air. When the extremity of the spring during a vibration is moving in the direction of the tube, it suddenly condenses the air before it, and when it vibrates back in the opposite direction, it rarefies the air immediately contiguous to the previously condensed portion; and these two portions of air, extending from the extremity of the tube to a certain distance within it, are equal, and together compose a *wave*, whose length is entirely dependent, not on the extent, but on the duration of the vibration of the spring. At every *complete vibration* backward and forward of the spring, a similar and equal wave is propagated, and a consecutive series of such waves constitutes what is technically termed an *undulation.*

4. When the number of vibrations in any given time—as a second, or the number of waves that reach the ear in a second—is confined within certain limits, the sensation of *sound* is excited. Waves capable of producing sound are called *sonorous waves*. The gravest sound that is perceptible to the human ear is produced by 32 half vibrations in a second, and is like a *whisper;* the highest, on the other hand, is caused by as many as 16,384, the resulting sound being like a *hiss*. There is, however, some difference in the compass of audible sounds by different ears. Some persons, for instance, cannot hear the shrill note of the grasshopper or cricket.

VELOCITY AND INTENSITY OF SOUND.

5. If the air were dry, and its temperature at freezing, the velocity of sound would be 1090 feet per second. When the air is in a mean hygrometric state, and the temperature 60 degrees, the velocity is 1125, and when the temperature differs from 60, if the number 1·25, or $1\frac{1}{4}$ feet, be multiplied by the difference, and the product added to 1125, or subtracted from it—according as the temperature is greater or less than

60—the sum or remainder will be the velocity of sound for that temperature.

6. Example.—Find the velocity of sound when the temperature is 54, and also when it is 66:—

Here the difference between 54 and 60 is 6, and the product of 1·25 by 6 is 7·5; hence the velocity is the difference of 1125 and 7·5, or 1117·5 feet per second.

For the temperature 66, the product is the same, or 7·5, which is to be added to 1125, and the sum 1132·5 is the required velocity.

7. The flash of a cannon is seen for some seconds before hearing the report. At the distance of half a mile, for instance, the report would be heard $2\frac{1}{3}$ seconds after seeing the flash; for the distance, 2640 feet, divided by 1125, gives $2\frac{1}{3}$ very nearly. So a flash of lightning is generally seen several seconds before hearing the thunder; and as the agitation of the air producing the sound of thunder begins at the instant of the flash, the distance of the thundery discharge will be found by multiplying the number of seconds in the elapsed interval by 1125. So the distance of an inaccessible rock or wall that causes an echo, can be computed by observing accurately the interval between the sound and the echo; for the elapsed time will correspond to double the distance of the reflecting surface, as the waves must advance to that distance, and then return, before the echo can be heard.

8. Both the velocity and the loudness of sound are considerably greater when conveyed through certain liquids and solids than through air. Thus through water it moves with a velocity of 4708 feet in a second, or more than four times that in air; through tin it is $7\frac{1}{2}$, through copper 12, oak $10\frac{2}{3}$, beech $12\frac{1}{2}$, elm $14\frac{2}{3}$, and through brass and iron $16\frac{2}{3}$ times quicker than in air. A very weak sound—as the scratch of a pin, or the ticking of a watch—made at one end of a log of wood, can be heard by an ear applied at the other end, though it would produce no audible sound at the same distance in air. Savages apply their ear to the ground when they wish to hear any noise that is weak or distant, such as the approach of men or horses.

9. The intensity of sound, like that of light, heat, and the force of gravitation, varies inversely as the square of the distance from the centre of propagation; so that at double the distance, it is four times less; at triple the distance, nine times less; at ten times the distance, 100 times less. Sound, therefore, diminishes more rapidly in intensity than the distance increases, and soon becomes comparatively weak. The transmission of sounds is modified or altogether suspended

when aqueous meteors disturb the homogeneity of the medium through which they pass. The following is a fine illustration:—The British and American troops were encamped on either side of a river, and the outposts so near, that individual figures could be distinguished. A drummer was seen to appear on the American side, and though the motions of his arms were perfectly visible, not a sound was audible. It seemed to have been obstructed by a coating of newly-fallen snow, and the thickness of the atmosphere. The opposite effect is produced by glazed or hardened snow, ice, or water. Thus the sound of cannon was distinctly heard booming over the ocean at a distance of nearly 200 miles from the scene of action, in the famous engagement with the Dutch in 1672. A few years ago, the noise of artillery was heard at Calais and Dover, which proceeded from a field exercise of 12,000 troops at Denderleeuw, about twenty-five miles from Brussels—a direct line of nearly 130 miles. At a similar distance the sound of cannon was heard at sea, on the landing of the British troops in Egypt. The noise of the eruption of Tumbora, in the island of Sumbawa, in the Pacific Ocean, which continued active from April to July 1815, on the authority of Sir Stamford Raffles, was heard clearly in a circle whose radius was 850 miles, consequently over an area of 2,269,806 square miles. The roarings of Cotopaxi in 1744 were heard as far as Honda, on the Magdalena River, in New Granada, a distance of more than 600 miles. These extraordinary sounds must have been conveyed through the ground not by atmospheric pulses. According to Ellicot, the sound of the Niagara Falls is often audible twenty miles off; and that of the stupendous cataract of the Missouri first fell upon the ear of Lewis when seven miles from the waterfall. Sound may also be heard at sea at very great distances when collected by the sails of the vessel, and reflected to a focus. The following case in point is related by Dr Arnot:—" It happened once on board a ship sailing along the coast of Brazil, 100 miles from land, that the persons walking on deck, when passing a particular spot, heard most distinctly the sounds of bells varying as in human rejoicings. All on board listened, and were convinced; but the phenomenon was mysterious and inexplicable. Some months afterwards, it was ascertained that at the time of observation the bells of the city of St Salvador, on the Brazilian coast, had been ringing on the occasion of a festival: the sound, therefore, favoured by a gentle wind, had travelled over 100 miles of smooth water; and striking the wide-spread sail of the ship, rendered concave by a gentle breeze, had been brought to a focus, and rendered perceptible." Lieutenant

Foster conversed with a sailor across Port Bowen Harbour, in the Arctic regions, at a distance of a mile and a quarter. The human voice, it is stated, has been heard at the amazing distance of ten miles over water, from New to Old Gibraltar. It is stated that the voice may be heard on Table Mountain from Cape Town, about a mile off; and on the testimony of the late Dr Jamieson, he once heard the words of a sermon preached at the distance of two miles. The goat-herds of the Alps, by using a falsetto intonation, exchange sentiments nearly at a similar distance. The Arabs, whose extraordinary power of distinguishing impressions on the sands, and thereby reading the *news* of the desert, are likewise skilled in the detection of sounds at incredible distances. Expecting the arrival of a ship from India, the Arab will, morning and evening, hasten to the shore, and kneeling, listen for a few minutes with his ear upon the water. Suppose the ship at that moment 150 miles from land, he hears the signal-gun, or perceives the vibration of the ground, and setting off in his skiff, finds he was not mistaken.

DURATION OF SONOROUS IMPRESSIONS.

10. The duration of impressions of sound (as in the case of light) is a sensible period of time. If sounds succeed each other at intervals of one-twelfth of a second, they seem to be one continuous sound. The fact of sounds rapidly succeeding each other, forming an uninterrupted sound, is well known to the drummer, who, by a rapid quivering motion of his hand, gives at least twelve strokes with his drum-stick in a second, and the effect is that of a single continuous sound, like a musical note, which is itself only a rapid succession of single sounds.

NATURE OF WAVES OF AIR.

11. The nature of a wave of air in passing along a pipe is represented in the annexed figure, in which that half of the

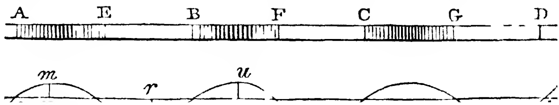


Fig. 1.

wave whose density exceeds the ordinary density of the air is strongly shaded, and the half that is of less than the ordinary

density is not shaded. Thus AB, BC, CD represent these waves, one-half of each of which consists of denser, and the other half of rarer air; AE is the denser half of the first wave, and EB the rarer half. In the second portion of the figure, *ab*, *bc*, *cd* represent the same waves, only the excess of the density of the more condensed parts above the medium density is represented at any point in the wave—as *n*—by the perpendicular line *nm* drawn to the curved line, and the defect of the density of the rarer half below that of the medium density at any point *r*, is denoted by the perpendicular *rs* drawn downwards to the curved line. The deviation of the waved line *amesbu* . . . . from the straight line *abc* . . . . indicates the deviation of the density from the mean density of the air along the line of the undulation. The wave moves with great rapidity, but the particles of air have a comparatively slow reciprocating motion, and to a very small extent on each side of their natural position of rest. The particles move, for instance, from the dense towards the rare portion, as from *n* towards *a* and *e*, and from *v* towards *b* and *f*, so that the effect is the same as if the condensation *ame* moved uniformly along the line *acd*, followed by a rarefied portion.

12. Solid elastic bodies of an elongated form, as strings and rods, have three kinds of vibration—*transverse*, *longitudinal*, or *tortive*. The *transverse* mode of vibration is like that of a string of a violin or piano, as in fig. 2. If the points CD are supposed to be the extremities of the string, and fixed, and if it be drawn a little out of its position of rest, as C*m*D, and then set free, it will vibrate to a nearly equal distance on the other side of the line C*n*D, and will continue in a vibratory state till it be gradually reduced to rest by the resistance of the air and its want of perfect elasticity. The *longitudinal* vibration is, in one case, exactly like that of the column of air O_3B, explained in art. 45. The vibration of *tortion*, or the tortive vibration, may be understood by conceiving a thin square rod of steel fixed firmly in a vice at one end, and twisted by a force applied at the other end; when set free, it will, by its elasticity, return to its original position, and by the circular momentum its particles have thus acquired, they will rotate nearly as far in the opposite direction, and again return; and thus the circular vibrations will be, like the other two sorts, continued for some time, till they gradually cease. A rigid rod of wood or metal is also capable of transverse vibrations, like an elastic cord when fixed at both ends, or when fixed only at one end, or when not fixed at all. These vibrations may be excited by means of a rosined bow. Longitudinal vibrations may also be excited in metallic rods, by

rubbing them in the direction of their length with a piece of rosined cloth, and in glass rods by moistened cloth.

13. The three kinds of vibrations can be easily exemplified by a simple apparatus. Thus C W is a brass wire suspended from the top of a stand at C, and kept tense by means of a weight W; L D is a movable clamp for securing any portion of the wire intended to vibrate transversely. Being thus secured at D, if it be drawn to a side at the middle *m*, it will exhibit the transverse vibrations. If the wire be previously made of a spiral form, the spiral being of small diameter, and the weight be raised, and then suddenly let fall, it will exhibit the longitudinal vibrations, the weight in consequence vibrating up and down. Again, if the weight be turned round several times, so as to twist the wire, and be then suddenly let go, the wire will perform the vibrations of torsion.

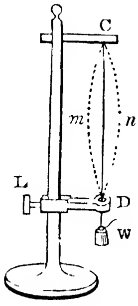


Fig. 2.

14. The cause of vibration is the elasticity of the vibrating material. Were it inelastic, it would remain dead after a stroke. When an elastic cord is fixed at both ends (fig. 3), and is drawn a little to a side, by its elasticity it is capable of some distension, which is essential to a transverse vibration. Forces of tension act upon it at every point, as at *v*, in the directions of a tangent to the curve. Take any minute portion of the cord, as *r m s*, then it is acted on by forces of tension in the directions *r t*, *s t'*, and its own weight downwards in the direction *m g*; for we may, without sensible error, suppose the weight of the elementary portion *r s* to be concentrated at its middle point, and that *r m s* is rigid. By constructing a parallelogram *r v s*, the action of the two tensions, denoted by the lines *m r*, *m s*, will be equivalent to a single force *m v*; and were the string so heavy as to rest in the position S*m*T, the weight of the portion *r s* would be represented by *v m*. But the force of tension is in most cases immensely greater than the weight, and therefore the latter may be neglected; the only resistance, then, that the resultant of the tensions *v m* has to overcome, is

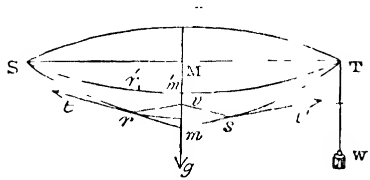


Fig. 3.

the mass of *rs*. Now when the point *m* is in the position *m′*, say half way to M, the angle *vrm* is half the size, and the resultant *vm* half as great, and therefore the acceleration at *m′* is half as much as at *m*, since the mass of *rs* is invariable, at least if it be supposed that the vibrations are of small sweep. It thus appears that at any point of the cord, the force drawing it to its natural position is proportional to the distance of the point from this position; and it can be proved that the accelerating forces on every element are equal at equal distances from the line SMT. The same is true of all the other points; but when the forces that act on a body are proportional to the distances of its position from its state of rest or equilibrium, the body vibrates like a pendulum, *isochronously* *—that is, the vibrations are of the same duration, whether large or small; so that whatever be the excursions of a vibrating cord within a certain limit, they are performed in equal times, and therefore cause waves of sound of equal length. The same is true of the vibrations of rods, whether entirely free, or fixed at one or both ends; and it is also true for longitudinal vibrations.

15. In experimenting with an elastic string stretched horizontally, it is usual to fix it at one end S (fig. 3), and to stretch it by means of a weight W attached to its other extremity, the string passing over a pulley at T; then ST is evidently the vibrating portion of the string. If the string possessed no elasticity, it is evident that it could not be stretched, and therefore it could not vibrate.

16. The square of the time of vibration of a pendulum is inversely proportional to the force acting upon it; or the square of the number of vibrations in a given time, as in one second, is directly proportional to this force. If, therefore, the force for one pendulum be four times that for another, the squares of the numbers of vibrations in a second will be as 1 to 4, or the numbers themselves as 1 to 2; if the forces are as 9 to 16, the numbers will be as 3 to 4; and so on. The same, therefore, is true for the numbers of vibrations of elastic cords.

17. EXAMPLE.—One string performs 120 vibrations in 1 second, and another is under the influence of a tension 4 times as great; what number of vibrations will the latter perform in a second?

The force 1 is to the force 4 as the square of 120 to the square of the required number of vibrations. Now the square of 120 is 14,400, and the fourth proportional to the three numbers 1, 4, 14,400, is evidently 57,600, which is therefore the

* See MATTER AND MOTION, article 268.

square of the required number of vibrations; the square root of this number is 240, the required number. A quadruple force, therefore, causes only a double number of vibrations.

18. If two strings are in every respect of the same kind, and stretched by equal weights, but one of them twice the length of the other, the latter will perform half as many vibrations as the former.

19. If two strings, the same in every respect, be stretched by weights in the ratio of 1 to 4, the number of vibrations produced by the action of the greater weight will be twice as many in a given time as by the smaller.

20. If the diameter of one string be double that of another of the same kind of the same length, and stretched by an equal weight, the thicker string will vibrate only half as fast as the thinner.

21. The last three theorems may be stated in general terms thus:—When two strings of the same kind and diameter, but of different lengths, are stretched by equal weights, the numbers of their vibrations in a given time are inversely as their lengths; when two strings are of the same kind and dimensions, but stretched by different weights, the numbers of their vibrations are directly proportional to the square roots of the weights; and when two strings of the same kind and length, but of different diameters, are stretched by equal weights, the numbers of their vibrations are inversely as their diameters.

HARMONIC DIVISIONS OF AN ELASTIC STRING.

22. When a string vibrates transversely in its whole length, it is found that its aliquot parts—as its halves, its thirds, and so on—also vibrate separately, and with a degree of rapidity exceeding that of the string itself, according to the principles just explained. An experienced musical ear is sensible of the higher notes produced, by the vibrations of these parts accompanying the fundamental or principal note produced by the entire string. These divisions of a string are called the harmonic divisions; the half length may, for convenience, be called the *second harmonic* division; the third part, the *third harmonic* part; and so on.

23. When a rope fixed at one end, and stretched by the hand at its other extremity, is agitated at equal short intervals of time, a series of waves will be formed along it, and it will at last assume the form of a string divided into a number of equal parts vibrating separately, the vibrations of every two consecutive parts being in opposite directions, and the vibrating parts

separated by stationary points, called *nodes*, or *nodal points*. Such a system of waves is termed a stationary undulation.

24. The form assumed by the undulation described in the last paragraph, can be communicated to a string vibrating in its *whole length*, by gently touching it at one of the nodal points. If it is wished to make it divide itself into two harmonic parts, this form is given by touching it in the middle point, which becomes a node; if into three harmonic parts, it must be touched at one of the corresponding nodal points; and so on for any other harmonic vibration.

25. That a string can vibrate in its whole length, and also at the same time in some of its aliquot parts, may be shown to be a possible and natural thing by the following considerations:—The whole string, supposed, when at rest, in a horizontal line, may be moving from right to left, its middle point being in a horizontal plane; while at the same time one of its halves may be seen sidewise moving up and down as if in a vertical plane, and of course the other half has a similar motion, only its vibrations would be in an opposite direction. As the half-string vibrates twice as fast as the whole, the air will receive two impulses from its vibrations up and down, in the time that it receives a single impulse by the whole string in a lateral direction; and consequently two musical notes are produced in the relation of octaves (art. 100). So a third of the string may be moving in an intermediate direction, producing a different note; though certainly the waves produced by the various kinds of vibrations will weaken each other, and produce in some parts a blending of sounds resembling a resonance (art. 67).

VIBRATION OF ELASTIC RODS.

26. Rods of wood, or metal, or of any other elastic materials, when fixed at one or both ends, or when free at both ends, are capable of transverse vibrations bearing a general resemblance to those of elastic strings, but observing very different laws. The vibrations of rods, however, are isochronous, as in the case of strings; and they vibrate either entirely or in parts, forming nodes of vibration like strings. For experiments on transverse vibrations of rods, they must be of uniform dimensions throughout their length—that is, either cylindrical, like wires, or square, or prismatic.

27. The simplest mode of transverse vibrations of a rod, is when it vibrates in its whole length with one end fixed, as in a vice. This sort of vibration is represented in fig. 4, in which VF is the rod in its position of rest, and VM, VN its posi-

tion when at the limits of a transverse vibration. The law of vibration in this case is this:—The times of the vibrations of perfectly elastic rods vibrating in their whole length, and fixed at one end, are directly proportional to the squares of their lengths; and the numbers of vibrations in a given time are therefore inversely as the squares of their lengths.

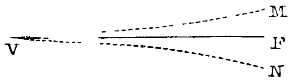


Fig. 4.

28. EXAMPLE.—Two rods of the same material, and of uniform thickness, are respectively 2 and 3 feet long; required the ratio of their times of vibration?

The ratio of these times are directly as the squares of 2 and 3—that is, as 4 and 9; or the time of a vibration of the latter is more than double that of the former: so the numbers of vibrations of the former, compared with those of the latter in the same time, are as 9 to 4. If the former, for instance, performs 36 vibrations in a second, then 9 is to 4 as 36 to 16, the number of vibrations of the latter in one second.

29. When a polished metallic knob is fixed on the end of a vibrating rod, it is observed that the vibrations are seldom performed in one plane, as the bright point is seen to move in paths sometimes nearly circular, at other times elliptical and lemniscatic—that is, resembling the figure 8, or in very complex intersecting curved lines, perpetually changing their form and position.

VIBRATIONS OF ELASTIC PLATES, RINGS, BELLS.

30. When elastic plates of uniform thickness and regular figure are put into a state of vibration by means of a rosined bow, or by percussion, there is a series of nodal points regularly arranged in lines, called *nodal lines*. These are easily rendered visible to the eye by strewing the plates with fine sand, which, by the agitation of the vibrating parts, is removed from them, and accumulates along the nodal lines. They vary in form with the form of the plate and the part where they are put in vibration; the point at which they are held will always lie in at least one of the nodal lines.

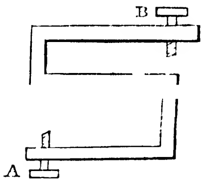


Fig. 5.

31. Plates may be considered as composed of straight fibres;

so that a rectangular plate, whose length is great compared with its breadth, would vibrate like an elastic rod, and would have nodal lines transverse to its length—that is, across it in the direction of its breadth, at exactly the same parts as the nodal points of a vibrating rod. The instrument used for holding the plates is a sort of wooden vice, like that in fig. 5, which can be fixed on a board or table by the lower screw A, and holds the plate by the upper one B.

32. If a square plate of glass or metal be held by the centre C, and be put into vibration by means of a bow applied near to one of its angles A, there will be two nodal lines passing through its centre parallel to its sides, and therefore perpendicular to each other. The sound thus produced is the gravest that can be obtained from the plate. The sound next in depth is produced when the plate is held at the centre as before, and the bow applied to the middle of one side. In this case the nodal lines are in the two diagonals of the plate.

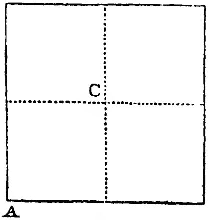


Fig. 6.

33. By holding the plate at other points, an unlimited variety of different nodal lines may be successively formed.

34. When the plate is circular, and the edge free, the nodal lines may be of two species—*diametral* or *circular*. The number of diametral nodal lines is always even, the least number being two. The reason why the number is even, is, that when one quarter (A E D) vibrates towards one side, the two contiguous quarters must vibrate in opposite directions; and in general every two contiguous divisions must vibrate in opposite directions, which cannot happen when the number of divisions is uneven. So, in the case of a ring, as ABCD, when one quarter (AB) vibrates outwards, its contiguous parts (AD, BC) must vibrate inwards, and it must divide itself into an even number of equal parts; and supposing the circular plate composed of rings whose quarters vibrate to opposite sides, instead of outwards and inwards, the nature of the vibrations of the plate will be easily understood.

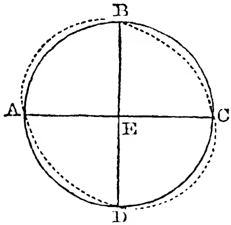


Fig. 7.

35. In the diametral species of nodal lines, the gravest sound is produced when there are two lines, the next grave when there are four, and so on.

36. The circular nodal lines are either one or two, concentric with the plate; or one with a single diametral line; or one with two diametral lines. These two types may degenerate into a variety of curved nodal lines, according to circumstances.

37. The immense variety of nodal lines that can be formed on a plate of any form, are proved to result from the superposition—that is, the co-existence of different modes of simple vibration, analogous to the simultaneous existence of the vibrations of a musical string, or of the air in an organ-pipe, and of their harmonies; or still more resembling the resultant note arising from the periodical coincidences of the vibrations producing two or more co-existing notes.

38. Nodal lines are found to exist in the air of a room when put into a uniform state of vibration by the sound of an organ-pipe, and these are observed by placing fine sand on any elastic tense membrane, as paper or parchment, which becomes agitated everywhere excepting in the quiescent parts.

39. The vibrations of a bell are exactly analogous to those of rings, and are easily conceived by considering it as composed of a series of rings parallel to its rim, and having, therefore, the same axis as the bell. The divisions will be equal portions formed by drawing lines from the apex to the points of equal division of the rim, and as in the case of rings, their number will always be even.

40. The parchment of a drum, cymbals, the tom-tom or gong, are familiar examples of sonorous plates.

STATIONARY UNDULATIONS IN CANALS AND PIPES.

41. When a series of equal waves are produced in a canal containing water, and bounded at one end by a plane perpendicular to its length, the reflected and direct series of waves, by mutual interference at equidistant points, destroy each other's influence in elevating or depressing the water at these points, and thus is formed a stationary undulation.

42. The mutual action of the opposing series of waves upon each other that causes this result is very easily understood. Let MEB (fig. 8) be the level surface of water when still in a canal, of which BH is the end; and let BcM, MSE be the elevation and depression constituting one wave, whose beginning has just reached the end of the canal. If the canal were unobstructed, the sinus of the wave in advance of EMB would at

the same instant have the position Bn'; but being reflected, it would actually, if not interfered with, be in the position BnM; and consequently, as the depressions of every point in

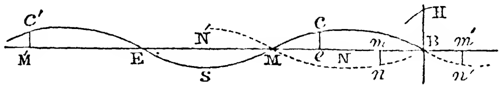


Fig. 8.

this half wave are equal to the elevations of the half (McB) of the other wave, the effect of their interference will be entirely to destroy the wave along the distance MNB, which would therefore cause the surface of the water to remain at its natural level over that space. But it is evident that this condition of the surface would continue only for an indefinitely short time, for the direct and retrograde motion of the two series of waves would immediately bring them into new positions, where they would not destroy each other's action except at certain points. It will not be difficult to ascertain the position of these singular points. One of them is in the *middle* of the *half wave* MB at N. For it has just been seen that the point N is brought to its natural level when B is the beginning of a wave. Take any point c in the surface of the elevation McB, and a corresponding point n in the reflected sinus; these points are evidently at the same distance from the middle point N; and consequently, since their motions are equal and opposite, they will both reach the middle point at the same time; and the elevation being exactly equal to the depression, the vertical motions will neutralise each other, and the surface at N will be in its quiescent position; and whatever other corresponding points are taken in the advancing crest and the retrograde sinus, the same effect will happen. Likewise, if MnB were an advancing sinus, and McB a reflected crest of a wave, the same consequence would ensue; the surface at N, therefore, is always stationary. It could be similarly shown that at N', the middle of EM, also the surface is constantly at the same height. These stationary points or *nodes* are therefore at a distance from the extremity of the canal, equal to $\frac{1}{4}$, $\frac{3}{4}$, $\frac{5}{4}$, ... or generally an *uneven* number of quarters of the length of a complete wave EMB.

43. At the intermediate points M, E, ... between the nodes, are formed the *loops*, or alternating elevations and depressions. For when the middle of the crest Ec' of the succeeding wave would reach the point M, the middle of the crest McB of the

preceding one would, after reflection, just reach the same point; so when any point c' in the former wave would reach M, the corresponding point c in the same phase with the latter would exactly reach M after reflection; and in this way it is shown that every direct wave will meet the immediately preceding one, returning, after reflection, in the same phase at M and E, and all the other loops. When the middle points of the crests thus coincide at M, the elevation there will be twice as great as that of the original undulation before reflection; and when the middle of two sinuses meet, the depression would be twice as great as that of the original waves. The resulting undulation is called *stationary*.

44. There is an exact analogy between this action of water in canals and the motions of air in *organ-pipes*. In the vibrations of air, there are points called *nodes* and *loops*, and the horizontal motions of the particles of air are exactly like those of water; but instead of the terms elevations and depressions of waves of water above and below the mean level, *condensations* and *rarefactions* of the air above and below the mean density must be used.

45. Let FB be a close organ-pipe, of which FT is the foot, O_3T the mouth, and B the flat closed end. Let a system of undulations be excited in the pipe, exactly like the advancing system of aqueous waves in fig. 8; let the system advancing from O_3 to B be represented by $O_3 r$, of

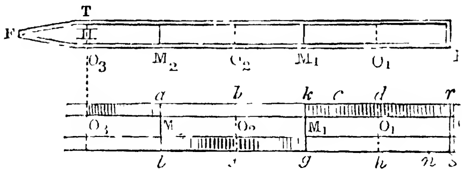


Fig. 9.

which O_3 and O_1 are the middle of dense portions, and b of a rare portion. This system corresponds to $c'EsMcB$ in the figure referred to. Also let the lower system $O_3 s$, in which O_3 and O_1 are at the middle of rare waves, denote the reflected system. Now when any point c of the advancing dense wave kr reaches O_1, the corresponding point n of the returning rare wave will also reach it, and their differences of densities from the mean being equal and opposite in kind, the density at O_1 will be rendered of the medium density, which will be the permanent condition of the point O_1. The same can be similarly

proved of the points $O_2 O_3$ .... Again, at the point M_1, corresponding points of *similar* half waves—that is, both dense or both rare—always meet; and therefore at one time the excess of density will be doubled there, at another time the defect of density. The point M will therefore be of continually varying density, whose limits will extend twice as much above and below the mean as those of the original wave; but the two meeting waves lying in opposite directions, the tendencies of their corresponding points to produce motion when they meet at M, will be equal and opposite, and therefore the particles of air have at this point no *horizontal* motion. The same things can be proved of the similar points $M_2 M_3$, &c.

46. The mouth of the pipe O_3 must be one of the nodes, because, from its communicating with the external air, there can be no excess or deficiency of density there as at the loops. This is proved by experiment; for by cutting the tube entirely round at any node, such as O_1, and separating the parts to a minute distance, the pipe continues to produce the same sound. The point O_1 might thus be made the mouth of a closed organ-pipe of the length $O_1 B$; and by exciting in it a wave, one extremity of whose dense portion would just reach O_1 after reflection at B, while its other extremity is entering at the mouth, a stationary undulation could be kept up. Hence it appears that the longest complete wave that can produce a stationary undulation in a closed organ-pipe, is 4 times the length of the latter. Since sound moves 1125 feet in a second, the number of waves propagated in a second through such a pipe would be 1125 divided by 4 times the length of the pipe. The longest wave corresponds to the lowest or gravest note that the pipe is capable of producing.

47. EXAMPLE.—In a closed organ-pipe $6\frac{1}{4}$ feet long, what is the number of complete vibrations produced in a second corresponding to its gravest or fundamental note? Ans. 45.

48. If the mouth of the organ-pipe is at the second node O_2, its length will be 3 times greater than before; but the number of vibrations is the same, and the length of the wave is $\frac{4}{3}$ that of the pipe. The waves in this case belong to the third harmonic of the fundamental note of the pipe $O_2 B$; for its gravest note would be produced by a wave 4 times its length, whereas the wave just considered is only $\frac{4}{3}$ of its length; and the number of vibrations produced by the fundamental note is to that of the other as 1 to 3. So if $O_3 B$ were the length of the pipe, the length of the first-mentioned wave would be $\frac{4}{5}$ths that of the pipe, and it would produce the fifth harmonic. In this way it appears that if the number of vibrations produced by a closed organ-pipe, when it sounds its

gravest note, be denoted by 1, the vibrations of the whole series of notes that it is capable of producing are represented by the series of *odd* numbers—

1, 3, 5, 7, 9, ... and so on.

A closed organ-pipe, therefore, cannot produce any of the even harmonics; and this result is conformable to experiment. The different notes are excited in an organ-pipe by the regulation of the blast; the gravest requiring a very moderate current of air compared with the harmonics.

49. In the open organ-pipe there is a mouthpiece at one end, and a stationary undulation can be excited in it, similar to that of the closed pipe, with this difference, that the open end in the open pipe is a node, and the closed end of the other a loop.

50. Let TB be an open pipe, and let a stationary undulation be excited in it such that $O_1\,O_2\,O_3$ shall be nodes, and $M_1\,M_2$

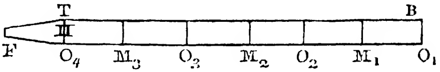


Fig. 10.

loops. The ends O_1 and O_4 are as if in direct communication with the atmosphere, and their uniform density is not disturbed. Since O_2 is a node, the pipe can be cut through there without altering the tone (article 46); and therefore a stationary undulation, consisting of waves of the length $O_1\,O_3$ may be formed in an open pipe equal in length to $O_1\,O_2$; but those waves are the longest that can thus be formed. Hence the gravest note that an open pipe of the length $O_1\,O_2$ can yield, is produced by a wave of *double* the length of the pipe.

51. Were the pipe the length of $O_1\,O_3$, a stationary undulation, formed by waves of the same length, could exist in it; but as the wave producing the fundamental note of this pipe would be double the length of $O_1\,O_3$, the former undulation would give the second harmonic of the pipe $O_1\,O_3$.

52. Again, a pipe of the length $O_1\,O_4$ would be capable of producing a stationary undulation with a wave twice the length of $O_1\,O_2$; but its fundamental note would be formed by a wave twice its own length, or six times the length of the half wave O_1O_2; the latter wave, therefore, would give the third harmonic of the pipe $O_1\,O_4$.

53. Thus it appears that the half wave producing the *fundamental* note of a pipe is *once* its length; that forming its *second* harmonic is ½ its length; that forming its *third* har-

METHOD OF ESTIMATING THE NUMBER OF VIBRATIONS.

60. The number of vibrations in any note may be determined in the following manner:—Take an elastic string, and keep it tense by a given weight, and let it be so long, that its vibrations will be slow enough to be seen distinctly, and counted, which can be done by observing how many there are in 20 or 30 seconds; then, by means of a bridge, shorten the string till it produce a known note, ascertained by the aid of a well-tuned musical instrument; then the part of the string will be to the whole length as the number of vibrations of the latter in a second to that of the former.

61. When air is put into a state of uniform undulation, it is capable of communicating vibrations in a sensible degree to any elastic body capable of sounding isochronously with it; therefore when a note is sounded on one instrument, any other musical instrument near it will have those parts of it put in vibration that are capable of producing the same note: these are called *sympathetic undulations*. Thus, when a note is sounded on a flute with sufficient loudness, a string of a violin, or of a pianoforte capable of producing the same note, is instantly put into a state of vibration strong enough to produce an audible note; this appears to take place instantly, though in all probability 20, or 30, or 100 vibrations of the air must strike the string before its sound becomes sensible. A tumbler will sometimes be heard to sound loudly in sympathetic unison with a strong flute note. If two strings are tuned in unison on an Æolian harp, and one of them is made to sound, the other will immediately sound in unison; when the two strings are only nearly in unison, they strengthen each other's vibrations for a short time, till they reach a maximum, and then gradually weaken each other till the sound is nearly destroyed, when again they conspire to corroborate each other's vibrations; and so on, periodically, affording an illustration of the principle of interference.

REFLEXION OF SOUND.

62. When a progressive wave in its course meets with the surface of a solid body, its direct motion is stopped, and it returns, according to the same law of reflexion, as elastic balls, heat, and light; so that when a series of circular waves, diverging from a centre, meets a plane surface, the reflected waves have the very same form as if they diverged from a point on the other side of the reflecting surface, directly opposite to the origin of the waves, and equally distant from the surface.

63. A straight line drawn from the origin or primary centre of a wave to any point in its surface is called a *ray of undulation*, or an *undulatory ray*; thus, O C, O F are undulatory rays of the wave D C G.

64. Let O be the origin of undulation, S R the reflecting plane, O′ the point equidistant with O from the plane, and

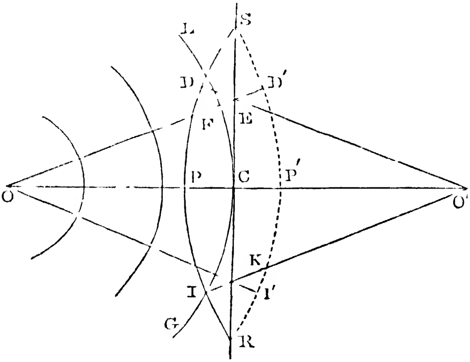


Fig. 11.

directly opposite to O, so that O O′ is perpendicular to R S; and let the wave D C G just reach the plane in C, any ray, as O F, when it reaches E, will be reflected in the direction E D, making the same angle with E S that O E does with E C; so that if O E be joined, O′ E D will be a straight line. But the velocity of the reflected wave being the same as that of the direct wave, while the ray O E would move from E to D′, the reflected ray will move from E to D, making E D equal to E D′. If, now, from the centres O and O′, circles S P′ R, S P R be described with the respective radii O D′, O′ D, while the direct ray would move from C to P′, the reflected ray would go from C to P; so while the ray O K would move directly from K to I′, when reflected it would move from K to I in the same time. Thus, every point of the reflected wave is now found to lie in the circular arc D P I, having O′ for its centre. The point O′, therefore, is analogous to an imaginary or conjugate focus.

65. When the origin O is far distant from the reflecting plane, the waves will be for some extent apparently rectilinear, being arcs of large circles. When the sea or any surface of water is bounded by a straight rocky shore or wall, if the

direct waves impinge on the shore obliquely, the reflected waves will be sent off also in an oblique direction, each ray making the angle of reflexion equal to the angle of incidence; and the reflected thus crossing the direct system of waves, will form beautifully-reticulated or chequered figures on the surface of the water. The rays of sonorous waves being reflected in exactly the same manner as those of water and light, exactly analogous effects will ensue. The following are some of the remarkable results of the reflexion of rays:—

66. Let O, O′ be the two foci of a shallow flat vessel ACBD of an elliptic form; let the vessel have a thin stratum

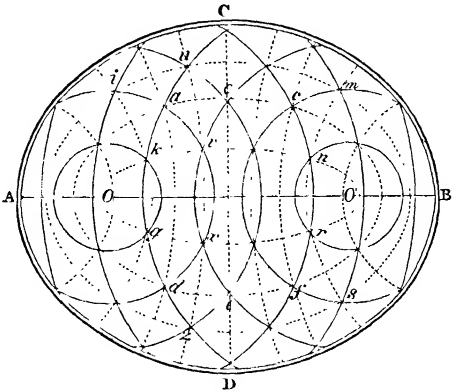


Fig. 12.

of mercury poured into it, then if any agitation be produced at one focus O, the direct series of waves proceeding from it will be reflected at the sides, and will converge towards the other focus; there will thus be excited two similar systems of undulations, which will exhibit the lines of interference arranged in elliptic and hyperbolic curves.

RESONANCE.

67. When the air is put into a state of sonorous vibration by any means, as by a note sounded on a musical instrument, it puts every elastic body into a similar state of vibration. The sounding-board of a pianoforte, for instance, is put into such a

state of vibration by sounding any note on it, that its simultaneous vibrations increase those of the air, and the original note is sensibly strengthened in intensity by the resonance. Even when several notes are sounding together, the sounding-board is put into a state of complex vibration by the simultaneous existence of different vibrations corresponding to the several co-existent notes. So, when the handle of a tuning-fork is placed on a table, the table serves as a sounding-board, and the resonance is very clearly heard, and serves to make the note distinct after it has otherwise become inaudible. If a sound be produced over the mouth of any vessel or cavity, not too wide in proportion to its depth, a certain degree of resonance will be sensible; and if the note corresponds with the vibrations of the internal air, the intensity is much increased by the resonance. From the same cause it happens that when a note is sounded in any room, it immediately produces the same note and its harmonics on any musical instrument in the room, such as a violin, a piano, an organ, or even a tumbler whose note is in unison with the former.

68. The structure of musical instruments in general illustrates this principle of resonance. The effect of a vibrating string by itself would be very feeble; but when it is stretched upon a massive wooden instrument, it communicates vibrations to the whole of that mass, and thus gives birth to a powerful and voluminous sound. In the violin and pianoforte, the body of the instrument constitutes the sonorous substance whose vibrations act upon the surrounding air: the strings are partly the medium through which the instrument is struck, and partly the regulators of the sounds. If the body of a violin were struck by a sharp blow, it would vibrate and sound according to its own natural tendency; that is, the pitch of the sound would depend upon the size, shape, and material of the instrument, exactly as in the case of a tuning-fork or a bell: but when, instead of applying the stroke direct, we set the instrument a vibrating by means of one of the strings, the vibrations of the mass are completely controlled by those of the string, and the resulting sound is therefore a voluminous expansion of the note determined by the string's vibrations. In place of a thin piece of wire or catgut, the air is acted on by an extensive mass of wood, and the result is proportionally great.

69. The sound of the human voice is dependant on the resonance of the bony mass of the skull. The effect of the vocal strings sounding by themselves is observed in a *whisper*. But when true voice is produced, it is by the vibrations of the vocal chords being communicated to the solid mass of the

head, exactly as the vibrations of the violin-strings are communicated to the entire body of the instrument. Hence the power and character of a person's voice depend on the size, form, and texture of the skull, to the same extent as the fineness of a violin or pianoforte is regulated by the wooden part of their structure. The feeble screech of the infant voice arises from there being as yet no bony matter in its head; but according as the ossification of the skull proceeds, the voice becomes firmer and stronger.

70. Sound may be conducted by means of rods of wood, or metal, or wires, from an instrument or from a sounding-board, to a sounding-board at a considerable distance; and in this way the combined harmony of an orchestra may be transmitted from the sounding-board that is influenced by the whole of the instruments to another at a distance, and be distinctly heard by an ear placed close to the second board; all the varieties of cadences and peculiar qualities of the sounds of the various instruments being distinctly preserved.

REFRACTION OF SOUND.

71. When sounds pass from one medium into another, they are refracted, and, in consequence, they suffer a considerable dispersion in passing through various media of unequal elasticities. At a certain angle of medium, in passing from a more to a less elastic medium, sound, like light, suffers a total reflexion.

72. When waves of sound impinge directly—that is, perpendicularly on a plane—they are reflected also perpendicularly; and if a person is standing at the origin of the sonorous waves, the reflected will be nearly as strong as the original sound, unless the plane be far distant, in which case it will be weakened. Since, however, when 10 single sounds succeed each other in a second, they form one continuous sound, and as sound moves over 112 feet in the tenth of a second, any single sound from a distance must precede another by at least 112 feet, in order that the latter may not be blended or confounded with the former, or may appear a distinct sound. Since the reflected waves in the case of an echo have to return over the same path as that passed over by the direct sound, a reflecting surface must be distant from the observer at least the half of 112—that is, 56 feet—in order that a distinct echo be heard; though at a less distance a species of resonance is audible, arising from the sound and echo forming one conti-

nuous sound, though of short duration when the sound is single or momentary.

73. The distance of the reflecting surface that causes an echo, can easily be computed from the known velocity of sound, when the interval of time between the sound and echo are known; or, conversely, the interval can be computed when the distance is known. The interval of time that elapses between the instant of the emission of a sound at any place, and the return of the echo to the same place, is evidently just double the time required for the transmission of sound from the place to the reflecting surface.

74. EXAMPLE 1.—The distance from an observer of a rock that gives an echo is 400 yards; what is the interval between the sound and the echo?

The sound must move over twice 400 yards during the interval, or 2400 feet; and this number, divided by 1125, gives 2 1/[illegible] seconds nearly for the interval required.

75. EXAMPLE 2.—A rock whose distance is unknown, causes an echo at an interval of $1\frac{2}{3}$ seconds after the sound; what is its distance?

The distance is that corresponding to half this time, or half the distance due to the whole interval. Now, 1125, multiplied by $1\frac{2}{3}$, gives 1875 feet, or 625 yards, and the half of this, or 312 yards, is the required distance of the rock.

76. When the observer is situated at or near the centre of a circular surface—as at the centre of the arch of a bridge, or of a circular apartment, or of a hollow in a rock of the form of an upright cylinder—the echo is then much stronger than when the reflecting surface is plane; for the reflected waves in such cases all converge towards the place of the observer, just as light incident on a concave mirror, when the radiant point is in its centre, would all be reflected towards the centre. Whatever be the form of the surface, if the observer be situated at its centre of curvature—that is, the centre of the circle or sphere most nearly coinciding with the reflecting portion of it—the waves will be reflected from it so as to converge towards the place of the observer.

77. But it is not necessary, for the production of an echo, that the origin of the sound should be at the place of the observer; the origin of sound, the place of observation, and the reflecting surface, may be situated anywhere, provided they are not too far distant. For when the reflecting surface is of an elliptic form, and the origin of waves in one focus, the reflected waves converge towards the other focus, and consequently if the observer is placed there, he will hear the re-

flected sound after hearing the direct sound, the latter passing over a shorter path than the former; and if the difference of the paths of the direct and reflected waves exceeds 56 feet, a distinct echo will be heard; but if less than this, there would be only a continuous sound of somewhat longer duration. As any portion of a concave curved surface very nearly coincides with some elliptic, or rather spheroidal, surface, if the origin of the sound and the observer be in the two foci, the reflected sound will converge towards the observer, and will be louder than if it were reflected from a plane surface. But diverging waves reflected from a convex surface would produce a weaker echo than a plane surface, because the waves would, by the reflexion, be made still more divergent than the incident waves.

78. From these principles, it is evident that the echo from a distant rock may be louder than that from one nearer, if the latter be plane, and the reflecting portion of the former nearly circular or spherical, the centre being at the origin of sound.

79. When an apartment is of a circular form, or of the form of a regular polygon—that is, octagonal or hexagonal, &c.—and the origin of sound is near the wall, an observer situated close by any other side of the apartment will hear the reflected sound much better than the direct sound. In this case, as in elliptic surfaces, the quantity of direct undulatory rays that reach the observer's ear is very small compared with that of the reflected rays; and on this account two persons situated in the foci could converse, and yet be inaudible to a company at any place between them.

80. On this principle is explained the phenomenon of whispering galleries, as that around the base of the dome of St Paul's cathedral in London, the elliptic cupola in the baptistry of the church of Pisa, the whispering gallery in the cathedral of Gloucester, and others. That of St Paul's is a well-known curiosity: it is 140 yards in circumference, and is just below the dome, which is 430 feet in circumference. A stone seat runs round the gallery along the front of the wall. On the side directly opposite the door by which visitors enter, several yards of the seat are covered with matting, on which the visitor being seated, the man who shows the gallery whispers with the mouth near the wall, at the distance of 140 feet from the visitor, who hears his words in a loud voice, seemingly at his ear. The mere shutting of the door produces a sound like a peal of thunder rolling among the mountains. The effect is not so perfect if the visitor sits down half way between the door and matted seat, and much less if he stands near the man who speaks, but on the other side of the door.

81. There are many famous *natural echoes* to be met with, that repeat two, three, four, five, six, or many times. One of the most remarkable natural echoes is that heard on the banks of the Rhine at Lurley-Fels. If, in favourable weather, a musket be fired on one side, its report is reflected from crag to crag, and is thus alternately repeated on opposite sides, as represented in the subjoined figure. P is the origin of the

Fig. 13.

sound, the rays of which crossing the river, strike the crag at 1, and are then reflected to the crag 2, next to 3, and so on to subsequent points, till it dies faintly away, or finally ceases opposite E, after seventeen repetitions.

82. There is an echo at Aldernach, in Bohemia, that repeats seven syllables thrice. In ancient times there was a famous echo known at Capo-di-Bove, and at the Villa Simonetta, near Milan, the latter of which repeated thirty times; as did also another in a building in Pavia. There was said to be an echo in the tomb of the Metelli in ancient Rome that repeated eight times distinctly the first verse of the "Æneid," which is a hexameter line consisting of fifteen syllables. It is obvious that the tomb must have been of great length, to cause an echo with so long an interval as was necessary for pronouncing a whole hexameter line articulately, which would require at least two seconds, corresponding to an echo caused by a reflecting surface 375 yards, or 1125 feet distant. One of the most singular echoes known is that described by Barthius, situated on the Nahe, near Bingen, not far from Coblentz, which repeated seventeen times the sound. About this echo there are several peculiarities besides: the voice is indistinctly heard, but the echo is very clearly audible, and

in surprising variety. A tower at Cyzicus repeated seven times; at Brussels an echo responded fifteen times; one at Thornby Castle, Gloucestershire, repeats ten times distinctly; one on the north side of Shipley Church, in Sussex, repeats twenty-one syllables; one in Woodstock Park, mentioned by Dr Plot, repeats seventeen syllables by day, and twenty by night. White records an echo near Selborne, in the king's field in the path to Norehill, which repeated ten syllables, and the last as distinctly as the first, if quick dactyls were chosen; as, for example—

"Tityre tu patulae recubans."

The reverberation took place from a stone building at a distance of 258 yards. Some time after its discovery, a hedge planted for the protection of a hop-garden obstructed the voice of the speaker, and silenced the response. In the centre of Königsplatz, at Cassel, where six streets meet in a large oval, there is an echo which is said to repeat six times distinctly. An echo at Roseneath, near Glasgow, described by Dr Birch, but now lost, repeated three times a tune played with a trumpet. Near to Samson's Ribs, near Arthur Seat, Edinburgh, there is a good echo, where from the foot of an isolated rock the voice is distinctly returned by a southern wall. The echo of Westminster Bridge is said to be heard in the arch-roofed sitting-places, from the dry arches below, and *vice versâ*. Sir John Herschel has noticed a remarkable echo at the Menai Bridge, which he describes: the percussion of a hammer upon one of the main piers is successively returned by each cross-beam which supports the roadway, and from the opposite pier, distant 576 feet; and in addition, the sound is often repeated between the water and the bridge. In an Irish grotto near Castle Comer, in Kilkenny—a fit locality for such a witch-like sound—at a distance of eighteen feet from the inmost recesses of the cavern, an echo falls upon the ear. One of the most delightful echoes in the sister kingdom is met with in the Bay of Glena, on the side of the lower Lake of Kilkenny, called the "eagle's nest:" it sends forth the percussion of a fowlingpiece like the voice of thunder bellowing among the lofty reeks of Kerry. At Paisley there is a fine echo in the burying-place of Lord Paisley, Marquis of Abercorn: musical notes rise softly, and swell till the several echoes have reverberated the sound, and then they die away in gentle cadence. The caves of Torridon, between Applecross and Gairloch, on the north-west coast of Scotland, possess singular echoes, subject to extinction by atmospheric influence. That

from the old castle of Lochaneilan, Inverness-shire, is a good echo. Southwell describes a remarkable one, which is probably that at the Marquis Simonetta's villa near Milan, which repeats the *vox humana* above forty times, and the report of a pistol about twenty times more: it is in the morning and evening that the effect is most apparent. Hundreds of other instances—differing according to locality, state of atmosphere, temperature, &c.—might be recorded.

83. When an echo is produced by a concave surface of sufficient extent, or by several surfaces that cause the waves to converge towards one place, this remarkable effect sometimes happens—namely, that the echo is decidedly stronger than the original sound heard directly without reflexion at the same distance; or, in other words, the echo is louder at the observer's place than the original sound is to a person standing at the reflecting surface when the origin of sound is at the former place. Thus the voice of a person calling aloud is with some difficulty heard at the distance of half a mile; whereas an echo is often very distinctly heard when caused by a surface a quarter of a mile distant, in which case also the waves pass through a distance of half a mile in their direct and reflected course.

ACOUSTIC INSTRUMENTS.

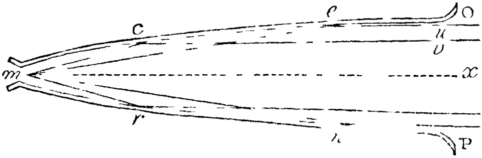
84. Any instrument that is capable of producing or modifying sound may be called an *acoustic instrument.* Thus if a person speak through a cylindrical pipe—one of the simplest of acoustic instruments—it will be found that the sound emitted at its farther extremity, especially if it be of considerable length, has much more intensity there than if no such instrument were used. The greater intensity in this case obviously arises from the circumstance, that the sound is not weakened by being diffused all around, but that its strength is preserved by the repeated reflexions of the rays from the internal sides of the tube. As an experimental proof of this fact, Biot and Martin conversed in a whisper along an empty iron water-pipe in Paris at night, through a distance of 3000 feet; though, however, the perfect tranquillity of night was favourable to the experiment.

85. Although musical instruments are, properly speaking, acoustic, yet the former more specific term is generally used in reference to them. Of the ordinary acoustic instruments, none is more interesting or remarkable in its effects than the *speaking-trumpet.* Any tube used for strengthening the voice, and also for confining it to a particular direction, may be called a *speaking-tube*—as, for instance, the fixed tubes used in offices and warehouses for conveying information from one room or flat to another.

86. Although a cylindrical tube strengthens the voice very much, still the rays of sound being reflected from one side of it to the other, they must emerge from its extremity in directions inclined to the axis, and must therefore diverge considerably. The best form of the speaking-trumpet is a parabola, that has its length great compared to its breadth; for with this construction, if the focus of the parabola be in the middle of its narrow end, the voice being emitted there, will diverge in all directions within the tube, and after impinging on its sides, will be reflected like the rays of light, in directions that are all parallel to the axis, and in this course it will emerge from the further extremity; and consequently the sound will be more concentrated than by any other means.

87. Thus if the sides ceO, rnP (fig. 14) of a trumpet are parabolic, m being the focus at which the mouth is applied, the

rays of sound mc, me, mr, mn diverging from m to the sides of the tube, are reflected in the lines eu, ev, &c. parallel to the



axis of the tube, with no sensible divergence; though in their progress through the tube they will be weakened by the lateral resistance of the air. The tube of course is supposed to be of the form that would be produced by the figure revolving about its axis mx, which would be a paraboloid.

88. Such is the force given to the voice by the speaking-trumpet, that Kircher, by means of it, about the end of the seventeenth century, read the Litany at a convent situated on the top of a hill, to a congregation of 1200 persons, who heard it at the distance of from two to five Italian miles. With a properly-constructed trumpet twenty feet long, a strong voice can be distinctly heard at sea at the distance of three miles.

89. An inverted speaking-trumpet forms a *hearing-trumpet;* the point m being applied to the external opening of the ear; but it is usually made of a small size, or from a few inches to a foot in length.

ACOUSTIC ARRANGEMENTS IN PUBLIC BUILDINGS.

90. Theatres, music-halls, churches, and lecture-rooms, ought to be constructed with reference to the acoustic effect of their various surfaces, in order that there may be no perceptible echo or resonance; for when these accompany the speaker's voice in any great degree, indistinctness and confusion of sound are the inevitable consequence. But, on the other hand, if there is no reflexion of the sound at all, and if the place is very large, the speaker's voice may be too weak to be distinctly heard. The best form for distinctness of sound would be that of the speaking-trumpet; but as this form is very inconvenient, and expensive to construct, only a distant approximation to it is practically attainable.

91. The interior of the building, therefore, when large, ought to be long compared with its breadth and height; for in this case the sound reflected from the side walls, and the

comparatively low ceiling, at small angles, would unite in strengthening the sound. In most cases the floor, when it is irregular, or has some soft covering, or when the building is filled with people, can have no sensible effect, except to absorb any direct or reflected sounds that may reach it. Generally speaking, that form of apartment is best suited for the proper distribution of sounds in which the length is from a third to a half more than the breadth, the height somewhat greater than the breadth, and having a ceiling bevelled off all round the sides, called technically a coved or *coach roof*, from its being lower at the sides than centre.

92. When a distinct echo is heard from the distant parts of a building, it can be stifled by suspending loosely on the wall any sort of cloth; or, in some cases, by hanging short curtains around the cornices; or merely by hanging small pieces of cloth from different parts of the side cornices, so as to project inwards a few feet towards the middle of the ceiling. When a building is crowded with people, they generally serve the same end as curtains and drapery in preventing echoes.

93. Confusion of sound is frequently occasioned by projections and recesses in the interior of the building. The former interrupt, and consequently weaken the sound; the latter cause a considerable resonance from the repeated reflexions, and an echo when they are of great extent. The confusion caused by an echo behind the chancel in a church has been corrected by merely erecting a concave parabolic surface behind the pulpit, so that the speaker might be in the focus; by which means the echo is partly interrupted, and the sound strengthened and more distinctly heard. The confusion caused by recesses like those in the transepts of a church, can scarcely be corrected by any means that do not at the same time weaken the sound so much as to make articulation inaudible.

MUSICAL SOUNDS.

94. Sounds excited by vibrations succeeding each other at irregular intervals of time, excite a disagreeable sensation in the ear. Such sounds are distinguished by various names in common language, according to their peculiar characters—as roar, rattle, hiss, buzz, crash, noise, and so on. When, however, the vibrations are isochronous, their regular periodicity produces an agreeable sonorous impression, and the sound is then called a *musical sound.*

95. Besides the quality of isochronism of the vibrations of a musical sound, another essential circumstance in its constitution is, that there should be at least 16 complete vibrations or waves, or 32 half vibrations in a second; and on the other hand, that the number of complete vibrations should not exceed 8192 in the same time. A less number of vibrations than the former, or a greater number than the latter, in a second, could not produce an audible note. When the vibrations, however, have a more than ordinary intensity, it is found that the limits of audibility are extended to notes caused by 8 and 16,384 complete vibrations respectively in a second.

96. That musical notes are produced by a rapid succession of aërial impulses at equal intervals, is very clearly illustrated by an instrument called the *syren*, the invention of Cogniard de la Tour. A blast of air is forced through a narrow aperture in a pipe, and a flat circular disk perforated near its circumference, with a number of small holes equidistant, and in a circle concentric with the disk, is so applied to the pipe, that the blast is interrupted by it, excepting when one of the holes in the disk is opposite to that in the pipe; and when the former is made to revolve rapidly, the resulting aërial impulses cause a series of isochronous vibrations that produce a musical note, and the corresponding number of its vibrations can very easily be computed, from knowing the number of holes and of revolutions of the plate. The results obtained by this instrument agree exactly with those found by other methods.

97. All the elastic bodies that are capable, by their vibrations, of producing sounds, may also be made to produce musical sounds of some quality or other. Musical strings, which are made of lamb-gut, steel and brass wires, elastic membranes, are capable, when under sufficient tension, of

producing musical vibrations, and also the columns of air in organ-pipes and other wind-instruments.

98. The notes that are caused by more than a medium number of vibrations belong to the shrill or high kind of notes, and are called *acute;* the others, that have fewer vibrations, are low or *grave* notes. The quality of a note that depends on its acuteness or gravity is called its *pitch;* the acute being of a high, and the grave of a low pitch. Besides the pitch, the character of musical notes comprehends also the properties of *quantity*, and what is specifically termed *quality*. The quantity depends on the extent of the vibrations, and is the same with strength, loudness, or intensity; and by quality is meant the peculiar property of musical sounds, whereby, when they are exactly equal in pitch and quantity, they are still distinguishable from each other; as the same note sounded equally loud on a trumpet and a flute. From its peculiar quality, the sound of the human voice is easily distinguishable from the sound of any artificial instrument of music, or from that of the most perfect speaking machine.

99. The note termed the middle C on the harpsichord is produced by nearly 256 vibrations in a second; hence, if this be the exact number to be assigned to that note, then its numerical expression or designation is 256. Now this number is just the 8th power of the number 2—that is, its continued product by itself repeated 8 times, or, as it is more concisely expressed, 2^8; hence, using the sign of equality (=), this value is expressed thus: $C = 256$, or $C = 2^8$.

100. The note produced by twice as many vibrations as another note, is called its next higher *octave;* thus the note whose vibrations are 512 in a second, is the octave of C, and is denoted by the subscript figure 1, or C_1; hence $C_1 = 512$, or $C_1 = 2^9$; for 512 is the continued product of 2 repeated 9 times. So a note produced by half as many vibrations as another, is called its next lower octave; thus C is the lower octave of C_1. The note that is an octave higher than C_1—that is, the octave of C_1 itself—is denoted by C_2, the next higher by C_3, and so on. So the note that is an octave lower than C is denoted by C_{-1}, and its number of vibrations is the half of 256, or 128; that is, $C_{-1} = 128$, or $= 2^7$. All the octaves of C, from the lowest to the highest, within the ordinary compass of the musical scale, are therefore represented, and their numerical values expressed thus:—

$C_{-3} = 32 = 2^5$	$C_1 = 512 = 2^9$
$C_{-2} = 64 = 2^6$	$C_2 = 1024 = 2^{10}$
$C_{-1} = 128 = 2^7$	$C_3 = 2048 = 2^{11}$
$C \quad = 256 = 2^8$	$C_4 = 4096 = 2^{12}$

101. The length of the waves producing the note C_{-3}, is found by dividing the velocity of sound 1125 by 32; the result is 35 feet nearly, which is just the length of an open organ-pipe capable of sounding this note. The next note C_{-2} would of course be produced by a pipe of half the length, or $17\frac{1}{2}$ feet, and so on; and the highest note C_4 by a pipe 3 inches; and the middle C by a pipe $4\frac{3}{8}$ feet.

102. Any number of notes of an intermediate number of vibrations may be inserted between any two successive notes of the above series. The series included between two notes that are each other's higher and lower octaves respectively, including these notes, is called an *octave*. Whatever number of notes may be inserted in an octave, the same number may be inserted in the next higher octave, and the number of vibrations corresponding to the latter series of notes, would just be double of those producing the corresponding notes of the former octave; so that the numerical values of the notes in one octave would have the same relation to each other as those of any other. There is one particular series of notes that has been in almost universal use for centuries, to which, therefore, our attention must be chiefly confined. The number of notes between C and C_1 in this system is six, and with C and C_1 they constitute an octave. If the number of vibrations for the note C be denoted by 1 (which is, in fact, the number in the 256th part of a second), then those for the other notes, which are named D, E, F, G, A, and B, are as follow:—

$$C = 1,\ D = \tfrac{9}{8},\ E = \tfrac{5}{4},\ F = \tfrac{4}{3},\ G = \tfrac{3}{2},\ A = \tfrac{5}{3},\ B = \tfrac{15}{8},\ C_1 = 2.$$

Or giving C its proper value, 256, the values will then be—

$$C = 256,\ D = 288,\ E = 320,\ F = 341\tfrac{1}{3},\ G = 384,\ A = 426\tfrac{2}{3},\ B = 480, \text{ and } C_1 = 512.$$

103. Since the lengths of strings of the same kind, and under the same tension, are inversely as the number of vibrations in a given time, if the length of the string that gives the note C be denoted by 1, as 1 foot or 1 yard, or any other dimension, the lengths of the strings that would give the above series will be—

Name of Note.	C	D	E	F	G	A	B	C_1
Length of String.	1	$\frac{8}{9}$	$\frac{4}{5}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{3}{5}$	$\frac{8}{15}$	$\frac{1}{2}$

These numbers also denote the comparative lengths of the organ-pipes capable of sounding the corresponding notes.

If C_1, D_1, E_1, F_1, G_1, A_1, B_1, C_2, be substituted in the table, the numbers will require to be halved throughout; and so on for any number of octaves.

104. The above scale of notes has been named the *diatonic scale* or *key*. It will be afterwards seen that the numbers of vibrations of the notes in an octave on this scale have the simplest possible relations, which renders them well adapted for melody. Any note, as C, to which the succeeding notes forming an octave are referred, is called the *fundamental* note, the *tonic* or *key-note;* and the others in succession, the *second, third, fourth, fifth, sixth,* and *seventh*. Thus E is the third of C, G is its fifth, B is its seventh, and so on. The fifth is also termed the *dominant*, and the third the *mediant*. The notes immediately above and below the tonic and dominant are named by prefixing to them *sub* and *super;* thus—

Key-note.	Second	Third	Fourth	Fifth.	Sixth.	Seventh	Octave.
Tonic.	Super-tonic	Me-diant.	Sub-domi-nant.	Domi-nant.	Super-domi-nant.	Sub-tonic.	Tonic.

105. The *interval* between any two successive notes of the scale is expressed by the ratio of their numbers of vibrations, and may be called a *scale-interval*, to distinguish it from other intervals. Thus the interval between C and D is expressed by the ratio of 1 to $\frac{9}{8}$—that is, by $\frac{9}{8}$; that between D and E by the ratio of $\frac{9}{8}$ to $\frac{5}{4}$—that is, by $\frac{10}{9}$; that from E to F by the ratio of $\frac{5}{4}$ to $\frac{4}{3}$—that is, $\frac{16}{15}$; and so on. The interval $\frac{9}{8}$ is called a *major tone;* $\frac{10}{9}$ a *minor tone;* and $\frac{16}{15}$ a *major semitone*. All the scale-intervals belong to one or other of these three denominations.

106. The interval between any two notes—called in general a *musical interval*—is measured in the same manner as the scale-intervals—that is, by the ratio of the numbers of vibrations corresponding to them, or of any other two numbers proportional to them, as those in the table in a former article. Thus the interval between the tonic and its fifth, called also a *fifth*, is the ratio of 1 and $\frac{3}{2}$—that is, $\frac{3}{2}$.

107. When any two notes are sounded together, the compound sound is called a *chord*, which is either agreeable or disagreeable to the ear; the former are called *perfect chords, concords*, or *consonances;* and the latter *imperfect chords, discords*, or *dissonances*. Thus all musical intervals whatever are divided into *concordant* and *discordant intervals*.

108. When any musical interval can be expressed by the

ratio of two numbers not exceeding 5, it is concordant; all others are discordant.

109. From the numerical value of a concordant interval as thus limited, it is evident that the two notes separated by such an interval will, if sounded together, produce at least one coincidence of their vibrations for every 5 of one of them, and for a number of the other, not exceeding 5 at most. Thus, for the tonic and dominant, the ratio or interval is $\frac{3}{2}$; so that at every second vibration of the former, and every third of the latter, there will be a coincidence; and the resulting chord, from its frequency, will be one of the most agreeable of the consonances. *Harmony* arises from the coexistence of concordant sounds, and consists of a succession of consonances.

110. The subjoined table shows the various intervals between the key-note and the other notes of the diatonic scale, the names of the notes, the ratios of the corresponding numbers of vibrations, and the ratio of the lengths of the strings or organ-pipes capable of producing them:—

Names of Notes.	Tonic.	Super-tonic.	Mediant	Sub dominant	Domi-nant	Super-domi-nant.	Sub-tonic.	Tonic.
	Key note	Second.	Third.	Fourth	Fifth.	Sixth.	Seventh.	Octave.
Nos. of vibrations.	1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2
Lengths of strings.	1	$\frac{8}{9}$	$\frac{4}{5}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{3}{5}$	$\frac{8}{15}$	$\frac{1}{2}$
Intervals and their names.		$\frac{8}{9}$ Major tone.	$\frac{9}{10}$ Minor tone.	$\frac{15}{16}$ Major semitone.	$\frac{8}{9}$ Major tone.	$\frac{9}{10}$ Minor tone.	$\frac{8}{9}$ Major tone.	$\frac{15}{16}$ Major semitone

111. That the agreeable or disagreeable quality of chords depends on the number of coincidences of vibrations, in reference to the whole number performed in a given time, may be shown by a reference to other well-known facts. The regular recurrence of any sensation not naturally disagreeable, produces an agreeable feeling. When two people are walking together, if the steps are made in the same time, the effect is agreeable; whereas if they do not keep the step, the irregularity in the movement produces a disagreeable feeling. If,

however, in the case of unequal movement, the coincidences are at every second of the slower step—that is, if two steps are made by one of the people, while the other makes three—the frequency of coincidences gives an idea of a periodic relation still simple, yet more complex than in the case of equal movement. The first case is analogous to that of two coexisting unisonal notes, or the case of *unison*, the most simple and perfect consonance; the latter is analogous to the chord of the key-note and its fifth. The dissonance of the chord of the key-note and its second is manifest from the fact, that 8 vibrations of one coincide with 9 of the other, so that there is only one coincidence for every 8 vibrations of the former; and on this unfrequency of coincidence, and consequent irregularity of relation, the discordant feeling depends. The same may be said of the chord of the key-note and its seventh, the number of vibrations in the same time being as 8 to 15, and of course there is a still greater discordance than in the preceding case.

112. Were the ratios of the numerical values of the successive notes equal, so would the intervals; but this is not the case, as the lowest line of numbers in the preceding table shows. If these intervals were equal, it would be found that there would be fewer consonants. It would be easy, however, to adjust the lengths of the strings so that these seven intervals would be equal; but though each note in the scale would, when sounded separately, be as agreeable as any other musical sound of nearly the same pitch, yet they could not be so effectively arranged in the production of *melody*, which consists of the succession of single sounds, each having an agreeable relation to those immediately preceding. There must be a species of harmonic relation between the impression of the note called the key-note and those preceding it, within an interval of time not sufficient to efface these impressions. In accordance with this view of melody, it may be remarked that the closing note of every measure in a piece of music is generally the key-note, to which the others, therefore, are subordinate, or related according to the diatonic scale. In melody, therefore, the relations of the notes must generally be of the consonant kind, and no notes must be admitted into the scale that would cause great discords, and very few even that would cause moderate discords. Hence the diatonic scale is preferable to one with equal intervals.

113. From the key-note to its third, as from C to E, is an interval termed a *major third*, and expressed by the ratio 1 to $\frac{5}{4}$; that is, $\frac{4}{5}$. The interval from C to G is called a *fifth*, and its value is $\frac{2}{3}$. There is an interval, not contained in the above

form of the diatonic scale, which is much used in music, and expressed by the fraction [illegible]; and as this fraction is a little greater than the major third [illegible], this new interval is less than the preceding. The major third, multiplied by [illegible], will produce this interval, which is, in consequence, called a *minor third;* and the multiplier inverted [illegible], which is again the interval between these thirds, is termed a *minor semitone.*

114. When a note is made higher or lower by a minor semitone, it is said to be *sharpened* or *flattened* respectively. A note is sharpened or flattened by multiplying its value respectively by the fractions [illegible] and [illegible]. A flattened note is represented by the mark ♭ affixed to it, and a sharpened one by ♯. Thus, C being represented in value by 1, when sharpened, it becomes C♯, or [illegible]; and when flattened C♭, or [illegible]. So G sharp is G♯, or [illegible], or [illegible], and G flat, or G♭ is [illegible] or [illegible].

115. The minor semitone is the least interval used in music; any less interval is called a *comma*, though this is more specifically applied to the interval [illegible]. When the higher note forming an interval of a major tone is flattened, the interval is made less; but being nearer to the value of a major semitone than of a minor semitone, it receives the former name. Thus, when D is flattened by multiplying [illegible] by [illegible], the resultant note is [illegible], or D♭, and the interval between C and D♭ is thus [illegible], which differs from the major semitone [illegible] by only a comma.

116. When the lowest note of the interval of a minor tone is sharpened, the reduced interval is still a major semitone. Thus, D♯ is [illegible] × [illegible], or [illegible], and the ratio of D♯ to E is therefore [illegible] : [illegible], or [illegible], which is the semitone major. So if the lower note of a major tone be sharpened, the resulting reduced interval is [illegible], which is reckoned a major semitone; and if the higher note of the interval of a major tone be flattened, the reduced interval is a major semitone. When a major semitone is multiplied by the fraction [illegible], the product is the minor semitone, so that they differ only by a comma; consequently, since the intervals of the scale (excepting the semitones) are each composed of a major and a minor semitone, or of these and a comma, the sharp of one note can be taken as the flat of the next higher, and conversely. The great convenience resulting from this fact in the practice of music will be afterwards seen. At present, we have to consider what change must be made in the intervals of a scale of several octaves of permanent notes on a musical instrument, when some other note than C is assumed as the fundamental or key-note.

117. If eight strings AB, CD, EF, GH, &c. be taken of

any equal length, and if AB be stretched by a weight till it can sound a note which we may assume as a fundamental note, as, for instance, C; then if a bridge be fixed at *a*, at the distance

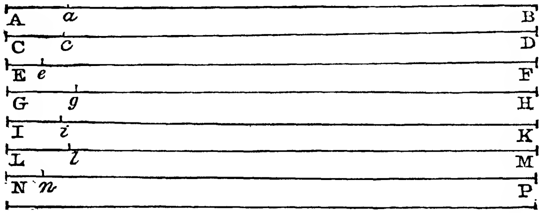


Fig. 15.

of $\frac{1}{9}$ of the length of the string from A, the remaining portion *a*B being $\frac{8}{9}$, will produce D; for C being denoted by 1, D will be denoted by $\frac{9}{8}$. If, now, the string CD, equal in length to AB, be made tense till its note is in unison with the note produced by *a*B, it will then of course sound D; and if a bridge be now placed at *c*, at a distance from C of $\frac{1}{10}$ of the length of CD, the remaining portion *c*D will sound E; for *c*D is $\frac{9}{10}$ of CD, and the ratio of the notes given by CD and *c*D is therefore $\frac{9}{10}$, which is the interval of D and E. So if the string EF be in unison with *c*D, and E*e* be $\frac{1}{16}$ of EF, the part *e*F will give F. Next make GH in unison with *e*F, and place a bridge at *g*, so that G*g* shall be $\frac{1}{9}$ of GH; make then IK in unison with *g*H, and take I*i* $\frac{1}{10}$ of IK, and place a bridge at *i*; make LM in unison with *i*K, and let L*l* be $\frac{1}{9}$ of LM, and then similarly make NP in unison with *l*M, and take N*n* equal to $\frac{1}{16}$ of NP, and make A′B′ in unison with *n*P. The notes made by the eight strings in order will then be the octave C, D, E, F, G, A, B, C_1. The interval CD (that is, from C to D) is evidently the ratio of the length of *a*B to AB; the interval DE is the ratio of the length of *c*D to CD; of EF that of *e*F to EF; and so on. The portions A*a*, C*c*, E*e* afford a tolerably good though not quite correct representation of the successive intervals, or of the major tone, the minor tone, and the major semitone. Were E*e* only $\frac{1}{24}$ of EF, then the interval denoted by the ratio of *e*F to EF would then be only a minor semitone. The ratio of the length *a*B to AB gives the major tone; of *c*D to CD the minor tone; and of *e*F to EF the major semitone; and so on.

118. Suppose now that the scale is extended over a part of two octaves—

C		D		E		F		G		A		B		C_1		D_1		E_1		F_1		G_1
	$\frac{8}{9}$		$\frac{9}{10}$		$\frac{15}{16}$		$\frac{8}{9}$		$\frac{9}{10}$		$\frac{8}{9}$		$\frac{15}{16}$		$\frac{8}{9}$		$\frac{9}{10}$		$\frac{15}{16}$		$\frac{8}{9}$	

and instead of C being assumed as the key-note, that G is so. Then, according to the order of the intervals of the diatonic scale, there must first be two tones, then a semitone, next three tones, and then a semitone. Now, from G to A is a tone, but instead of being a major, it is a minor tone; the difference, however, between these tones is less than a comma, and they may be interchanged indifferently; the next interval, from A to B, is a tone, from B to C a semitone, from C to D and D to E are tones; but from E to F is only a semitone, whereas this interval (the sixth) ought to be a tone. Now this half-interval can be enlarged to a whole tone, either by flattening E or sharpening F; the former expedient is inadmissible, as it would change the interval DE to a semitone. Let F, then, be sharpened, and the interval EF♯ becomes a full tone; but the next (F♯ G) is now only a semitone, as it ought to be, being the seventh interval. When F is sharpened, then $\frac{15}{16} : \frac{25}{24}$ gives $\frac{9}{10}$; so that EF♯ is a minor tone. The interval F♯G is now $\frac{8}{9} : \frac{24}{25}$, or $\frac{25}{27}$, which is the same as $\frac{15}{16} \times \frac{80}{81}$; that is, it differs from the major semitone $\frac{15}{16}$ by only a comma, and is therefore admissible. The series of notes, then, for the key of G becomes

C		D		E		F♯		G		A		B		C_1		D_1		E_1		$F♯_1$		G_1		A_1
	$\frac{8}{9}$		$\frac{9}{10}$		$\frac{9}{10}$		$\frac{25}{27}$		$\frac{9}{10}$		$\frac{8}{9}$		$\frac{15}{16}$		$\frac{8}{9}$		$\frac{9}{10}$		$\frac{9}{10}$		$\frac{25}{27}$		$\frac{9}{10}$	

The key of G is therefore called the major key of G with one sharp. By assuming any other note as the key-note, it would be found that one or more of the notes would require to be sharpened or flattened in order to preserve the diatonic intervals.

119. Were it convenient to adjust all the notes of any musical instrument to any particular note assumed as the key-note, so that the intervals succeeding it would be the usual diatonic intervals in their proper order, then there would be no necessity for the sharpening or flattening of notes. But this adjustment would require very complex apparatus for most instruments, in order to alter the lengths of strings or pipes, and another method is therefore adopted, much more convenient, and sufficiently correct in practice, though not so in a theoretical point of view. The method is by employing a

$\frac{5}{4}$, and $\frac{3}{2}$; the least whole numbers that are proportional to these are 4, 5, and 6, so that the coincidences are at every 4 vibrations of the key-note, 5 of the third, and 6 of the fifth. Take now the triple chord C, F, G, or 1, $\frac{4}{3}$, and $\frac{3}{2}$; the least whole numbers proportional to these are 6, 8, and 9, so that the coincidences are only at every 6 vibrations of C, 8 of F, and 9 of G; the former, therefore, will be the more agreeable chord.

124. The diatonic scale has two forms or *modes*, according as it begins with C or A. In one mode, the third of the key-note is a *major* third; in the other, a *minor* third; and consequently the modes are accordingly denominated the *major* and *minor mode.* In the major mode, the interval of the third CE is major; and in the other, the third AC is minor.

125. In the minor scale, though the intervals are the same as in the major, the first is a full tone, the second a semitone. the third and fourth are tones, the fifth a semitone, and the sixth and seventh are tones. There is, however, this peculiarity in the minor scale, that in ascending from the seventh to the octave, the former note is made sharp, which of course makes the interval immediately below—that is, the last interval of this scale—a semitone, as in the major mode. The interval between the sixth and seventh would thus be increased to a tone and a-half. This sharp seventh is called the *accidental seventh* of the minor mode; it is called also the *leading* note, and is considered to be *essential* to the scale in ascending. This irregularity may, however, arise from a mere prejudice, on account of the habit of hearing the last interval of the major mode as only a semitone, the latter mode occurring much more frequently than the former.

TRANSPOSITION OF MUSICAL SCALES.

126. When an air is to be changed in pitch from one key to another, the alteration is called *transposition.* The flats and sharps, if there are any, are to be determined according to the method formerly mentioned in article 118. Thus, if it is required to write an air on the scale of F major; since the third interval from F—namely, that between A and B—ought to be a semitone, the B will have to be flattened, and then all the intervals will be the same as in the key of C, which is called the *natural key* of the major mode, and is the type of the major scale. For the same reason, the key of A is called the *natural key* of the minor mode. If it is required to transpose to the minor scale of D, for example, then since the fifth interval in the minor scale is a semitone, the fifth interval from

D must be a semitone; therefore B must be flattened. Accordingly, B♭ is the signature for this key.

127. Transposition is easily effected by means of a very simple instrument (which is new), consisting of two scales of pasteboard PN, MQ; or a scale of wood PN, with a slider MQ moving in a groove. On the principal scale, PN is the series of diatonic notes extending from C to G′, the major and minor tones being, however, made equal, and the semitones equal to the half of the tones, so that the series of intervals,

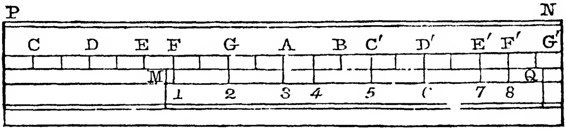


Fig. 16.

when the tones are halved, as represented in the figure, constitute the chromatic scale. On the slider MQ are the intervals for one octave, and by moving it till its first note (1) is opposite to the proposed key-note on the principal scale (suppose F), its divisions 2, 3, 4, 5, 6, 7, 8 will show what notes must be flattened or sharpened in the upper series of notes; for only the intervals in the latter series that do not coincide with those on the slider require to be changed. In the position of the scales in the figure, it appears that only B requires alteration; for if it were flattened, which would bring it to the position 4, the scales would coincide; the key of F, therefore, requires B to be flattened. For the minor scale, it is found in the same way that for the key of F the notes A, B, D, and E would require to be flattened. Were 1 put opposite to D, then it would be seen that the key of D major requires F and C to be sharpened; and that D minor requires only B to be flattened. Generally, it will be found, that since every major scale for any key-note, when its third and sixth are flattened by a chromatic semitone, becomes a minor scale for the same key-note, the latter scale will have three flats more or three sharps less than the former.

MUSICAL INSTRUMENTS.

128. The instruments or machines employed for producing musical sounds and their combinations are in modern times very numerous and varied, and great perfection has been attained in their construction. They may be classified in several

different ways; but the following considerations will assist in determining the groups that they most naturally fall into:—

129. In every musical instrument there are three things to be distinguished. These are, first, the *striking body*, or the means of setting the instrument a-vibrating; secondly, the *regulating medium*, which controls or determines the number of vibrations, and the pitch of the sound; thirdly, the *sounding mass*, or the body of the instrument, which acts upon the surrounding air, and through it to the ear of the listener.

130. The *striking body* may be a hammer of some solid material, as the clapper of a bell or gong. In the pianoforte, the strings are struck by a hammer with a surface of leather or felt. In the harp and guitar, the fingers are used to stretch the string, and then suddenly to let it go, whereby the elasticity of the material is brought violently into action. A rubbing surface is found very powerful in imparting the blow in the violin; this has the advantage of yielding a prolonged sound, besides giving the player a great command over the strength and quality of the tone. It is by the friction of the wet fingers that the beautiful sounds of musical glasses are kept up. The rubbing action is essentially a series of rapid and severe blows, of very small range individually, but in the aggregate possessing a very great degree of power. Friction is extremely energetic in acting upon the minute forces of cohesion and elasticity which bind the atoms of bodies into masses, as we see in its power of abrasion, and in its causing the evolution of heat, which never takes place without producing some atomic change in the structure where the heat is developed.

131. In one class of instruments an air-blast is the exciting cause of the sound; whence has arisen the designation of Wind Instruments. In some of these the blast is caused by the human lungs, and in others a bellows is made use of. The simplest form of wind instrument is the tube closed at one end, where the sound is produced by blowing at the mouth: a row of these, made of different lengths, forms what is called Pan's Pipes. In such a case, the air-blast from the mouth may be supposed to communicate a vibratory motion to the air within the tube, which, acting by friction on the solid mass of the tube itself, brings the whole into resonance, and yields the resulting sound. But the action may be explained differently; for it is not unlikely that the air-blast, acting on the sharp edge of the mouth of the pipe, creates at once the sonorous vibrations in the solid materials; and these vibrations will be governed by the size of the pipe. On this supposition, the air within the pipe will play merely a subordinate part, contributing only to the regulation of the note. In

several kinds of wind instruments, as in the common child's whistle, we are compelled to resort to this supposition of the excitement of sonorous vibrations by an air-blast properly directed on a sharp edge. Just as, in the musical glasses, the friction of the wet finger on the lip of the glass creates the sounding action, so the friction of a strong current of air upon the exposed edge of the whistle, or on the mouth of a Pan's pipe, can set the solid mass into the requisite state of vibration.

132. Reed Instruments are also wind instruments, the exciting cause in them being an air-blast. A reed has already been described as a thin slip of elastic substance, fixed at one end, and free to move at the other. If a rectangular opening is made in a brass plate, and if this opening is covered by an elastic slip of brass or steel, the slip may be made to vibrate by blowing against it with the mouth applied to the other side of the plate. A sound will be produced whose note will depend on the size and material of the slip, or on the rapidity of its natural vibrations. The intensity of the note will of course be increased by the resonance of the plate.

133. The clarionet is a wind and reed instrument, the reed being a slip of cane set in the mouthpiece, and vibrating to the player's blast. But when a reed is set in a pipe or tube in this way, its rapidity of vibration does not entirely depend on its own natural swing; for if that were the case, it would yield only one single note. The column of air vibrating within the tube controls the openings and shuttings of the reed, and therefore determines the note that it will communicate to the instrument by resonance. The vibrations of the reed itself, although reinforced by those of the air column, would have very little sonorous effect of themselves; but the flutter of the reed sets in action a similar set of vibrations in the mass of the pipe, and there are thus three separate vibrating masses all kept in unison by their mutual contact—the reed, the air, and the solid pipe—the last of the three being what gives to the sound its intensity and volume, or, we may say, its very existence as sound; for in all probability the pulses produced by the other two would be too feeble to reach the ear of a listener.

134. The organ-pipe is likewise a wind and reed instrument, with the parts somewhat differently arranged from the clarionet, but embodying the same essential principles. The air-blast is caused by a bellows, and the air, after entering the tube by the pedal, or opening at the lower end, passes along and escapes at the upper end by forcing open a tongue, thereby producing a series of vibrations, whose rapidity depends on the length of the tube—the vibrations of the enclosed air-

column being the means of regulating the tongue vibrations, and through them the audible vibrations of the mass of the pipe.

135. The *regulating medium* in musical instruments has been unavoidably noticed in the foregoing remarks upon the exciting cause or primary stroke. In stringed instruments, the vibrations of the string are determined by its length, thickness, and material; and when strings are stretched in a solid box, the resounding or communicated note of the box is governed by the natural note of the string. The box has no doubt a note natural to itself, which it would sound if struck directly; but this note, whatever it may be, is made to succumb to the note imparted from without, or to the regular series of tugs arising from the action of the string. We may easily suppose that the box would take in some sounds better than others; but in fact it is obliged to subordinate in its own natural note to the coerced note of the governing string. The strength of the player is exerted not merely to set the string in motion, but to keep up the additional thrill of the mass of the instrument.

136. In wind instruments, as already stated, the vibrating column of air is the regulating power. A column of air of a certain length has a fixed rate of vibration, and this rate is imparted to the tube that contains it, as well as to the tongues and reeds through which the tube is acted on. In the drum, the vibrations of the membrane are the regulating power, and these determine the rapidity of the acquired vibrations of the body of the instrument.

137. It thus appears that the *sounding material*, which is always the great mass or body of every kind of musical instrument, is in many cases forced to submit to a system of forced vibrations, and hence may be said to derive its merit from its readiness to take on any note that it is desired to impose. In the pitchfork, the bell, or the gong, which are struck directly, the note depends solely on the form and structure, and cannot be varied; the regulating medium and sounding mass are one and the same thing, and the resulting sound is that native to the instrument. But when a variety of notes has to be produced on one machine, the apparatus of a regulating power is superadded, and the plan of inducing foreign notes upon a sounding mass is resorted to. With this view, the form of the instrument needs to be very considerably modified.

138. In giving a short sketch of the principal musical instruments, it is convenient to commence with the simplest forms that are capable of yielding distinctive musical tones.

Of these, none can be simpler than a solid metallic *rod* suspended by a string, and struck by another piece of metal. A note is thus produced dependent on the size and material of the rod. The vibrations must be conceived as something analogous to those of an air-column of a limited length—that is, a series of condensations and rarefactions of particles will be passed from end to end backwards and forwards. But this class of longitudinal vibrations is not unlikely mixed up with others of a lateral kind; that is, the two ends of the rod may shake or vibrate while the other action is going on. Moreover, the stroke may produce also vibrations of twisting or torsion, and all the three sorts being coincident, a mixed sound will be the result. It is only in some one proportion of the dimensions of the rod that these three motions would yield the same note: hence it very rarely happens that a pure musical tone is produced from a metallic rod struck in this way. The only instrument in use that corresponds with this simple form is the *triangle*, which is a rod bent into a three-sided figure, and struck from side to side with another metallic rod; but the effect thus produced is not so much a set of musical tones as of rhythmical beats, which give time and emphasis to a march performance.

139. Next in simplicity to the rod is the *ring* or circle, whose principal vibrations, when struck, are in all probability a succession of flattenings and elongations of the circular figure. These may, however, be combined with lateral vibrations, and perhaps with those of torsion as well as with others propagated round and round. The want of a clear distinguishable note in this case is the proof that a complex sonorous action goes on, and that the different sounds are rarely in unison.

140. A third elementary form is the *plate*, or a plane surface of metal, glass, or wood, whose vibrations have been studied, in the manner already described, by using pieces of glass strewed with sand, and put in motion by a bow. This has shown that the mode of vibration is by opposite halfs or fractions of the plate vibrating in opposite directions, leaving a quiescent line of division. The effect of the *cymbals*, as compared with the *triangles*, gives us the means of judging the difference between the plate and the rod. The cymbals would appear to be not only more voluminous and powerful, but also more pure and simple in tone than the triangles.

141. The combination of these elementary forms in various ways gives rise to several well-known instruments. The *pitchfork* is a modification of the rod, and is so shaped, that the vibrations are chiefly of one kind—namely, the swing of the

legs backwards and forwards. Although it is impossible to extinguish entirely all other vibrations in favour of this one class, yet the lateral motion predominates so much, as to cause one distinct note to be discernible, which is the note belonging to the peculiar size and form of the instrument. The pitch-fork is formed to yield a particular note or pitch on the scale, and is hence employed as a standard of pitch in the tuning of instruments, or in starting off in a vocal air. The most familiar illustration of resonance, as already mentioned, is furnished by resting the fork on a table after it is struck. The whole mass of the table is in this way brought into unison with the note of the fork, and gives that voluminous expansion and exaggeration of tone which is the characteristic effect of resonance.

142. The *bell* may be looked upon as a combination of the tuning-fork with the circle. It is sufficiently massive to give a powerful tone without the aid of resonance from a superadded mass. Being struck with a metallic knob or clapper, it is set a-vibrating nearly on the principle of the ring—that is, by alternate flattenings and elongations. A bell can have no more than one note, determined by its size and dimensions; and the purity of the note must depend on the perfection of the form and proportions. The longitudinal vibrations of the pitchfork section must be in harmony with the vibrations of the circular sections, in order to produce an accordant tone. The bell is a very ancient instrument, and owes its extensive use to the powerful and piercing character of its sound. The large bells constructed for churches are often audible for miles; and the sound, which to a person close at hand is harsh and painful, becomes by distance soft and melodious.

143. The *gong* is a Chinese instrument of very great power, and its uses in China correspond to a great degree with the employment of the bell in other countries. Being of the shape of a tambourin, but made entirely of metal, it is a combination of the plate and circle. It is struck on the middle of the plate, and this part is sometimes swollen out into a spherical or round projection. The power and volume of the tone seem to the listener to be immensely greater than in proportion to the size of the instrument. But there would appear to be a very great advantage in this peculiar combination. The cymbals are an example of the power of the plate; they are, for their size, perhaps the most noisy apparatus that exists. And the rim or *cylinder* is seen in many ways to be a very advantageous form of instrument. In the tambourin and drum it is the resonant mass. It is, moreover, the type of the greater number of wind instruments. We are not to

wonder, therefore, at the effect that may be produced by a proper combination of so effective a pair of elements as the plate and cylinder. The noise and roar of a Chinese village on a festive day, when gongs are sounding in every direction, are such as to astonish and confound any foreigner that may happen to be present.

144. Both with the bell and the gong each instrument has but one note, and for a melody or tune there must be a series provided of different sizes, according to the desired range of notes.

145. The *musical glasses* are of the cylindrical form, and are excited into action by the friction of the wet fingers. The manner of their vibration is not very obvious. The tone arising from the rubbing of the edges is considerably different from the effect of striking the side in the manner of a bell, in which last case it may be supposed that the vibration is of the same character as in the bell and in the ring. Possibly, therefore, the internal movement of the particles, when under friction, is not the same as under a stroke; and it is not unlikely that the peculiar action may be an instance of *torsion*, which would correspond well enough with the nature of the sounding impulse.

146. The *drum* and *tambourin* have a greater alliance to the foregoing group than to any of the other principal divisions of musical instruments. A cylinder, with a tight membrane stretched over both ends, is the characteristic description of the drum. The chief power of the instrument resides in the resonance of the cylinder, which is put in motion by the vibrations of the membrane. If we suppose that a blow on the membrane draws together and contracts the circular edge of the drum all round, then there will be a corresponding expansion when the effect of the stroke ceases, and the circular edge will alternately contract and expand, and this will be the mode of the vibration. Doubtless these alternate contractions and expansions will be duly propagated from end to end of the cylinder, and the number of them will depend partly on the rate of vibration natural to the dimensions of the instrument, and partly on the rapidity of the stroke. Moreover, the air within and without the drum will be acted on by the direct impulses of the membrane, and the interior air may further affect the vibration of the solid cylinder. The double head is essential to the full tone of the drum.

147. This explanation, if at all near the truth, would indicate a different sonorous action in the drum from what we have assumed in the ring and bell under the agitation of the kind of blows communicated to them. If the effect of the

stroke on the drum-head be to bring together and contract the rim all round, this contraction being propagated from end to end, and regularly returned from the dumb end to the sounding one, the vibratory motion would be a sort of vermicular series of swellings and contractions, instead of the alternate flattenings and elongations that characterise the ring.

148. The drum, like the triangles and cymbals, is wanting in purity of tone, and is therefore incapable of contributing to the melody of musical choirs; and, like those, it is used to render prominent the *rhythm* or time of the composition, and to produce emphasis of effect in particular passages, there being musical compositions expressly adapted to bring out the power and efficacy of such accompaniments. As one part of the excitement arising from music is owing to this rhythmical effect, or the recurrence of beats at equal and regulated intervals, the instruments that excel in yielding the effect perform an important function, although they may be incapable of contributing to the stream of *melody*, which is a thing of a totally different nature, acting upon a distinct susceptibility of the human constitution.

149. The *tambourin* cannot be considered as specifically different from the drum. The agitation of the stretched membrane by the hand is the cause of the sound, and the effect is reinforced by the resonance of the solid rim and the rattle of the brass rings. Being a light and easily-handled instrument, it excels even the drum in rhythmical effect, and is the accompaniment of dance, and frolic, and gesture of the gayest and wildest description. In the frantic exhibitions of bacchanalian orgies and ecstatic worship, the tambourin and cymbals are appropriate instruments, being capable of leading to the highest pitch the intoxication and frenzy of rhythmical movements. The interposition of melody on such occasions would be quieting and soothing; hence the clangour and noise of the unmelodious instruments of maddening iteration were the source of the inspiring effect. In ballet music, the instruments of rhythm and emphasis are likewise indispensable.

150. After having adverted to the musical instruments of a more simple kind, we must now proceed to notice those of a complex description. The two great classes of instruments capable of yielding an extensive range of musical sounds, and of performing melodious compositions, are designated *stringed* and *wind* instruments respectively.

151. The principles regulating the sounds of strings have already been explained, and we have farther explained the influence of the appended solid mass in increasing the sound

by multiplication or resonance. The least complicated stringed apparatus is exemplified in the *harp* and in the ancient *lyre*. A series of strings, of different size, quality, and length, are stretched in a solid framework, each being adapted to produce a separate note, and the series forming a continuous scale. On being struck by the fingers, they sound their several notes, and melodies may be played by touching the proper strings in due succession. Also, by the use of two, three, or more fingers, contemporaneous tones may be sounded, and harmony superadded to the melody. The ancient lyre had very few notes; but the harp, as now constructed, is a large instrument, with notes extending over a wide compass.

152. The *guitar* is in principle the same as the harp and lyre, but it embodies the innovation of artificially shortening the strings, so as to bring more than one note out of each, which is done also in the violin. In this way, from a small number of strings a great number of notes may be produced, and the compass of the instrument is enlarged, while its dimensions remain the same. To suit this arrangement, the strings are chosen of such an amount of inequality, that an interval of (say) a fifth occurs between each adjoining pair, and the intermediate notes are formed by shortening the graver of the two by the fingers of the left hand.

153. In the harp and guitar, which are sounded by the fingers stretching and then suddenly letting go the strings, although melody may be derived in a very high degree, yet the sharpness and brevity of the tones are such as to make the beat and rhythm prominent. Hence they serve as accompaniments to rhythmical action and expression, although not so violent or intense in this particular as the drum or tambourin.

154. The *pianoforte* is a stringed instrument of still more extensive powers. Its strings are struck by means of hammers, and the wooden box enclosing the stringed apparatus is the resounding substance. To prevent the confusion that would arise by the too-long-continued vibration of the strings, a damper is mounted over every string, which is lifted by the same action that strikes the blow, and falls down again when the finger is removed from the key. Although the tones of the pianoforte are less sudden and more continuous than in the harp and guitar, they are still of an emphatic character, rendering beat and rhythm conspicuous along with the melody. The extensive popularity of the instrument has led to the composition of a large amount of music solely for its use. The mixture of melody and emphasis peculiar to it probably meets the taste of a large section of the listeners to music; but the preference given to the violin in the orchestra shows,

among other things, that the rhythmical beat of the pianoforte is considerably above what would constitute the golden mean. In dance music, both this instrument and the harp are extremely effective; the stimulus of their rhythm is not too great for the sobriety of the modern domestic dance. In combination with the voice, the pianoforte serves the purpose anciently served by the lyre, in giving emphasis and marking time, while it also contributes to the performance a melody of its own.

155. In slow and sacred music the pianoforte fails; and as it requires a constant percussion of its wires to sustain its full tones, its powers are most advantageously displayed in rapid and brilliant compositions, such as waltzes, quadrilles, variations of melodies, and sonatas. Beethoven and Mendelssohn have produced some of the most beautiful and classical music for this instrument.

156. In passing from the pianoforte to the *violin*, we make another step in that transition from rhythm and emphasis to pure melody which marks the succession of instruments above enumerated. The action of the bow being to produce a sustained tone, in place of a sharp and sudden beat, it is possible to subordinate rhythm to continuous flow of sound as far as we please. This is the grand characteristic advantage of the violin. Moreover, in the act of playing, the tact and skill of the player have the utmost possible scope, both in the handling of the bow and in the shortening of the strings. Whatever the ear may desire, it is possible for the hand to execute. In no other stringed instrument are the vibrations of the string under such perfect control. A kind of movement that seems, by its very nature, to be passing and momentary, is rendered continuous and durable. The transitions from note to note, which give the perception of time and rhythm, may be softened to almost any degree; while, on the other hand, it is possible to mark them with such emphasis and sharpness, as to produce all the effects of the other stringed instruments. The gay and animated dance may be supported with an equally animated accompaniment, and yet the instrument is capable of any degree of melody or solemnity.

157. The pianoforte, being an orchestra in itself, is liable to the defects of such an arrangement. But the violin has but a limited range, and each instrument is intended for only one single succession of sounds. The purity and clearness of the effect are in this way much better preserved. Probably, too, the compact body of the instrument forms a better resonant mass than the lumbering wooden structure of the piano. The action of the string upon the shell of the instrument is through

the bridge resting on the belly, and the belly and back are connected both at the edges and by a pillar in the middle. The sweetness of the tones may be supposed to depend partly on the form of the instrument, and partly on the material, but in what manner, seems not to be well understood. The gradual change that is wrought in the structure of all solid bodies by continual wear and tear, and by the motions of their particles consequent on shocks and agitation, seems favourable to their sonorous qualities, for it is remarked that violins improve with age.

158. To form a complete orchestra with the violin, four different sizes are required, corresponding to the four voices distinguished by musicians; namely, treble, contralto, tenor, and bass. With these it is considered that a perfect orchestra might be made up, capable of almost all the finest effects of the musical art.

159. The instruments above described are the leading examples of the stringed class. There are several others belonging to the same class that are now fallen into disuse, as the harpsichord and the dulcimer of the ancients. The *Æolian harp* is a peculiar example of the class. Placed before the narrow slit of a window, opened to about an inch or two, the concentrated wind-draught playing in the strings communicates sonorous vibrations of the sweetest natural melody. The same action may be sometimes observed in the wires of the electric telegraph. By listening near one of the posts, a very audible sound may be felt. In these cases the wind, or an air-current, is the striking body, and the delicacy of the stroke may be readily imagined to surpass every kind of blow from solid masses, even the most skilled performance of the violinist. Possibly instruments may yet be contrived in imitation of the Æolian harp, where a wind-blast may be the actuating impulse of the string-caused notes.

160. We come next to the *wind instruments*, in which the vibration of a column of air acts the part of the vibrating string in the preceding class, being the regulating medium, and the connecting link, between the primary impulse and the resonant mass.

161. The *Pan's pipes* we have already alluded to as the simplest form of wind instrument. A concentrated stream of air acts upon the sharp edge of the open end of the pipe, and by the strong friction, or impulse, imparts a vibrating state to the solid mass. At the same time a series of undulations are created within the pipe, whose rapidity depends on its length. These undulations act on the interior walls of the pipe, and

produce in its mass a corresponding note, or at all events regulate the vibrations caused by the blast, so as to make the two sets in unison. The note of each pipe would depend strictly on its length, if the undulations of the air had the entire control of the vibrations of the resonant tube.

162. It must never be forgotten, in the discussion of wind instruments, that there are very few examples of sound produced by a vibrating mass of air acting alone, or without being reinforced by resonance. We are therefore hardly in a position to appreciate the intrinsic feebleness of an aërial sounding substance. The most remarkable instance of a pure aërial sound is furnished by the thunder of the sky, which is no doubt a sound of immense power; but if the extent of the commotion is taken into account, the magnitude of the effect will not seem surprising. A thundery discharge is known sometimes to be several miles in length, and if a corresponding breadth and thickness of the agitated stratum be assumed, the bulk of air set in action will be many times greater than all the musical instruments of the world put together. With a solid resonance coextensive with the aërial agitation, the sound would be sufficient to rend the ears of a whole empire.

163. The *whistle* is a modification of the Pan's pipe, wherein the sound is produced with less difficulty, in consequence of a distinct direction being given to the air-blast. The sharp edge played upon by the current is expressly adjusted so that the stream passing through the narrow mouth of the whistle plays on it with perfect precision: in fact it is rendered impossible to miss the sound. The structure of the whistle thus yields a confirmation of the view given above of the action of the blast on the open pipe.

164. The whistle further differs from the simple pipe in producing a variety of notes. The manner of effecting this, in it and in similar instruments, is well known to be the virtual shortening of the tube by the holes pierced along the side. Since the condensation and rarefaction of the air are destroyed as effectually by a hole in the side as by cutting off the tube at that point, the lengths of the undulations are limited by the escape thus afforded into the outer air, and therefore the same sounds can be produced in a single tube as in a succession of tubes of different lengths. A range of notes, extending to an octave, are produced from a single pipe, and a second octave may be formed by increasing the stress of the blast till the column of the tube is broken up into its harmonic of half the length; and if, by a still greater exertion of voice, the harmonic undulation of one-fourth were brought into action, a third octave of notes might be given forth.

165. The *flute* is a wind instrument, differing from the whistle in the manner of exciting the sound. The air-blast is thrown upon the edges of a hole pierced in the side of the tube, in a way somewhat similar to the action in the Pan's pipe. As in the whistle and simple pipe, a sharp edge is obviously the place for concentrating the action of the current; and the vibrations produced in it are communicated to the mass of the instrument, and to the air-column within. The undulations of the air-column being the regulating power which determines the pitch of the note, their action upon the interior of the resonant mass of the tube must control the rate of its vibrations, even although we suppose these vibrations to be principally excited by the strong blast acting with a mechanical advantage on the edge of the hole. The air-stream from the mouth must tell directly both on the solid mass of the tube and on the air-column inside. The lowest note that the instrument can yield is made by blowing gently, and keeping all the holes shut with the fingers or keys. The upper notes are formed by successively opening the several holes, and by increasing the stress of the voice at each note. It requires a stronger mechanical impulse to produce the quicker vibrations of the higher parts of the scale.

166. The *octave flute* and *fife* are instruments of a smaller calibre than the concert flute, and therefore produce a series of notes higher up in the scale. It is to be observed of the flute, in common with the whole class of wind instruments, that these are truly and properly speaking instruments of melody, and not of beat or rhythm, being thus decisively contrasted with the class of stringed instruments. The nature of a wind-blast being to produce a continuous mellow sound, and not a sudden jerk or sharp stroke on the ear, wind notes coincide exactly with the very essence of melody, just as string notes coincide more particularly with the essence of rhythm. In wind instruments, the transitions from note to note, or the successive impulses of the air-blast, are the only means of indicating time or making beats; but such indications are very faint in comparison with the sharp strokes that excite stringed instruments into action. Hence in accompaniments to the dance or the military march the wind class are less applicable; and if employed at all, they must be used in conjunction with instruments of the other class. In military music, the fife and drum are the usual combination.

167. The brass instruments played by the mouth form a genus by themselves, and are all remarkable for the power, sharpness, and impressiveness of their sounds. They include the bugle, the cornopean or cornet-à-piston, the trumpet, the

Sax-horn, the French-horn, the trombone, the bass-horn, and the ophicleide. The simplest form of this class of instruments is seen in the *coach-horn*, which has a mouthpiece contracted to a narrow hole, and joining a tube which gradually swells out towards the other end. In observing the mode of bringing out the sound in such a structure, the principle of the sharp edge is manifestly detected. The air-blast acts, as in the cases already noticed, at once upon the solid tube and on its aërial contents; while the aërial undulations themselves act on the walls of the tube, and bring its vibrations into unison with theirs. That the friction of thin air is able to control the movements of particles of solid metal, seems at first sight strange, but the fact, nevertheless, is undeniable.

168. The *bugle* is the coach-horn curled round upon itself, so as to become more compact; but the principle of the structure is the same. Neither of the instruments can produce a great range of notes, inasmuch as the only means of varying the pitch is by altering the stress of the blast, one effect of this being to break up the air-column into its several harmonics. It is possible, however, to produce a few additional notes by changing the mode of the blast, or the mode of applying the lips to the mouthpiece. By thrusting the hand into the mouth of the tube, a further modification of the tones can be effected. But notwithstanding all these varieties of action, the range of the instrument is only about an octave. Were it not that the individual tones have great richness, and may be managed with a highly-melodious effect, the smallness of range of the instrument would reduce its performances to the extreme of meagreness and penury.

169. The *trumpet*, in its primitive shape, was exactly what we have called the coach-horn, being originally formed of a cow's horn, although afterwards made of metal. But the modern trumpet surpasses the bugle in range, in consequence of the adoption of crooks, keys, and valves, which serve the same purpose as the holes and keys of the flute; that is, they shorten at discretion the contained air-column, and thereby change the pitch of the notes.

170. The *trombone*, or the *sackbut* of ancient times, is a very powerful instrument, and may be described as a trumpet, the tones of which are regulated by a tube of brass sliding within another, so as to shorten or lengthen the column of air. An instrument made on this principle was discovered among the remains at Pompeii.

171. The above examples of the wind class of instrument are all formed on the principle of an air-stream playing on a sharp edge and on a contained air-column simultaneously; but

we have now to notice a different species, characterised by the use of reeds, the construction of these having been already mentioned. The *clarionet*, although not unlike the flute in its general structure, has a great superiority in the richness of its tones. The vibrations of the reed fixed in the mouthpiece seem to communicate a far more powerful agitation to the mass of the tube than it is possible to effect by blowing on the dead edge of the breath-hole of the flute. The mechanical advantage in the case of the reed may be supposed to be very considerable.

172. The *bagpipes* is a very ancient instrument of the wind and reed class, being of Arabian origin, although the Romans called it Etruscan. It has the peculiarity of keeping a bag of air constantly full, which is made to supply the instrument by being squeezed by the arms of the player. Like the clarionet, this instrument may sound very harsh if badly played; and great skill, as well as a good quality in the structure, is requisite to bring out the strong rich tones to perfection.

173. The greatest of all the wind and reed instruments is the *organ*, which is a vast collection of pipes brought into connection with a common wind-chest and bellows, whose blast supplies them with the exciting current. As each pipe plays only one note, a separate pipe is required for every note that it is wished to produce. But the organ is not content with one series of pipes extending over several octaves of the musical scale, and sufficing to make it as complete an instrument for melody or harmony as the piano; it comprises many such series, each having a peculiarity of effect distinguishing it from the rest, and constituting, as it were, a separate instrument. The more extensive the scale of the organ, the greater the number of these distinct sets of pipes. There is a peculiar machinery for bringing the different series into active connection with the bellows and keys called the *stops*—that is, each stop corresponds to a separate range of pipes, and, on being drawn, brings that range under the power of the performer. Thus by employing several stops simultaneously, the sound of many instruments is produced. The organ in the new church at Amsterdam contains fifty-two whole stops. Some of the stops are named from their imitating various instruments—as the *cornet*, the *trumpet*, the *bassoon*, the *flute*, the *cremona*. The most important stops of the more ordinary class are those named the *open diapason*, the *stopt diapason*, the *principal*, and the *fifteenth*. The compass of the individual stops may be extended from the lowest to the highest notes appreciable by the human ear, for this purpose it being requisite merely to enlarge or diminish the size of the pipes. Each pipe being

excited by a reed apparatus, the greatest possible mechanical advantage is allowed in the action of the blast upon the solid walls of the tube.

174. The organ is the extreme example of the instruments of melody and harmony, as contrasted with such as are conspicuous for beat and rhythm. Hence its characteristic effect is overpowering grandeur and solemnity, and its function in human life is to assist in public worship, or to produce vast and solemnising impressions. Anything approaching to gaiety or sparkling animation is totally excluded from the range of its powers.

175. The organ of voice in the human subject, as well as in the inferior animals, is a wind instrument of very remarkable power, considering the simplicity of its parts. With a single tube it can produce a range of notes of from two to three octaves, depending in part on the different degrees of tension of the two edges or strings known as the vocal chords, which are acted on by the air-blast. These chords seem to have something of the character both of the string and of the sharp edge.

176. The organs of speech comprehend the lungs, the wind-pipe, and the mouth. The first serves as an air-chest to supply the current of air necessary to the production of sound; the second is a pipe extending from the lungs to the throat; and the cavity of the mouth, with the lips and tongue, are capable of modifying the sound into what is termed *articulate* sounds. The wind-pipe, reckoning from its upper extremity, consists of the *larynx*, the *trachea*, and the two *bronchial tubes;* the latter branching off from the lower end of the trachea, and proceeding, the one to the right, the other to the left lung. At the upper extremity of the larynx, called the *glottis*, sound is produced. It arises from the passage of the air from the lungs through a narrow slit called the *rima*, or *cleft* of the glottis (fig. 17), the edges of which consist of fibrous membranes called the *vocal chords*, by whose contraction so much tenseness or tightness is acquired, that they are capable of sonorous vibrations when a strong enough current of air is forced through the slit, either in inspiration or expiration, the sounds being more perfect in the latter case. When the vocal chords are not tense, the air passes through

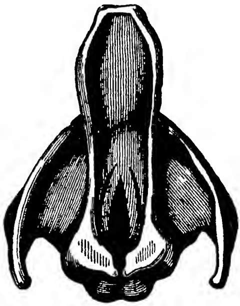


Fig. 17.

the glottis freely and silently, as in ordinary respiration. The glottis, with the cavity of the mouth, operate as a reed-pipe, and the different degrees of tension of the vocal chords has the samè effect as different reeds in producing notes of different pitch.

177. The machinery requisite for the production of vocal sound includes, in the first place, the muscles of respiration, which are situated partly outside the chest, on the back and breast, and partly in the abdomen, as the diaphragm. The muscles that contract the chest, and force air outwards, are the most powerful of the two sets, whence arises the superior energy of the sounds of expiration, or of the outward current. For the stronger class of utterances a powerful blast is necessary, and hence the importance of a large powerful chest in vocal exertions.

178. The machinery in the vocal tube is chiefly the cartilages for stretching the vocal chords, and a series of connecting muscles capable of tightening or slackening the chords according to the note that is to be formed. The regulation of the note is left dependent on the different degrees of tightness and distance of the chords, and on the modifications of the tube itself, such as its being widened or constricted, lengthened and shortened. What is most certainly understood on this point is, that by stretching the vocal chords through the action of certain of the muscles, a high pitch is produced, and according as they are relaxed, the pitch is lowered. That a slack string yields a graver tone than a tight one is a well-known fact; but in ordinary strings the whole range that could be produced from this cause would be very trifling; in fact, hardly amounting, by any possibility, to two or three notes of the scale, whereas in the human voice a range of two octaves is quite usual. We must therefore suppose that the elongation of the vocal apparatus, which is sensibly felt as we ascend in pitch, is essential to the production of high notes. To produce a low bass note, the chin is lowered, and the neck contracted, so as to shorten the tube as much as possible, and alter the form of the cavity of the outlet; on the other hand, for an acute note, the windpipe is drawn out by elevating the head, and is thus rendered more slender and contracted in all its parts.

179. The whole of the *musical* apparatus of the voice is contained in the windpipe with its cartilages and vocal chords. By the various modifications of these, made under the guidance of the ear, we may produce all the degrees of pitch and intensity that come within the compass of the voice. The capacity of fine execution depends partly on the strŭcture and

flexibility of the vocal organs, and partly on the form and structure of the resounding skull. But it is only in proportion to the delicacy of the ear or musical taste that exquisite organs of execution can be cultivated or adjusted to a high order of performance; a fact which holds in relation to every capacity within the range of the human constitution.

180. Although the voice shares the peculiarity of the whole class of wind instruments, in being more favourable to melody than to rhythm, it nevertheless has great power of rhythmical execution. This, however, in part depends on its articulate character, or in the use of language in connection with its performances, which gives it a manifest superiority over all instruments whatsoever.

181. The articulate capacity of the voice depends on certain additions to the organs already mentioned as supporting its musical power. It is found that the sound, in passing through the *mouth*, may have its character altered, not in respect of musical pitch or strength, but in a way to give it a distinguishable effect on the ear. If a person singing any one note of the musical scale with the mouth gaping open, were to continue the same note with the mouth nearly shut, the sound would be identical in its musical effect, but in respect of character or expression it would appear to be different. There would seem to be a change of shape in the sound itself. This peculiarity of sounds, which is dependent on the form and movements of the mouth during their utterance, is termed their articulate character; and sounds strongly marked with it are called *articulate sounds*. The musical and the articulate characters of sounds arise from different organs, and are governed by totally different principles. Their connection with the general framework of body and mind is also totally different. The windpipe sounds are combined into melodious successions, according to one class of feelings, while the mouth sounds are connected under the guidance of sensibilities which have very little in common with musical taste.

182. For articulate sounds, therefore, we have to refer to the construction and movements of the mouth. Every one knows its general form and parts, and we need only call attention to the movements performed in it. These are—1*st*, The movement of the lower jaw, which enlarges or contracts the height of the cavity, or its dimensions from above downward, and opens or closes the aperture of the teeth; 2*d*, The movements of the cheeks, which distend or lengthen the mouth in the cross direction, and, along with the lowering of the jaw, open the cavity to its fullest dimensions; 3*d*, The contraction of the ring of the mouth or lips, as exemplified in

the whistling position; *4th*, The elevations and depressions of the upper and lower lips, which combine with and modify the other movements; *5th*, The movements of the tongue. These are very various:—*1st*, It may be protruded outwards, or drawn in to the back of the mouth; *2d*, It may be bent or curled either up or down; *3d*, It has a free motion from side to side. By these motions the tongue can come into contact with any point in the cavity, and make the touch by different parts of its own surface.

183. All these movements tend to alter the shape of the mouth, and with this the expression of the sound which issues from it. Hence the possible variety of sounds that may arise is unlimited. The distinguishable sounds, however, are not very numerous. They are arranged into various kinds:—

184. *1st*, We have what are called the *vowel* sounds. When all the parts of the mouth are in one fixed position, giving a free opening outwards, and remain fixed during the emission of a sound, so as to exercise no other influence than arises from the mere shape of the cavity, a vowel is produced. Thus, in sounding *ah*, the mouth is opened, and the jaws, cheeks, lips, and tongue are fixed dead in one posture; so in sounding *uuh*, the posture, though different from the former, is still a quiescent or dead posture. By altering the shape, the sound is altered; but so long as it is an unalterable shape, a vowel is the result. The vowels that are most markedly distinguished from each other, are such as arise from the most widely-different arrangement of the parts of the mouth. The five vowels, *ah*, *ee*, *ay* (*say*), *oh*, *uh*, are the five most distinct sounds resulting from the various extreme positions of the organs, and may be called the five fundamental vowel sounds, having a greater difference from each other than any one of them has for any other sound distinct from them. Thus the English vowel sound *awe* arises from a middle position between *ah* and *oh*. The English sound of *i*, as in *sit*, is very little different from the fundamental *ee; set* is very near *say;* and even *u* in *but* is but one remove from the same sound. The *a* in *sat* is a modification of the fundamental *ah*. Every one of these sounds can be varied by a slight shading, so as to produce several that a fine ear can distinguish. In fact no two nations pronounce similar vowels exactly alike, and even in the individuals of the same nation slight differences are very common: sometimes the people of one province can be distinguished by the shade that they give to the fundamental letters of the alphabet. Thus the Scotch sound of short *i*, as in *sit*, is often too near the *ay* sound, whereas in correct English pronunciation it should be nearer the *ee*.

185. But the varieties of vowel utterance can be immensely extended by combinations of vowels, or by changing from one to another within the same breath, as in *boy*. This gives rise to what are called *diphthongs*. There are some of these diphthongs so natural and easy, that they are adopted as regular alphabetical sounds, on which differences of words are founded. In English there are three proper diphthongs: these are the sounds in *sigh*, *now*, *boy*. The first is a combination of *ah* and *ee;* the second of *oh* and *uh;* the third of *oh* and a sound approaching to *ay*. There are other diphthongs less perfect than these, or in which the sounds do not run together so completely. Thus the *ua* in *quake*, the *we* in *Tweed*, are regarded as diphthongs less pure than the others.

186. 2*d*, Of the class of sounds called *consonants*, a great many divisions have been made. They differ from the vowels in requiring some of the parts of the mouth to perform particular movements, in order to their being uttered. A certain play of the tongue, teeth, or lips, is necessary to each of them. This play may vary from the mere quiver of the tongue in sounding *s*, to the forcible shutting off of the sound by the sudden closure of the lips in *p* final. The sounds *p*, *t*, and *k*, are connected either with sudden closures or with sudden explosions of the sounding emanation, and are therefore called *mutes*, and also *explosive* letters: *p* is formed by the lips, *t* by the point of the tongue striking the roof of the mouth near the teeth, *k* by the back part of the tongue striking the back part of the roof. Of these, *p* is the easiest to sound, and the first learned by children, and *k* the most difficult. The *p*, being formed by the lips, is called a *labial*, *t* a *palatal*, and *k* a *guttural*, or throat-formed letter. And as all the consonants are formed more or less nearly in one of these positions, a general division can be made of them into labials, palatals, and gutturals. Six distinct labials are enumerated, depending on different ways of sounding with the lip closure. The mute or explosive *p* has been mentioned; next to it is *b*, produced by a less violent closure, which allows the voice to be heard during the act, as any one will feel by sounding *cup* and *cub*. The third labial is *m*, which is still farther removed from the sudden extinction occurring with the *p;* a free communication is opened with the nose for the egress of the air, and the sound can be made continuous like a vowel; in other words, we have the humming sound; this is the *nasal* labial, while *b* is called the *vocal* labial. The fourth labial is *f*, produced by the upper teeth and the lower lip coming together, and the breath passing through them without voice; this is the whispered or *aspirate* labial. When the vocal chords are tightened

up, and the hard sound of the voice sent through this closure, we have *v*, or a second vocal labial, called the vocal aspirate. Lastly, a sound may be sent through the closed lips, making them vibrate or shake like a reed, as in the sound *prr;* this is the *vibratory* labial, or the labial *r*. A similar series can be described in the palatals. The mute being *t*, the vocal is *d;* the nasal are *l* and *n;* the aspirates are *th* (*th*umb), *s*, *sh*, arising from slightly-differing positions of the tongue in its contact with the palate: the vocals, or audible forms of these, are *th* (*th*y), *z*, *j;* the vibratory palatal is the common *r*. The gutturals likewise show the same list of varieties. First, *k* mute; then the vocal *g;* the nasal *ng*, a simple sound, though spelt in our language with two letters; the aspirate *ch*, as in *loch*, together with the fainter form *h;* the vocal aspirate *gh* unknown, and almost unpronounceable by us; and the vibratory *ghr* occurring as a burr in some people's utterance. This classification, which was first proposed by Dr Arnott, may be summed up in the following table:—

	Labials.	Palatals.	Gutturals.
Mute, ...	*p*	*t*	*k*
Vocal, . . .	*b*	*d*	*g*
Nasal, ...	*m*	*l*, *n*	*ng*
Aspirate, . . .	*f*	*th*, *s*, *sh*	*ch*, *h*
Vocal Aspirate, . .	*v*	*th*, *z*, *j*	*gh*
Vibratory, . . .	*pr*	*r*	*ghr*

187. Besides these, there are two letters essentially of the nature of vowels, but having in many cases the force of consonants. These are *w* and *y;* the one a prolonged or double *u*, the other a prolonged *e*. The peculiar effect of each is brought out when followed by another vowel, so as to make a diphthong. The *w* has a labial character, the *y* a guttural.

188. The nasal letters may be so attenuated as to lose the character of consonants, and merely give a nasal twang to the vowel adjoining. This is the case in the French pronunciation.

189. Speech is generally a mixture of vowels and consonants. The utterance most easy to sustain, and most agreeable to the ear, is formed by a vowel and consonant alternating. Vowels alone produce too feeble an impression to make a distinct language. As a general rule, abrupt sounds have the most marked effect on the ear; so that a mixture of these is necessary to make a clear and intelligible series of sounds. Hence the mute consonants *p*, *t*, *k*, have a high value, as characteristic and unmistakeable letters; but the hissing sound of *s* is remarkable for its piercing effect on the ear, and for its being

so peculiar and distinct, that no other sound can be confounded with it; and it is therefore an exceedingly useful member of the alphabet. The same remark, in a less degree, applies to *r*, which leaves a vivid impression, and is not easily mistaken for any other sound. The aspirates generally, *f*, *sh*, *ch*, *h*, have a certain amount of the hissing peculiarity, but none of them are so intense as the pure *s*. They have all, however, a distinct and sharp effect on the ear.

190. The three mutes, *p*, *t*, *k*, and the three vocal sounds corresponding, *b*, *d*, *g*, cannot be pronounced without the help of some vowel; hence in their pure form they are abstractions rather than realities. Almost all the others permit of themselves a constant passage of the breath, and can therefore be sounded without the addition of vowels. Thus *m*, *n*, *l*, *r*, *ng*, *f*, *s*, &c. can all be sounded each by itself alone, although the addition of a vowel will in general make the exercise more easy. Thus *mmee* is easier and pleasanter than *mmm*. The passing into a vowel is a passing from a forced to a free posture of the parts of the mouth. But as these letters can be sounded with more or less difficulty by themselves, a number of them have been called *semivowels*, or we might call them thick or viscid vowels. They have a middle character between the vowels and the six consonants above-mentioned. They demand a less violent exertion than the abrupt consonants, but a greater exertion than the vowels.

SPEAKING MACHINES.

191. From the time that the statues of Memnon emitted their mystical tones on the banks of the Nile, and the oracular responses were delivered at Delphi, through the period when a speaking head was exhibited by the pope, towards the end of the tenth century, and others afterwards by Roger Bacon and Albertus Magnus, various surprising efforts have been made to produce a machine capable of articulating human words and sentences. The record left us concerning the Egyptian statues is by far too scanty to afford basis even for a probable conjecture; and with respect to the oracle at Delphi, the cave of Trophonius, and the like, we have every reason to suppose that the sounds emitted were merely those of some confederate, rendered more surprising by calling in the aid of acoustic principles in the construction of the oracular temple. Again, the speaking instruments of the middle ages were simple combinations of pipes and stops concealed by an external semblance of a human head, and capable of uttering only a few simple syllables.

192. It is but recently that ingenuity, aided by the numerous mechanical facilities of the present day, has been able to complete a machine capable of simulating the human voice in a tolerable manner. In 1779, Kratzenstein of Petersburg, and Kempelen of Vienna, constructed machines which pronounced articulately letters, words, and phrases. By adapting sliding tubes to a reed, Mr Willis of Cambridge has recently succeeded in the enunciation of the vowels *i, e, a, o, u* in order, by drawing out the tube gradually during the passage of a current of air through it from the bellows of an organ; on farther extending the tube, the same vowels, after an interval, are obtained in a reverse order; and on continuing to extend the pipe, the same series of vowels are, after similar intervals, obtained alternately in the direct and inverted order.

193. Of all speaking machines, that lately finished by Faber, which engaged him for twenty-five years, is the most perfect. It is constructed as like as possible to the human organs of speech. An elastic tube represents the larynx, a pair of double bellows the lungs, and a piece of caoutchouc the tongue. By means of a set of fourteen stops, like those of a pianoforte, motion is communicated to internal levers, which produce the pressure and movements necessary for modifying the simple vowel sounds into the various kinds of labial, nasal, guttural, and lingual consonants. So complete is the machine, and so entirely is its operation under the control of its inventor, that it can pronounce a variety of words and phrases in different languages, with a wonderfully-perfect articulation and intonation of voice. There is no doubt that the machine may be much improved, and more especially that the *timbre* of the voice may be agreeably modified. The weather naturally affects the tension of the India-rubber; and although the inventor can raise the voice or depress it, and can lay a stress upon a particular syllable or a word, still, one cannot avoid feeling that there is room for improvement. This is even more evident when the instrument is made to sing; but when it is remembered what difficulty many people have to regulate their own *chordæ vocales*, it is not surprising that Mr Faber has not yet succeeded to his wishes in this department.

ORGANS OF HEARING.

194. The organs of hearing are divided into three parts—the external, middle, and internal ear. The *external* ear consists of an external cartilaginous plate, curved in various directions; it is what in common language is called the ear or conch, *c*. The *external auditory passage*, *m*, or funnel-shaped canal, extends inwards about an inch. The *middle* ear consists of the cavity called the *tympanum*, or drum, *t*, and its *membrane*, *d*, which bounds it on the external side, at the inner end of the external passage. This cavity contains a machinery of four small bones, *b*, called the *hammer*, the *incus*, the *orbicular bone* (a very small one), and the *stirrup;* their Latin names are the *malleus*, the *incus*, the *os orbiculare*, and the *stapes*. The *internal* ear, or *labyrinth*, contains the several cavities named the *vestibule*, *v*, the *cochlea*, *k*, and the *semicircular canals*, *s*.

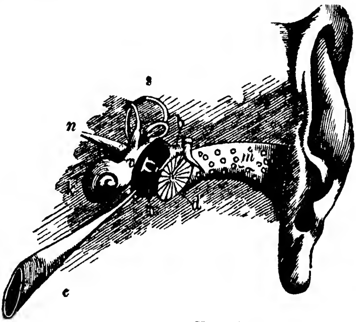


Fig. 18.

195. On the inner side or wall of the cavity of the tympanum, are two small holes, or foramens, the upper being named the *oval foramen* (*fenestra ovalis*), and the lower the *round foramen;* the former of these foramens communicates with the vestibule, and the lower with the cochlea, and both are closed by membranes. The membrane of the tympanum is similar to the top of a common drum, and can be kept in any required degree of tension by means of two small muscles that respectively stretch and relax it, called the *tensor* and *laxator* muscles. The four little bones are articulated in order into one another; the handle of the malleus is inserted into the membrane of the drum, and the sole of the stirrup is intimately connected with the membrane of the oval foramen. The ex-

tremities of the semicircular canals open into the vestibule; the cochlea is like a spiral shell, such as one of the elongated land-shells called a helix, and is divided into two compartments by a spiral thin partition in the direction of its length, excepting a small portion at its apex, at which there is a communication between these divisions; one of the divisions at its base opens into the vestibule, and the other communicates with the tympanum through the round foramen. In the interior of the bony labyrinth—that is, in the vestibule, the semicircular canals and the cochlea—is the *membranous labyrinth,* which is a membranous structure following the windings of the bony cavities, and having, therefore, nearly the same shape. In this membrane the ultimate branches and fibres of the nerve of hearing, *n*, are distributed like a fine network, and the membrane is surrounded on all sides with the liquids which fill the cavities of the labyrinth.

196. There is a communication between the cavity of the drum and the mouth, by means of a narrow canal called the *Eustachian tube, e,* terminating in the posterior nares near the pharynx; it is trumpet-shaped, the narrow end being next the drum.

197. The sensation of sound is produced by a series of different actions, commencing with the agitation of the *membrana tympani,* and ending in the excitement of the nerve of hearing. The sound passes through three different substances, and these are so chosen and adjusted, as to make the ultimate effect as perfect as possible. In the first place, it is found that nothing is so well adapted to receive the sonorous undulations of the air as a tight membrane or drum-head, like the membrane of the tympanum; in no other way could aërial impulses communicate themselves with so little loss. In the next place, the agitations of a membrane may be most effectively imparted to another body, if that body is a solid, and accordingly a framework of a compact bony structure receives the vibrations of the membrane of the tympanum at one end, and communicates them at the other to a liquid mass contained in the closed chamber of the labyrinth. The stirrup bone, by lying tight upon the membrane that closes the oval foramen of the labyrinth, communicates its own vibrations to the liquid contents of the cavity; in other words, a series of undulations are excited in the liquid where the nerve of hearing lies spread out on the membranous labyrinth. The undulations of the liquid are thus a series of compressions of the soft labyrinth and of its imbedded nerve-fibres; and this squeezing of the nerve-fibres is what constitutes their stimulus preparatory to the sensation of hearing. The effect ultimately to be brought

about being the successive compressions of the nervous network of the ear, corresponding to the undulations of the air without, and it being impossible to effect this by the direct action of the air, which would be too feeble for the purpose, an intermediate structure is contrived, consisting of membrane, solid and liquid, the first being the thing best adapted to receive the aërial impulses, and the last being the only means of producing, in a sufficiently delicate way, the effect on the nerve, while an intermediate solid framework is equally requisite to connect the vibrations of a membrane with the undulations of a liquid. The ultimate action on the nerves is precisely the same as in the sense of touch, being a mechanical pressure on a membrane where the nerves are imbedded, and equivalent to a pressure on the nerves themselves. But in the sense of hearing, the pressure is of a far more delicate kind; anything approaching to a *solid* compression of the membrane of the auditory nerve would probably cause unutterable agony, and the notion of a sound as of the crashing of the universe.

198. The muscles of the drum of the ear are an essential part of the organ, as the muscles of the eyeball are of the eye. The movements effected by the muscles are principally the tightening and relaxation of the membrane of the tympanum, *d*, to suit the different degrees of strength of the sound. The more relaxed the membrane, the feebler the impression made in it and on the ear; while, by tightening it, a much sharper effect is produced. The immediate effect of any sonorous impression is to send a nervous stimulus to the muscle that stretches the membrane of the tympanum, in order that this last may be better prepared for the reception of the sound, and according as the effect is grateful to the sense of hearing, the reflex action on the muscle is the more sustained. If the sound is too strong, the membrane of the tympanum is relaxed; if too weak, it is tightened, in order to magnify the effect; just as in the case of seeing, excess of light causes a contraction of the pupil of the eye, and a deficiency a corresponding enlargement. But since the muscles are sensitive as well as the membranous surface of the labyrinth, a series of muscular sensations are mixed up with the auditory sensations proper, and impart their own character to the feelings communicated through the sense of hearing. A constant relation is kept up between the auditory impressions and the muscular apparatus that governs the movements of the *membrana tympani*, and regulates its exposure to the sonorous vibrations of the air, and thus a complicated result is produced in the end, half auditory and half muscular.

199. The sense of the direction of sounds, which at best seems very imperfect, especially when we compare the ear with the eye, depends partly on the motion of the membrane of the tympanum, and the feeling arising from its muscular adjustment, and partly on the motion of the head. There is a certain position of the head that gives us the feeling of the sound being direct, or straight from the opening of the ear, while in all other positions more or less of obliquity is conceived. This is purely a feeling of the muscles, and the sense of it resides properly in the cavity of the tympanum, or with the apparatus of bones and muscles enclosed there: it is exactly as in the case of a stroke upon our body, the direction of which is known from the course of the movement that follows it. If the waves of air go straight towards the membrane of the drum, the vibrations of the membrane will give a corresponding straight course to the vibrations of the bones of the drum; and in like manner a slanting direction in the one will make a slanting direction in the other, and of this the muscles will be cognisant, just as the muscles of the leg are cognisant of all movements impressed upon it. In fact the vibrations of the small bones, which are destined ultimately to leave an impression on the auditory nerve, will, on the way, impart impressions of a different kind through the muscles that are attached to them for the regulation of their movements.

200. The individual character of so complex an instrument as the ear must depend on the peculiarities and modifications of its several parts, and thus very great differences may occur in the nature of the auditory sense. The variations in the delicacy and refinement of the membrane of the tympanum, the bony and muscular apparatus of the middle ear, and the nervous filaments of the membranous labyrinth, respectively, will constitute ears of different kinds of merit and perfection. A detailed account of the varieties of sounds, according to the distinctions that the ear aids us in perceiving, will be the best means of enabling us to judge of the specific qualities that may belong to the ears of differently-formed individuals.

CHARACTER AND VARIETIES OF SOUNDS.

201. In discriminating sounds, we rely partly on our sense of hearing, and partly on our knowledge of the instruments or machinery causing them; and an accurate account of their varieties being important in many points of view, an attempt at this will form the most appropriate conclusion to the present treatise:—

202. *First*, Tune, or *Pitch*. The nature and cause of this

peculiarity have already been repeatedly touched upon. Being dependent on the dimensions of the sounding body, or that portion of it acting as the regulator of the sound, it is measured by the number of vibrations made in each second by the particles of the instrument, and by the number of impulses given to the membranous labyrinth of the inner ear, and through it to the ramifications of the nerve of hearing. The rate of rapidity of these impulses is distinctly perceived by the sentient mind, although not alike by all minds. To have a keen and delicate perception of this particular property, is one of the very first requisites of a musical ear. It is probable that all parts of the ear are concerned in giving such a sensibility. The delicacy of the membrane of the tympanum, the fineness and susceptibility of the muscular and bony apparatus of the intermediate ear, and the sensitiveness of the nerve-fibres to the peculiar effect of a repetition of impulses, and to the degree of rapidity of that repetition, would each contribute to the perfection of the musical ear, apart from the inward feelings of the mind itself. The same power of discrimination that serves to determine differences of pitch or tone, would serve also to discriminate between the equal succession characteristic of a musical note and the unequal and irregular succession of unmusical sounds. There is a great satisfaction communicated by a recurrence of beats at equal intervals, but different persons are liable to it in very different degrees. Some enjoy the succession that exists in the musical note, which seems to form a continuous stream, and others prefer the more slow and palpable beats of one of the instruments of strongly-marked rhythm, which is naturally the most exciting of the two, the excitement, however, in all probability depending on the intensity of the strokes. Excessively high or acute sounds, if at the same time they possess any degree of intensity, are extremely painful and piercing; while grave sounds have generally an indifferent, if not an agreeable effect.

203. *Second, Intensity.*—This means literally the violence of the blow given by a vibrating medium in a state of sonorous excitement. The more powerful the undulations that strike on the drum of the ear, the more powerful the impulses that compress the nerve-fibres of the membranous labyrinth, and the more intense the sensation communicated inwards to the auditory centre of the brain. Up to the point of producing some disorganizing effect on the system, we generally prefer strong sensations to weak ones, and hence intensity may be considered as a desirable property of sound, if in other respects it is of the agreeable kind. The exhaustion or fatigue of the nerves and other parts of the ear is the limit to the enjoyment

of strong sounds. We have already seen that the instruments of the metallic kind yield the most powerful tones; and in the unmusical uproar of busy life, all kinds of work and traffic in articles of metal are noisy and fatiguing to the ear. When sounds are overpoweringly intense and hard, they produce not only pain, but positive fright and panic, if people have not been disciplined to endure them. The ringing of church bells close at the ear, or the firing of ordnance, are apt to deafen and discompose the listeners. To persons deriving enjoyment from sounds, but apt to be pained and fatigued by such as are intense, the calmer and quieter class of utterances are preferred—the ripple of the waters, the gentle breeze, or the hum of industry in the distance. In the silence of nature's solitudes, gentle sounds make a deep but not fatiguing impression.

204. The suddenness of sounds is connected with their intensity, as marking a contrast between two successive states of nerve, one little excited, and the other much. In producing terror and mental discomposure, or any of the other secondary effects of sounds, suddenness of action is very effective. A *prolonged*, intense sound will wear out the organs, but it does not naturally agitate the system to the same degree as one that is sudden.

205. *Third*, *Clearness*, or purity.—A clear sound is one that has a distinct, uniform character, and is not choked, or encumbered with confusing ingredients. A clear-toned instrument is one that yields, in their most unmixed and perfect shape, the notes that it is intended to produce. The perception of tone or pitch depends very much on the clearness of the sound, and we judge of clearness by our ability to discern the exact character of what is intended. In instruments, the purity of the sound must depend very much on the texture of substance employed. Silver is the clearest-toned metal. Glass, from the uniformity of its texture, yields remarkably clear tones. In instruments of wood, a hard and uniform tissue is indispensable. In the human voice, *musical* clearness and *articulate* clearness depend upon totally different qualities. The first arises from the structure of the larynx and the constitution of the resounding skull; the second depends upon the sharpness and suddenness of the articulate actions of the mouth. In every kind of expression clearness is an indispensable virtue, and the merit of musical or articulate performances must be exactly in proportion as the effect intended stands out apart from other effects not intended.

206. *Fourth*, *Simplicity* and *Complexity*.—This obviously means the contrast between few and many sounds concurring at the same moment. All undulatory actions may be exceed-

ingly complex without mutual destruction. The same surface of water may be agitated by numerous crossing waves; but each will proceed unimpaired to its destination, the same as if no others existed. So in sounds, the membrane of the ear may be affected with several series of vibrations, which it will transmit, with all their primitive distinctness, to the fibres of the nerve of hearing; and this nerve may also transmit them, without confusion, to the auditory centre. But in the mind itself the distinctness is not so well preserved: some degree of fusion takes place; and this may be grateful or otherwise, according to the nature of the separate sounds. Sometimes these may be such as to produce an agreeable concurrence or harmony, as formerly explained; at other times they give jarring impulses, and cause discord and pain. The essence of discord, in the innermost recesses of soul, is exactly what it means in its ordinary signification—pulling opposite ways, *distraction*. Sometimes a multitude of sounds will fall on the ear, and be perfectly indifferent to one another, neither concurring nor contradicting in the impulses they stimulate. This is the most common case in the ordinary sonorous din of life.

207. *Fifth, Volume.*—In order to understand fully the subject of complex sounds, we must consider them in the point of view of volume or magnitude. When the sounding body is a large, extensive mass, the sound is said to be voluminous. The vibrations of a great number of different points pour themselves all together upon the ear. This increases the effect and influence of the sound, without causing a pure increase of strength or intensity. The waves of the sea, the thundery discharge, the howling winds, are all voluminous sounds, and the effect of resonance and echo is to give this character to sounds originally of limited nature.

208. The effect of volume on the ear is analogous to the effect of expanse upon the eye—it gives a sensation of largeness and extension. The stimulus of the ear from so many directions at once produces an attempted sweep of the exposed surface round the whole, through the muscular movements of the tympanic apparatus, and this gives the feeling of dimension on every side. The susceptibility to this peculiar feeling is exclusively muscular, and is likely to be common to all the parts of the muscular system, and especially to be shared with the muscles of the eye, which have a parallel function.

209. It happens, as a matter of necessity, that grave sounds are voluminous. They require a large extent of sounding mass for their production, and hence the impulse they give to the ear must be multitudinous.

210. *Sixth, Timbre,* or *Quality.*—By this we mean the peculiar

distinction between the sounds of different instruments when they are precisely the same in pitch, intensity, clearness, and volume, and depending evidently on the nature of the sounding material. The notes of the violin have one quality, those of the clarionet a different quality, and even between instruments of the same nature we may detect differences. No two human beings have precisely the same quality of voice, and we apply a great many epithets to distinguish the merits and demerits of individual voices, apart from their power of producing certain tones, or a peculiar amount of strength or clearness. The virtues of sweetness, richness, mellowness, refer to the property we are now considering.

211. What peculiar difference is impressed upon the vibrations of the air, to convey to us the feeling of distinctness of quality, has not been precisely ascertained.

212. *Seventh, Expression.*—This relates particularly to the articulate quality of sounds, or to the effect produced upon us by the human voice, according to the shape given to the mouth during their utterance. It has been already explained that the varieties of the vowels are dependent on this circumstance. The effect of volume, or of sound proceeding from an expanded surface, is here refined upon, so as to give character and meaning to vocal utterance. It furnishes a faint resemblance to the power of the eye in ascertaining the shapes and proportions of visible objects. The difference of action in the sounding of *ah* and *ooh* is a difference in the area of the sounding stream issuing out of the mouth, and probably also involves some distinction in the intensity and direction of its different parts. But this portion of the subject is extremely obscure.